SOME ELEMENTS ON THE TRAINING IN HISTORY OF MATHEMATICS FOR TEACHERS IN FRANCE

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ABSTRACT

After a brief presentation of the main method of recruitment of secondary school teachers (in all disciplines, not only in mathematics), we will detail extracts of some official texts that provide the framework for the training of future teachers in mathematics: specifications for the master "MEEF", school curricula and the expectations of the national recruitment competition exam called the "CAPES". We will more particularly study the place of the history of mathematics in these texts and its theoretical impact on teachers' training.
In the second part of the article, we will specify what training (initial and in-service) is given in the history of mathematics for secondary school teachers. How many hours? During which years at university? How many training courses? On which themes? Etc. We will give examples of contents. We will also discuss the difficulties encountered by teachers, particularly in finding and working on documents that allow them to introduce the history of mathematics into their own courses.

1 Recruitment of secondary school teachers in France (some general elements)

In France, most of current teachers have been trained in two different schools. From 1990 to 2013, the initial training of French teachers was organized by the "IUFM" ("Institut Universitaire de Formation des Maîtres"), which first was independent institute and then integrated to university from 2010. Students started IUFM after the third year of university (licence degree). During the first year in this school, they prepared for the competitive exam named "CAPES"[1].When the student passed the exam, the second year was divided in two parts: part time course in a class as a trainee and part time initial training at IUFM (courses on evaluation, lesson planning, class management, …). From 2013 on, the "IUFM" was replaced by the "ESPE" ("École Supérieure du Professorat et de l'Éducation" that is to say "higher school of teaching and education") which became fully integrated in universities.

This change of schools mainly brought administrative changes with different statuses for trainers, university professors, a bigger place for research, etc. For future teachers, the concrete change had been linked with the reform of universities in 2008. From that year, the initial training of French teachers has conformed to the national LMD reform "Licence (years 1, 2, 3) – Master (years 4, 5) – Doctorate (years 6, 7, 8)". As the two years of school corresponded to years 4 and 5 of university, the creation of a Master's degree was necessary. So, now, during year 1, the ESPE prepares students for "the CAPES" exam and organizes (with the university) courses of a Master's degree in education ("Master 1 MEEF"). During year 2, the ESPE organizes courses of "Master 2 MEEF" and the students do an internship in in several classes (about 8 - 10 hours a week in total[2]). To

[1] The CAPES is a national competitive exam, consisting of written admissibility tests and, for about half of the candidates, oral admission tests. The number of candidates admitted to CAPES depends on the number of vacant positions throughout France. There is a CAPES for each discipline of the French curriculum.
[2] In mathematics, it usually corresponds to two or three classes.

become teachers, the students have to pass their Master's degree in the university and their internship has to also be validated by the "Ministère de l'Éducation Nationale".

This procedure is the normal recruitment of French teachers but there are other possibilities. Students can pass another Master's degree (year 4) and prepare alone "the CAPES" exam; they can then follow only year 2 (with adjustments) of ESPE. Some students can prepare for another competitive exam (the "agrégation") generally after a Master's degree of research; if they before passed a Master's degree, they only must validate their internship.

2 The Master degree "MEEF" in mathematics – some officials texts

2.1 General organization

The courses of a Master degree "MEEF" in mathematics are organized in each region by the "ESPE" and the department of mathematics of the University[3]. Different choices may be done in each region but many common points can be noticed. Indeed, the training provided by the "ESPEs" is defined by a national specification (Ministère de l'Éducation nationale, 2013-August). The following points can therefore be found in the organisation of MEEF Masters degree in mathematics:

In the first year, students go to a secondary school and shadow an experienced teacher in his or her classes. If all goes well, the student can implement some sequences with the teacher's help. There must be four to six weeks of internship in the year, which are followed by courses where the students analyse their observations and experiments.

Most of the students' time is devoted to theoretical mathematics courses as algebra, geometry, analysis, probability, etc. These contents are evaluated in the Master's final exams. They are also useful for the written exams of "CAPES" which are admission exams and take place in March or April. This first year ends with the oral exams of "CAPES" in June.

The second year, the students have a one-year training course (internship) as a trainee teacher. To obtain their Master's degree, they also follow courses at "ESPE" two days a week about evaluation, lesson planning, class management, etc. They must write a Master's dissertation on a theme linked with their experience as a teacher.

2.2 Place of history of mathematics in the contents of the Master degree

Another common point is that some courses of history of mathematics can be found in the schedules of some Master degree "MEEF" in mathematics[4]. In Poitiers, a new course was created in 2009, especially for the Master's degree "MEEF". What motivated this creation?

In addition to the decree of August 2013 which gives the specifications of the "ESPE", the choices of training content are linked to the "Référentiel des compétences professionnelles des métiers du professorat et de l'éducation" (Ministère de l'Éducation nationale, 2013-July) where we can read that teachers must "have a thorough knowledge of their discipline […]. Identify the fundamental points of

[3] I only present the case of mathematics but master degree "MEEF" do exist for all disciplines of French curriculum.

[4] I will detail in next paragraph some different choices.

reference, epistemological issues and didactic problems."[5]. The contents are also linked to the definition of the CAPES oral tests. Since its creation and until 2009, one of the aims of the CAPES oral test was to "detect that candidates: […] have reflected on the aims and evolution of the discipline" (Ministère de l'Éducation nationale, 1993, p. 19, translation)[6]. In 2009, the word "history" appears in the new official text:
"This oral examination allows the candidates to show:
– their mathematical and professional culture;
– their knowledge of teaching content and curriculum;
– their reflection on history and aims of mathematics and its relationship with other disciplines." (Ministère de l'Éducation nationale, 2009, translation)[7] This text may explain the choice of my university and several others.

In practice, few oral subjects explicitly contain references to the history of mathematics. But the candidate can make it on his or her own initiative. One of my students told me that, while being asked about probabilities, he presented the problem of the Grand Duke of Tuscany. However, it is worth noting that in 2018, one of the possible subjects for the oral examination is based on an extract of Elements of Algebra (Euler) "A troop of men and women spent a thousand cents at an inn. The men paid 19 cents each, the women 13. How many men and women were there?".

2.3 Place of history of mathematics in the curriculum

Of course, the CAPES oral exams are also related to the content of the teaching curriculum. What place does the history of mathematics have in it? It may be a other reason for the introduction of history of mathematics for the future teachers. In fact, since 2009, there has been less indication of the history of mathematics in the curriculum but it can still be found in those of two last years of "lycée" (16-17 and 17-18-year-olds). Even if there haven't been explicit contents linked with history, in the introduction, it is indicated:

"Elements of epistemology and history of mathematics naturally fit into the implementation of the curriculum. The knowledge of the name of some famous mathematicians, the time when they used to live and their contribution is part of the cultural background of any student with scientific training. The presentation of historical texts helps to understand the genesis and the development of some concepts." (Ministère de l'Éducation nationale, 2010, p.1, translation)[8]

In February 2018, the Villani and Torossian's report "21 measures for mathematics teaching" reasserts the importance of the history and epistemology of mathematics:

[5] Original text: "Connaître de manière approfondie sa discipline […]. En situer les repères fondamentaux, les enjeux épistémologiques et les problèmes didactiques."
[6] Original text: "déceler que les candidats : [...] ont réfléchi aux finalités et à l'évolution de la discipline".
[7] Original text: "L'épreuve permet au candidat de montrer:
- sa culture mathématique et professionnelle ;
- sa connaissance des contenus d'enseignement et des programmes ;
- sa réflexion sur l'histoire et les finalités des mathématiques et leurs relations avec les autres disciplines."
[8] Original text: "Des éléments d'épistémologie et d'histoire des mathématiques s'insèrent naturellement dans la mise en œuvre du programme. Connaître le nom de quelques mathématiciens célèbres, la période à laquelle ils ont vécu et leur contribution fait partie intégrante du bagage culturel de tout élève ayant une formation scientifique. La présentation de textes historiques aide à comprendre la genèse et l'évolution de certains concepts".

"First, epistemology and history of the construction of mathematical notions, which bring a real didactic richness, are little taught in initial training. […] By taking advantage of history of mathematics, teachers place their teaching in the evolution of knowledge. In addition, students are often sensitive to the "mathematics legend". Narrative can play a motivating role here. On the other hand, the epistemological lessons that emerge from history (the role of problems, the entanglement of concepts and techniques, the need of abstraction) are obviously made to contribute to training, in particular by overcoming short-sighted utilitarianism." (Villani & Torossian, 2018, pp. 35, translation)[9]

Almost a year after this report, in January 2019, new curricula for the "lycée" are published and history of mathematics indeed takes a real new place. The introduction always gives a general indication: "It may be judicious to enlighten the course with historical or epistemological contextual elements. History can also be seen as a fertile source of problems that clarify the meaning of certain concepts. The "History of Mathematics" items identify some possibilities in this direction. To substantiate them, the teacher can rely on the study of historical documents." (Ministère de l'Éducation nationale, 2019, p.5, translation)[10]. The real novelty is these items of history of mathematics! For each part of the curricula, some examples and indications are given. For example, in the part "Numbers and calculus" of the curriculum for the "seconde" (15-16 years old students), it is explained:

"The seemingly familiar notion of number is not self-evident. Two examples: the crisis caused by the discovery of irrationals by Greek mathematicians, the difference between "real numbers" and "numbers from the calculator". It is also a matter of highlighting the gain in efficiency and generality brought by literal calculus, by explaining that a large part of mathematics could only develop once this formalism had stabilized over the centuries. It is possible to study ancient texts by authors such as Diophantine, Euclid, Al-Khwarizmi, Fibonacci, Viète, Fermat, Descartes and highlight their algorithmic aspects." (Ministère de l'Éducation nationale, 2019, p.6, translation)[11]

In France, these new curricula may lead to an increase in training in the history of mathematics. For the moment, it is not well developed, in particular in the in-service teacher-training. Let us look at the situation in a little more detail.

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[9] Original text: "Tout d'abord, l'épistémologie et l'histoire de la construction des notions mathématiques, qui apportent une réelle richesse didactique, sont peu enseignées en formation initiale. […] En tirant parti de l'histoire des mathématiques, les professeurs inscrivent leur enseignement dans l'évolution de la pensée. De plus, les élèves sont souvent sensibles à la « légende des mathématiques ». La narration peut jouer ici un rôle motivant. D'autre part, les leçons épistémologiques qui se dégagent de l'histoire (rôle des problèmes, enchevêtrement des concepts et des techniques, nécessité de l'abstraction) sont évidemment de nature à contribuer à la formation, notamment en permettant de dépasser un utilitarisme à courte vue."

[10] Original text: "Il peut être judicieux d'éclairer le cours par des éléments de contextualisation d'ordre historique, épistémologique ou culturel. L'histoire peut aussi être envisagée comme une source féconde de problèmes clarifiant le sens de certaines notions. Les items «Histoire des mathématiques» identifient quelques possibilités en ce sens. Pour les étayer, le professeur peut s'appuyer sur l'étude de documents historiques."

[11] Original text: "La notion apparemment familière de nombre ne va pas de soi. Deux exemples la crise provoquée par la découverte des irrationnels chez les mathématiciens grecs, la différence entre «nombres réels» et «nombres de la calculatrice». Il s'agit également de souligner le gain en efficacité et en généralité qu'apporte le calcul littéral, en expliquant qu'une grande partie des mathématiques n'a pu se développer qu'au fur et à mesure de l'élaboration, au cours des siècles, de symbolismes efficaces. Il est possible d'étudier des textes anciens d'auteurs tels que Diophante, Euclide, Al-Khwarizmi, Fibonacci, Viète, Fermat, Descartes et mettre en évidence leurs aspects algorithmiques."

3 History of mathematics in teacher-training

To get some data on the situation in France, I sent two questionnaires. The first one was addressed to secondary school teachers. I first proposed it to all the teachers of Poitou-Charentes, the region where I teach. I got around 230 answers. To get more feedback, I asked some colleagues for help in others regions. So, the questionnaire was given to teachers both in Paris and in Franche-Comté. In those two regions, we only managed to contact teachers who are used to follow training courses (not all the teachers), but in the end, the results of the survey were similar and I therefore got 530 answers altogether. The main questions of this survey were: During your initial training did you follow a course in History of Mathematics? Do you use History of Mathematics in classroom? If yes, how? [a. in an anecdotic and occasional way? b. in a constant and integrated way? c. other option (specify)] If yes, what kind of use ? [a. some anecdotes; b. motivated introduction of a chapter, of a notion; c. reading historical texts; d. exercises at home; e. interdisciplinary works; f. other option (specify)] If no, why? [a. lack of personal interest; b. lack of formation; c. lack of knowledge; d. lack of documents; e. lack of time; f. lack of motivation; g. other option (specify)] Do you think that the use of History of Mathematics could improve your teaching? If yes, how?

Another survey was given in several universities in different regions. I send it to some members of the IREM[12], who often was secondary teachers or university teachers who teaches history of mathematics. There were two sets of questions. The first was on university courses in the history of mathematics: are there such courses in licence? If so, with what modalities (hours, semesters, contents, etc.)? Are there such courses in master degree "MEEF"? if so, with what modalities (hours, semesters, contents, etc.)? if not, why not? The second series concerned the history of mathematics courses in in-service training: number of courses offered by IREM in 2017-2018? on what subjects? courses offered by institutions other than IREM? number of courses in 2016-2017, 2015-2016 and 2014-2015?

3.1 The place of history of mathematics in studies at university

In the survey for secondary school teachers, the first question was: "During your initial training, did you follow a course in History of Mathematics?" Only 20% answered "Yes" and more than 70% of them have had less than twenty-hour courses. For example, in Poitou-Charentes before 2009, trainee teachers used to get only six hours of history of mathematics and after 2009 a new thirty-hour history course was created for them. But most teachers who have answered the survey have been recruited before 2009… In addition, the University of Poitiers does not offer courses in the history of mathematics during the three years of the "Licence de Mathématiques". What about the other regions?

In the second survey, one of the questions was: "Do you offer courses of history of mathematics? If yes: when? How many hours? required or elective?". Out of sixteen regions, fourteen of them offer history of mathematics in master degree "MEEF". If they

[12] "Institut de Recherche sur l'Enseignement des Mathématiques": The IREMs network is formed by institutes (about one per region) where primary, secondary and higher education teachers can work together to conduct research on mathematics education, and to prepare teachers' training. Some representatives of each region can meet up in "Commissions inter-IREM" on different themes including "epistemology and history of mathematics" (there is also "didactic", "popularization of maths", "informatics", etc).

don't, history of mathematics is a required course in "Licence". Six of them offer history of mathematics both in master degree "MEEF" and in "Licence".
Here is a summary of the answers[13] to the survey:

	Licence		Master	
Bordeaux	S3 or S5	25h		
Caen	S6	elective 50h	S2	30h required
Clermont	S5	elective	S4	21h required
Dijon			S3 ; S4	15h + 1 seminar + 1 training day
Besançon	S3 ; S5 ; S6	18h+(24h or 29h)+ 50h		
Lille	S3 ; S6	25h elective + 72h	S3 ; S4	28h + 28h required
Limoges			S1 ; S2 ; M2	10h + 10h required + 6h (elaboration of a course)
Montpellier			M1 ; M2	M1: 16h at least M2 : 12h elective + 6h required
Paris 13 - UPC			M2	Integrated in "contents for teaching" and/or elaboration of a course (possibly linked to the master's dissertation)
Paris 8	S1-S2-S3-S4-S5-S6	6*30h Elective (S1-S2) Required (others)	M2	
Paris - UPEM	S3-S4-S5-S6	conferences	M2	
Orléans			M2	(elaboration of a course)
Nantes	L1 ; L2	20h + 16h elective	M1	24h required
Amiens			S3	24h required
Poitiers			S1	30h required
Réunion			S3	24h required

3.2 Examples of contents of courses in master degree "MEEF"

Most common contents in the various regions are elements of epistemology and history of mathematics on some classical themes:

- History of numerations and numbers; calculus methods; algorithms
- Euclidian geometry; proofs; some elements of non-Euclidian geometry
- History of algebra; algebraic equations
- Infinitesimal calculus

Some courses are proposed to students to think about the interest and the limits of using history of mathematics in a class. The future teachers have to analyse examples of exercises for secondary students, based on history of mathematics. When these courses are given during the second year of the Master's degree, the trainee teachers are proposed to create sequences including history of mathematics. These sequences are tested in their classes and then, analysed.

[13] Table's caption: There are three years in "License" labelled L1, L2 an L3 including six semesters labelled S1 to S6 in the columns 2 or 3. There are two years in Master labelled M1 and M2 and thus four semesters labelled S1 to S4 in the columns 4 and 5; When I don't know the precise semester or when the course is a year-long one, I indicate L… or M…, otherwise I write the semester.

In the Master degree "MEEF" of Poitiers, as the students haven't had any history of mathematics courses in "Licence", the sequences are divided into different times with different aims. There are times of lecture course to give them bases and points of reference about mathematicians and history of mathematics. Students also have to read historical texts. That allows them to do mathematics through historical documents and confronts them with unfamiliar notations and vocabulary. The reading of historical texts can be followed by different types of questions as detailed by Evelyne Barbin in *De grands défis mathématiques d'Euclide à Condorcet,* (Barbin, 2010, p. 66): interpret in the historical context, compare texts, write in the manner of a mathematician, interpret in modern terms, etc. For example, in a course about infinitesimal calculus, I present some parts as problems of tangent or quadrature in Antiquity, as the method of indivisibles of Cavalieri or as the method of tangent circles of Descartes. However, the students have to read themselves some texts in different aims. About the tangent to a circle by Euclid (book 3, prop 16) they have to understand the type of reasoning; from a text on the quadrature of cycloid by Roberval, they have to write in modern notations the areas of the figure; from an extract of *Analyse des infiniment petits pour l'intelligence des lignes courbes* by the Marquis de l'Hospital, they have to calculate other differences in the same manner as $d(xy)=xd(y)+yd(x)$. The students also prepared brief oral presentations, from a historical text. Last years, the main texts they studied were extracts of *Euclids' Elements* as book I, proposition 47 (Pythagoras'theorem) or proposition 32 (sum of angles in a triangle), *La Disme* of Stevin, texts on the resolution of second-degree equations by Descartes or by Al Khwarizmi, etc. They had to present the author, explain the mathematical content of the text and analyse the link with contents of the current curriculum as can be seen in the example in annex 2.

3.3 History of mathematics in the in-service training: Difficulties and paradox

It is more difficult to find history of mathematics in the in-service training. Fifteen regions completed the survey about in-service training and only six of them answered they could offer a training in history of mathematics in 2018! For the past four years[14], only five regions have succeeded in maintaining their training in history of mathematics at the same quantity each year ("Académies" of Besançon, Paris, Lille, Montpellier, Limoges); several regions stopped providing such courses. In the "Académie" of Grenoble, the number of them has decreased each year but there is still one proposed in 2017-2018. Many others haven't given any for several years.

The different topics offered within in-service training in 2018 were "Focus on mathematics in Germany" and "Mathematics and philosophy: Model and reality" (IREM of Franche-Comté), "Ancient instruments" (IREM of Grenoble), "Mathematics and other disciplines throughout history" (IREM of Lille), "History of mathematics and algorithms" (IREM of Limoges), "History of astronomy in mathematics class" (IREM of Paris 7) and "Voting systems and general will" (IREM of Montpellier).

In the "Académie" of Poitiers, for years we have had fewer and fewer elective training (including those in history of mathematics) because there are more institutional trainings for example on algorithms and programming. So, we tried to include a touch of history of mathematics while studying other themes of training like "probability and models in

[14] In the survey, I asked them for 2014-2015; 2015-2016; 2016-2017 and 2017-2018 in order to see the evolution. Of course, many of them had proposed in-service trainings before 2014.

lycée" in 2015, or "teaching with magnitudes in collège" last year. We hope that the introduction of the history of mathematics into the new curricula will make it possible to offer more training.

Though there is little training related to history, paradoxically it seems to be appreciated. I do not have enough statistical data on this issue but in my own experience as a trainer, I have noticed some very positive reactions. For example, in the "Académie" of Poitiers, in 2014 and 2015, elements of history of mathematics were integrated in the training about probabilities whereas they weren't announced in the title of the course. And, at the end, when the colleagues had made an assessment of their daytime, it was one of the most appreciated aspects of the training! In Franche-Comté, the courses in history of mathematics have become a tradition as it can be seen in one of the answers to the survey: "The math history training at the IREM of Franche-Comté has been offered and accepted by the rectorate without interruption since 1986. It brings together between 25 and 35 participants every year. Its duration is traditionally 3 days (with sometimes a negotiation to 2 days for budgetary reasons). The mathematics and philosophy workshop has been taking place every spring since 2015 and is a continuation of a workshop from 2013 on Aristotle's syllogisms. It brings together 20 to 30 participants. Its duration has increased from one day to two days since 2017."

Secondly, there are few trainings but an actual use in class! In the survey for secondary school teachers, while 70% of teachers said they had no initial training in the history of mathematics, 70% said they use it in class. Of course, as colleagues responded voluntarily to the survey, it is likely that they did so willingly when they used the history of math, hence this high percentage. However, this result remains paradoxical in relation to their training.

3.4 Documentary resources on history of mathematics for teachers

The answers to this survey show that, even if teachers don't use history of mathematics, it isn't a lack of personal interest (only 3%) or lack of motivation (3% too). Some of them are afraid of wasting time in class (around 9%). The main difficulties indicated by the teachers are the lack of training, the lack of knowledge and the lack of references or sources. Let us look at the documentary resources that French mathematics teachers can access.

Many documents exist but it is not necessarily easy to find them and especially to study them alone. The in-service trainings do help colleagues to use these materials. Let us quote some examples of resources on history of mathematics in France.

For more than 40 years, the IREM have been linking university research and teachers, often through the organization of training courses. The IREM are quoted in the Villani-Torossian report as an important resource for the teachers. In history of mathematics, many of their members (professors of university or secondary teachers) take part to the initial training, previously described. The epistemology and history of mathematics inter-IREM commission has already produced numerous books for teachers. The first one was *Mathématiques au fil des âges* (Dhombres, Dahan-Dalmedico & Bkouche, 1987) and the last one *Passerelles : enseigner les mathématiques par leur histoire au cycle 3* (Moyon &

Tournès, 2018). Most of book references can be found in the Website of the IREM institutes[15].

One of the current issues is the way to access to these resources. As there are fewer and fewer in-service training sessions, how to make these resources known to teachers? Internet use must be taken into account. In the end, it becomes a way for the teachers to learn by themselves. In France, "Culture Maths"[16] and "CNRS - Images des maths"[17] are two institutional websites where you can find, among other things, articles on the history of mathematics. In the website of APMEP (National association of mathematics teachers), a working group on history of mathematics[18] lists documents, books, sites, videos, etc. that can help colleagues to learn about the history of mathematics and to find ideas for classroom activities.

And obviously, IREM-related websites can be found. Websites of certain IREM-groups provide documents on the history of mathematics prepared for training, such as the "IREM de Caen-Normandie"[19]. Publimath (managed by an inter-IREM commission) is a bibliographic database on mathematics education[20], including many references in the history of mathematics. Many of these documents are available online. Recently, a new use of website has been experimented on the website of the epistemology and history of mathematics inter-IREM commission. The new pages[21] created are linked to the chapters of the book *Passerelles : enseigner les mathématiques par leur histoire au cycle 3*. They allow readers to download documents for the class and they provide references and additional links.

4 Conclusion

Through the answers to the two surveys and some official texts, we have seen that history of mathematics has been maintained last years in the initial training, or even developed in some regions including mine. We can hope that the recent projects of the curriculum can contribute to the recovery of the in-service training in this field. For now, some teachers wanted to use history of mathematics, if the projects of curriculum is carried out, teachers will therefore need knowledge and ideas to construct activities for students. Particularly, the members of the IREM will be able to propose training sessions to help them.

However, even if there is more training, it will take time for as many teachers as possible to be trained. The issue of the diffusion of the resources is at stake. Of course, books will be written but a lot of teachers will keep on finding resources online. But how can we work on our own from a old text found online? How can we find additional information among the multitude of websites? How can we promote reliable and well-documented websites? These questions will be part of the challenges of the future training sessions.

[15] http://www.univ-irem.fr/spip.php?rubrique163&debut_article_numerotes=5#pagination_article_numerotes
[16] http://culturemath.ens.fr/
[17] https://images.math.cnrs.fr/-Histoire-des-mathematiques-.html
[18] https://www.apmep.fr/-Histoire-des-maths-
[19] https://irem.unicaen.fr/spip.php?article104
[20] see the article of Hombeline Languereau and Annie Michel-Pajus, in this volume (ch. 2.6).
[21] http://www.univ-irem.fr/spip.php?rubrique505

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Annex: Example of oral exam in Master degree "MEEF" (Poitiers)

Etude d'une méthode de résolution d'équation du second degré par Al Khwarizmi

(traduction Ahmed DJEBBAR dans *L'algèbre arabe, genèse d'un art,* Vuibert Adapt, 2005)

Quant à la justification de *un bien et dix racines égal trente-neuf dirhams*, sa figure est une surface carrée de côtés inconnus, et c'est le bien que tu veux connaître et dont tu veux connaître la racine. C'est la surface (AB), et chacun de ses côtés est sa racine. Chacun de des côtés, si tu le multiplies par un nombre parmi les nombres, quels que soient les nombres, sera des nombres de racines, chaque racine étant comme la racine de cette surface. Comme on a dit qu'avec le bien il y a dix de ses racines, nous prenons le quart de dix – et c'est deux et un demi – et nous transformons chacun de ses quarts < en segment> avec l'un des côtés de la surface. Il y aura ainsi, avec la première surface, qui est la surface (AB), quatre surfaces égales, la longueur de chacune étant comme la racine de la surface (AB) et sa largeur deux et un demi, et ce sont les surfaces (H), (T), (K), (J). Il <en> résulte une surface à côtés égaux, inconnue aussi, et déficiente dans ces quatre coins, chaque coin étant déficient de deux et un demi par deux et un demi. Alors ce dont on a besoin comme ajout afin que la surface soit carrée, sera deux et un demi par lui-même, quatre fois ; et la valeur de tout cela est vingt-cinq.

Or nous avons appris que la première surface, qui est la surface du bien, et les quatre surfaces qui sont autour de lui et qui sont dix racines, sont <égales à> trente-neuf en nombre. Si on leur rajoute les vingt-cinq qui sont les quatre carrés qui sont dans les coins de la surface (AB), la quadrature de la surface la plus grande, et qui est (DE), sera alors achevée. Or nous savons que tout cela est soixante-quatre, et que l'un de ses côtés est sa racine, et c'est huit. Si on retranche de huit l'équivalent de deux fois le quart de dix – et c'est cinq – , aux extrémités du côté de la surface la plus grande qui est la surface (DE), il reste son côté trois, et c'est la racine de ce bien.

Questions :

1) Présenter l'auteur.

2) Le problème posé est, en écriture moderne, la résolution de l'équation : $x^2 + 10x = 39$
Comment est désignée la quantité x^2 ? Comment est-elle représentée géométriquement ?
Comment est désignée la quantité x ?

3) Exposer la résolution proposée par Al Khwarizmi en langage actuel.

4) On trouve dans l'ouvrage d'Al Khwarizmi une deuxième méthode pour résoudre la même équation :

> **(traduction de J. Høyrup, reproduit dans KOUTEYNIKOFF Odile, « Regard historique sur la résolution des équations du second degré », Repère IREM n°28, juillet 1997)**
> Il y a une autre figure qui conduit à la même chose. C'est la surface (AB) qui représente le trésor. Nous voulons donc lui ajouter ses dix racines. Pour ce faire, nous divisons les dix en deux, ce qui devient cinq, et nous construisons deux surfaces sur deux côtés d'AB, à savoir les surfaces G et D, dont les longueurs égalent cinq, ce qui est la moitié des dix racines, tandis que la largeur de chacun d'eux égale le côté du carré AB. Alors cinq sur cinq nous manque opposé au coin de AB : ce cinq étant la moitié des dix racines que nous avons ajoutées à deux des côtés de la première surface. Nous savons donc que la première surface, qui est le trésor, et les deux surfaces sur ses côtés, qui sont les dix racines, font ensemble trente-neuf. Pour compléter la grande surface en carré, seul cinq sur cinq, ou vingt-cinq, fait défaut. Nous ajoutons à ceci trentre-neuf, pour compléter la grande surface SH. La somme est soixante-quatre. Nous extrayons sa racine, huit, qui est une des côtés de la grande surface. En lui enlevant la même quantité que nous lui avons ajoutée antérieurement, à savoir cinq, nous obtenons trois comme reste. Ceci est le côté de la surface AB, qui représente le trésor ; c'est la racine de ce trésor et ce trésor lui-même est neuf. Ceci est la figure

a) Faire la figure correspondant à cette deuxième méthode.

b) Comment peut-on utiliser cette figure en classe de 1ère S ?