THE CONTRIBUTION OF THE CHINESE ABACUS TO THE DEVELOPMENT OF THE NUMBER SENSE

Apostolos POPOTIS[1], Konstantinos NIKOLANTONAKIS[2]
University of Western Macedonia, 3rd km of National Road Florina-Niki, Florina, Greece
[1]apopotis@gmail.com, [2]knikolantonakis@uowm.gr

ABSTRACT

One of the multiple uses of the abacus in teaching mathematics is to illustrate the true nature of carrying and borrowing in addition and subtraction respectively. The understanding of carried and borrowed number requires deep knowledge of the place value concept in the decimal number system. Generally, students do not seem to understand these two concepts. In our workshop we tried through several activities to convince the participants that the Chinese abacus can help teachers to teach and students to understand and learn the aforementioned notions. The fact that the Chinese abacus enables someone to write up to fifteen in each place value and then to make exchanges between place values with his hand reinforces the understanding of carried and borrowed number. The visual and tactile perception, that the Chinese abacus provides, can support learning.

1 Introduction

Undoubtedly, one of the most important issues in the mathematics curriculum of elementary school is the understanding of place value concept in the decimal number system. However, it seems somewhat incongruous that such a system, which incorporates in its heart a small number of simple rules, would be so difficult for teachers to teach and for students to learn (Jones & Thornton, 1993; Ross, 1990; as cited in Price, 2002). In recent years, after a long period of formal and ineffective learning, mathematical education has focused more on the learning of numbers and operations (Tzekaki, 2007). The use of appropriate educational material contributes to better approach of mathematical concepts, since it is well integrated into the mathematical activity of students (Tzekaki, 2016).

One of the earliest calculating instruments, which has entered the classroom as a manipulative, is the abacus. It is an ancient tool through which one can process the place value concept in the decimal number system. According to Spitzer (1942), there are five important characteristics of arithmetic that are admirably demonstrated through this device. Firstly, the beads can represent various concrete objects. Then, the abacus can nicely illustrate that the value of each digit depends on its position to the number. Furthermore, it demonstrates the idea of a place-holder or the function of zero. Moreover, it illustrates the idea of collection. Last but not least, it reveals the true nature of carrying and borrowing.

As far as the Chinese abacus is concerned, Zhou & Peverly (2005) state that it provides a semi-concrete representation of number, children can manipulate it, and they can easily create a mental image of it. “With experience, mental calculation is enhanced by learning how to use an abacus because children learn to rely on internalized mental abacus which helps them create mental representations of numbers” (Zhou & Peverly, 2005, pp. 266-267). Stigler (1984) concluded that several pieces of evidence in his research indicate that the mental abacus incorporates important features of the physical abacus.

Bartolini Bussi states that a way to introduce the history of mathematics into classroom is through the exploration of copies of ancient instruments and artifacts (Nagaoka et al., 2000). This exploration is not limited to the visual contact only. It is extended to the tactile perception, too. Students do not just observe the instruments, they also use them. This tactile experience has the potential to motivate students and it is an important part of the cognitive foundations of mathematical activity (Nagaoka et al., 2000).

2 Background of the workshop

The workshop was based on a research study that was conducted during the school year 2016-2017 in a primary school of central Macedonia in northern Greece. The purpose of the research study was to examine if 3rd Grade students of primary school are familiar with the structure of the decimal number system and to ascertain whether the Chinese abacus, which as an artifact is in the core of the theory of instrumental genesis and semiotic mediation, can contribute to the understanding of the structure of the base 10 number system. The participants were 7 students (4 boys and 3 girls) consisting the 3rd Grade of the aforementioned primary school.

Data collection was attained through a questionnaire along with personal interviews before and after the instructional intervention, with the filling out of 35 worksheets, most of them with the aid of the Chinese abacus, as well as by observing the students' activities during the intervention. The interviews took place just before and just after the intervention, which was completed within 35 class periods.

In the questionnaire before the intervention, students were asked to read and write three-digit numbers, to perform additive analysis of numbers, to recognize place value in three-digit numbers, to make exchanges between classes of numbers, to compare and to set in order three-digit numbers, to execute addition and subtraction algorithms, as well as to explain the notion of carried and borrowed number.

During the instructional intervention, the students dealt with activities that firstly introduced them to the notion of number system in general, and then to the notion of the decimal number system. Moreover, everyone constructed his own Chinese abacus with materials provided by the teacher-researcher and they used it in several activities. The intervention's exercises and activities covered the whole range of the above aspects of the place value concept. The abacus, through its use, revealed the true meaning of the place value concept. The conceptualization of place value is the basis for number sense development.

Through a questionnaire after the intervention, students' knowledge of the above aspects was checked again.

The findings of the questionnaire before the intervention indicate that students do not have profound understanding of the notion of place value in whole numbers and in operations with them. During the intervention, the Chinese abacus was converted from an artifact into an instrument and mediated mathematical content to the students. Whereas before the intervention there were major difficulties especially in the execution of the addition and subtraction algorithm, and in the explanation of carried and borrowed number, after the intervention these difficulties were limited.

Similar research studies concerning the development of the place value concept in 6th Grade students with the aid of the Chinese abacus were conducted by Poisard (2006), and Tsiapou & Nikolantonakis (2013, 2016).

In the context of the ESU-8, we decided to organize a workshop based on a sequence of activities to introduce the Chinese abacus as a resource for the teaching and learning of number sense and place value system in primary school. According to Poisard (2017) it benefits both students' learning and teachers' classroom practices.

So, we focused on how teacher can foster the understanding of the paper and pencil algorithms for addition and subtraction work by means of the prominence of place value representation of the addends in addition and of the minuend in subtraction, and on the procedures for addition (carried number) and subtraction (borrowed number) on the abacus.

Most of the worksheets that were used in the workshop were originally designed for the aforementioned research study and our young participants filled most of them out with the aid of the Chinese abacus. In addition, the workshop's material was enriched with activities that emerged from the students' answers to the questionnaires' exercises before and after the intervention. The 11th and the 19th question (worksheet) below appertain to this category. It should also be noted that the 2nd question (worksheet) below has been modified so as to better correspond to the age of the workshop participants. The original one was asking the students to discover how the Chinese abacus works through its electronic version (http://cii.sesamath.net/lille/exos_boulier/exo1.html) as the structure and the function of the actual Chinese abacus differs a lot from the Slavonic and the spike abacus that they had already been using. The modified one asks the participants to discover the abacus' mode of use through the study of four figures depicting the successive stages of the Chinese abacus' evolution and a copy of it that was constructed by the workshop leaders and was distributed to everyone. To sum up, 21 worksheets and a copy of the Chinese abacus were available to everybody. Most of the participants were in-service teachers in different grade levels.

3 The way that the Chinese abacus works

The Chinese abacus, which is also called counting frame, known in China as "Suan Pan" (literally "counting disc"), consists of a rectangular frame. It has a height of about 20 cm and its length varies according to the magnitude of the numbers that are represented on it. It usually has more than seven rods. There are two beads in each of the top rods and five beads in each of the lower rods. The upper beads stand out from the bottom ones with a separating beam. The beads are usually round and they are made of hard wood (Korros, 2012).

The beads are activated when they are moved up from below the beam or down from above the beam. When they are moved towards the beam they are counted, while when they are moved away from the beam they are not counted.

Now, let's see how we "read" the Chinese abacus. Firstly, we will talk about the beads. There are two kinds of beads on the abacus, these of the upper and those of the lower deck. Each of the lower deck beads is worth 1. Sometimes they are called earth or water beads. The upper deck beads sometimes are called heaven beads and each of them has a value equal to 5. The rightmost rod of the abacus represents the units, the next one on the left the tens and so on. However, we may select an inner rod for the units' place, whence the rod directly to its left becomes the tens' rod and those to its right the tenths, the hundredths, etc. On the Chinese abacus there are always two heaven and five earth beads in each rod.

At the end of an action-operation on the abacus, we will never see five earth beads activated in the same rod. If this happens, they immediately return to their original position and a heaven bead is activated on the upper deck. The same thing happens if two heaven beads are activated in the same rod. They immediately return to their original position and an earth bead on the rod to its immediate left is activated.

4 Workshop activities

The workshop has the same structure as the instructional intervention of the research study. The workshop activities are divided into four parts:

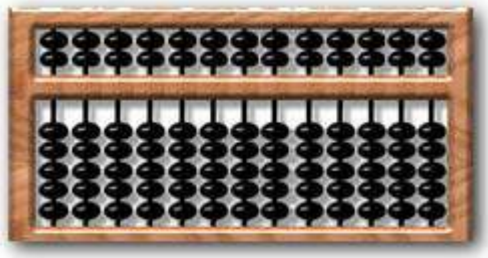
- the discovery of the structure and the mode of operation of the Chinese abacus
- the way that numbers are formed on the abacus
- the way that an addition (with or without carried number) is executed on the abacus
- the way that a subtraction (with or without borrowed number) is executed on the abacus.

The participants had the option to work either individually or in couples. Each activity was followed by plenary discussion. In general, the following pattern was applied: action-opinion formulation-validation-formalization.

Next we present the worksheets that were used in each part of the workshop with the correct answers to their questions.

4.1 First part

The first part of the workshop concerned the discovery of the tool. Initially, the participants were asked to guess what they could do with the Chinese abacus.



1. *Can you guess what you could do with the Chinese abacus?*

As most of the participants were teachers at various grade levels, it was easy for them to answer correctly that the Chinese abacus can support number representation, the execution of addition, subtraction, multiplication, division, and the finding of square and cube roots.

After studying some images representing the successive stages of the historical evolution of the Chinese abacus as a computational device, they were asked to guess how it works.

2. *After studying the images depicting the stages of the evolution of the Chinese abacus and a copy of it, which was given to you, can you guess how a number is formed on the Chinese abacus?*

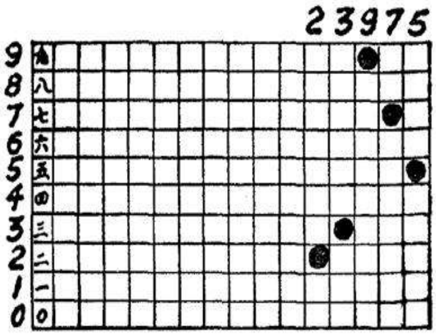


Fig. 1 Chu Pan or Bead Tray – The prototype of the Chinese abacus (600-300 BC)

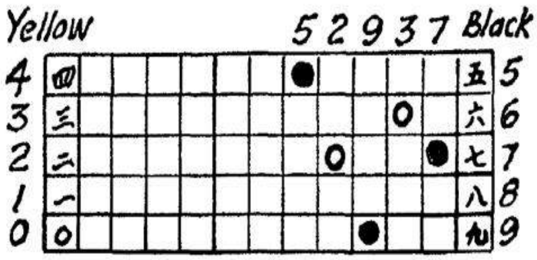


Fig. 2 Half-size tray with beads in two colors: yellow and black

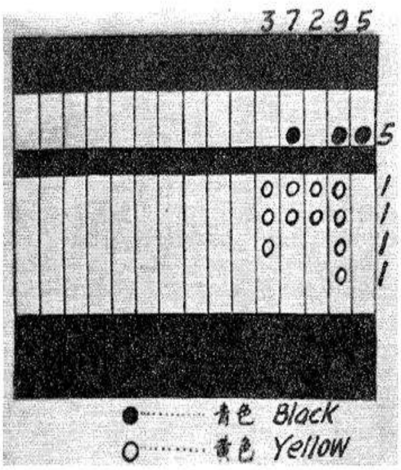


Fig. 3 The abacus with unattached beads (in use till the early 7th century)

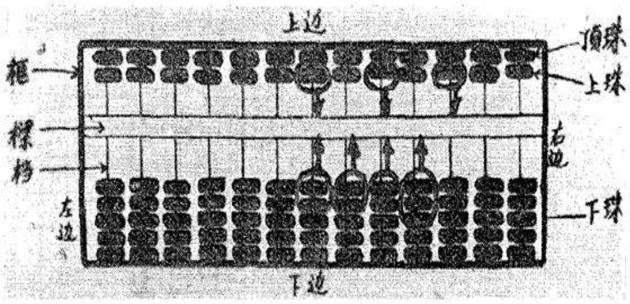


Fig. 4 The abacus having been in use for nearly 13 centuries

Chu Pan or Bead Tray (figure 1) was the initial prototype of the Chinese abacus. Its rectangular chess-board-like tray was divided into squares which had ten rows horizontally and as many columns as necessary. Each horizontal position denoted numerals 0 to 9 from bottom up. Each column represented a place value increasing from right to left. Numbers for operations were presented on the tray by placing beads in proper squares (Li, 1958).

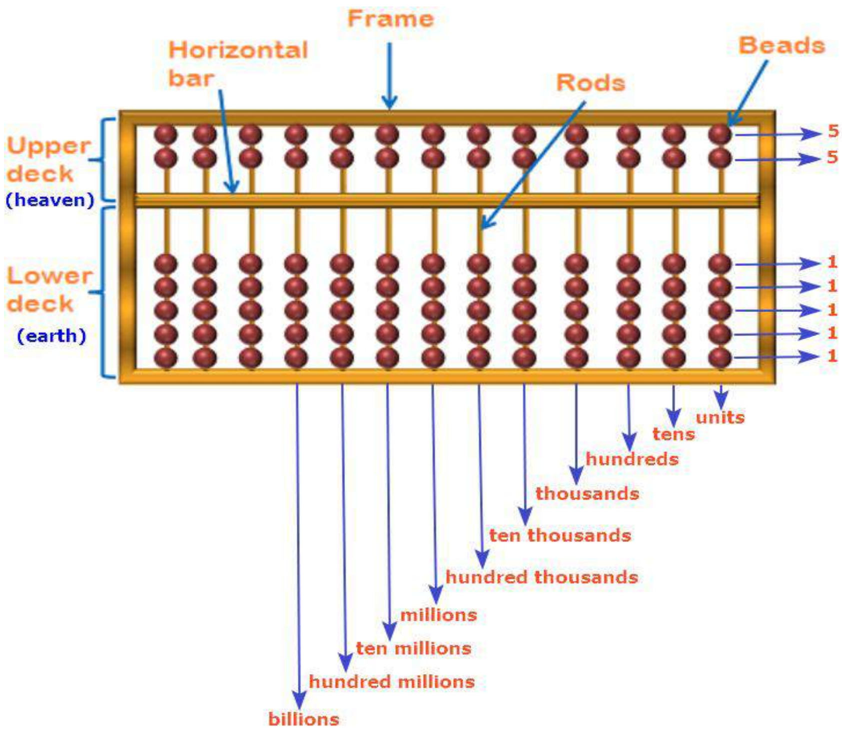
The next improvement of the Chinese abacus was the tray with beads in two different colors (figure 2). The yellow beads were denoting numerals 0 to 4, from bottom up and the black ones numerals 5 to 9, from top down. Place value was increasing from right to left, too. This improvement enabled the bead tray to be reduced in size as it was divided into five only horizontal rows of squares and as many vertical columns as before (Li, 1958).

Before the 2nd century AD Chu Pan had almost taken the form of the current Chinese abacus but with unattached beads (figure 3). The beads had not yet being attached to bamboo bar rails. Chu Pan consisted of a wooden board divided into rectangular column spaces. An upper horizontal partition separated the board in two unequal parts. On the top and the bottom of the board there were recesses for storing beads. In each column only five beads could be placed. One bead could be placed in the upper part and four beads in the lower part. The bead above the partition represented 5, while each of the four beads below the partition represented 1. Place value was increasing from right to left as in the previous figure. Though the beads that were used above and below the partition were differently colored, it was not absolutely necessary as the worth of each bead was determined by its position and not by its color. This model had existed until the early 7th century AD (Li, 1958).

The 4th figure represents the Chinese abacus that has been constantly in use for nearly 1.300 years without further improvements since the 10th century until very recently. At this stage the frame was perfected by providing a hole through the middle of each bead, by setting a round bamboo rod at each place value, and by placing an additional bead both above and below the partition at each rod. Thus the beads became attached, enabling the elimination of the top and bottom recesses for the storage of the beads. As in the earlier Chinese abacus with unattached beads, each bead above the partition is worth 5, and each below the partition denotes 1. When 5 is reached, it is exchanged with one upper bead replacing five lower beads. When 10 is reached, it is exchanged with one lower bead on the left place value replacing two upper or one upper and five lower beads on the right place value. From the point of view of simple number representation, one of the two upper beads and one of the five lower beads seem redundant. In fact both are necessary in the advanced techniques of multiplication and division operations (Li, 1958).

The workshop participants explained that the 3rd image was very helpful in discovering how the Chinese abacus works as it clearly shows the worth of each bead on both parts of the abacus. It should also be noted that some people already knew the mode of its operation, as they had already used it in the past. Then, the following picture that describes the parts of the Chinese abacus was given to them. Thus the organizers described in detail its structure.

The parts of the Chinese abacus

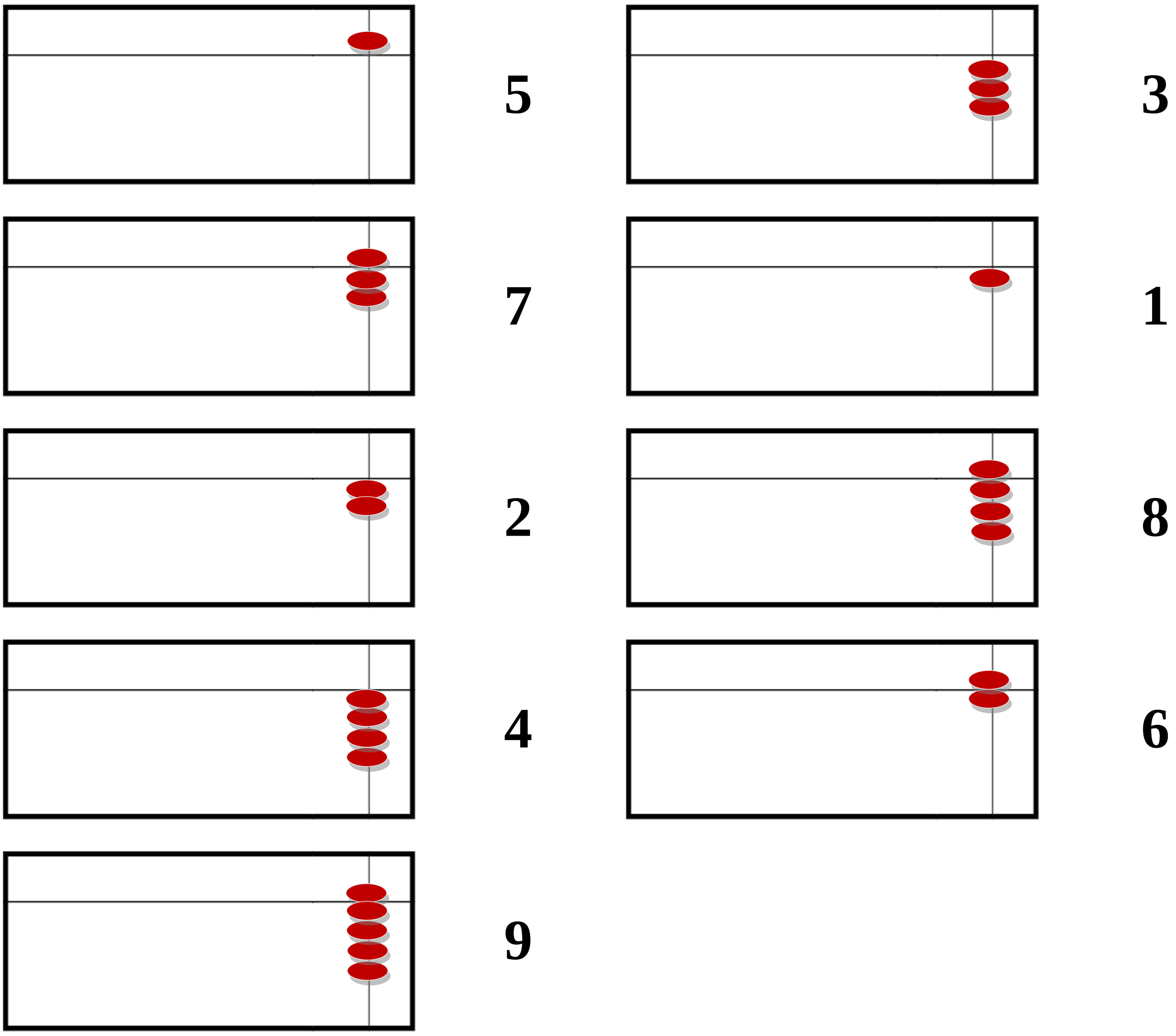


Afterwards, a video entitled “Chinese Zhusuan, Knowledge and Practices of Arithmetic Calculation through the Abacus” was presented to the participants. This video was submitted to UNESCO in the year 2012, so that the Chinese Zhusuan be inscribed on the representative list of the intangible cultural heritage of humanity. It describes the form and content of the Chinese abacus, its cultural value, its challenges and its protection measures. Zhusuan means bead computation, that is, a time-honored traditional method of performing mathematical calculations with an abacus, the Chinese abacus. Finally, in the year 2013 Chinese Zhusuan was inscribed in the list of the intangible cultural heritage of humanity.

4.2 Second part

In the second part of the workshop activities, the participants were asked to form single digit numbers both in their individual abacus and in the worksheet's sketched abaci.

3. *Can you draw beads on each abacus to represent the number written next to it?*

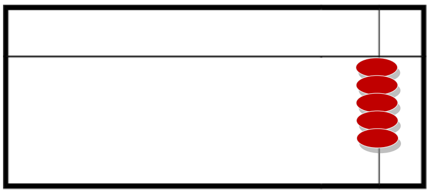


Firstly, they formed the given numbers in their individual abacus and then they drew beads in the worksheet's sketched abaci to represent the same numbers. The workshop leaders were walking around the room to check the participants' way of working. Most of them formed the given numbers correctly.

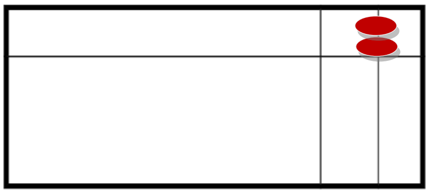
Then, they were asked to discover how the numbers 5 and 10 are represented on the Chinese abacus. They were suggested to study again the figure that shows the parts of the

Chinese abacus and the value of each bead, as it implies the different ways that the two numbers can be represented on it.

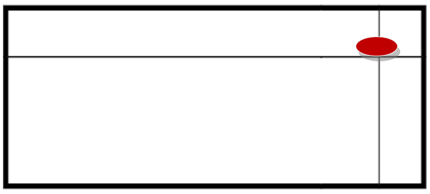
4. *Can you draw beads on each group of abaci to show the different ways the same number can be represented?*



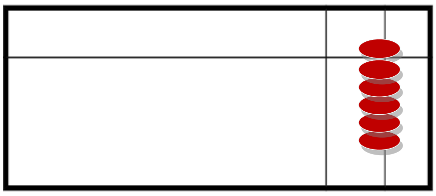
5



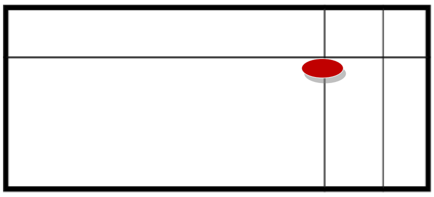
10



5



10



10

The participants identified all the possible ways that are presented above. Afterwards, they were suggested to think which one could be the most appropriate for the representation of each number on the Chinese abacus.

5. *Which do you think is the most appropriate way to represent the numbers 5 and 10 on the Chinese abacus? Can you justify your answer?*

The representation of the number 5 with an activated bead on the upper deck of the rightmost column of the abacus and the representation of the number 10 with an activated bead at the lower deck of the left column are the most appropriate answers. Thus there are available beads, which may then be needed in the same column of the abacus in case of adding, subtracting, multiplying or dividing with another number that, in turn, must be represented on the abacus.

Time was also devoted to the identification of the role of zero in a number. The following question was given.

6. *What do you think is the role of zero in a number?*

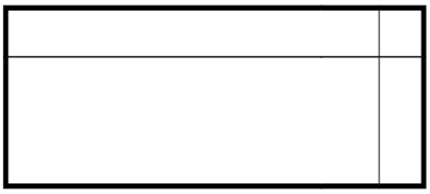
Its twofold role

- as an indication that there are no units, tens, hundreds, etc. in the number and
- as a place holder,

was highlighted by the participants.

Then, they were asked to represent zero both on their individual and the following sketched Chinese abacus.

7. *How would you represent zero in the following Chinese abacus?*

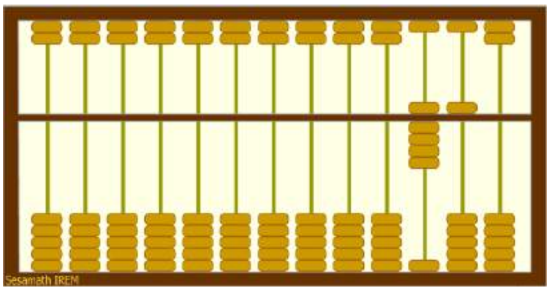


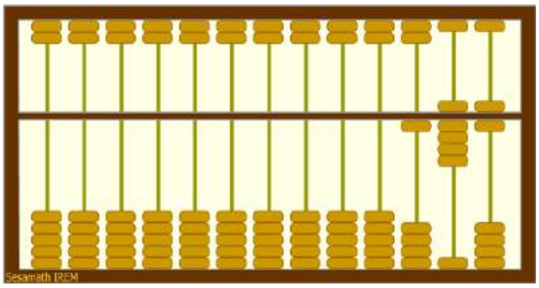
0

They easily suggested that zero be represented by activating no beads on the abacus. Most participants individually and successfully interpreted numbers that were represented in sketched abaci.

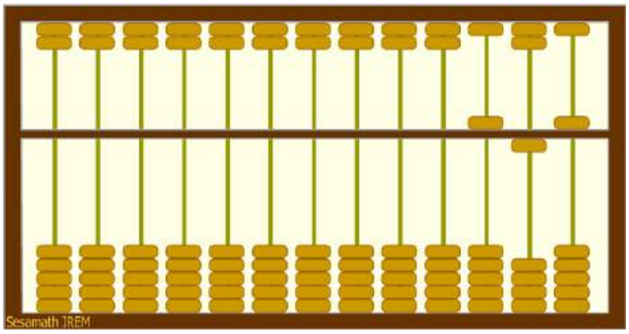
8. *Can you write above each abacus the number that it represents?*

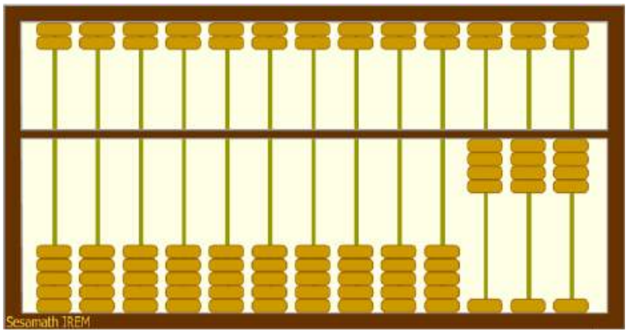
950 **196**



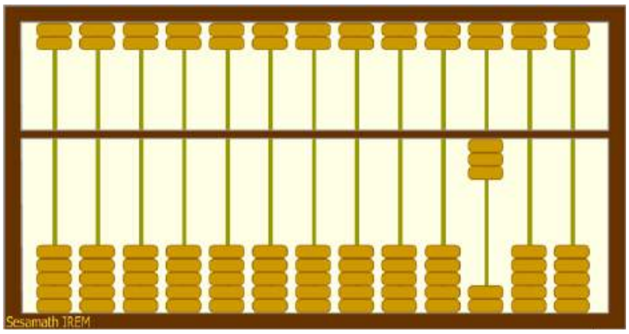


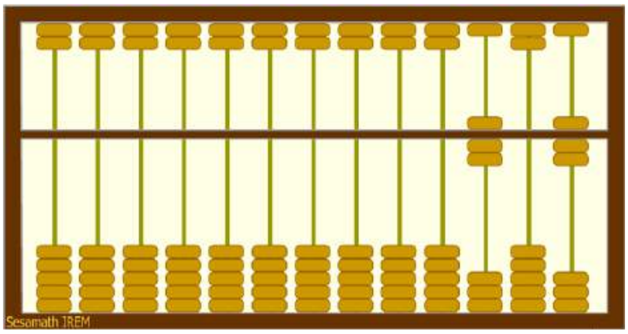
515 **444**



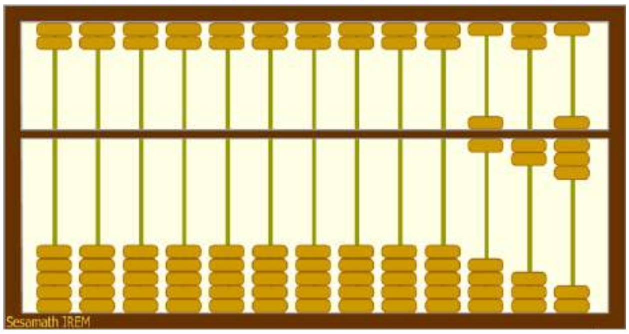


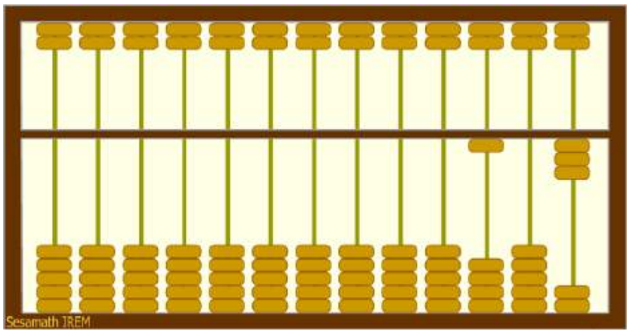
300 **707**





628 **103**

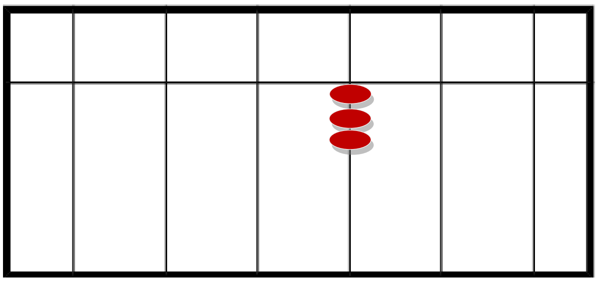




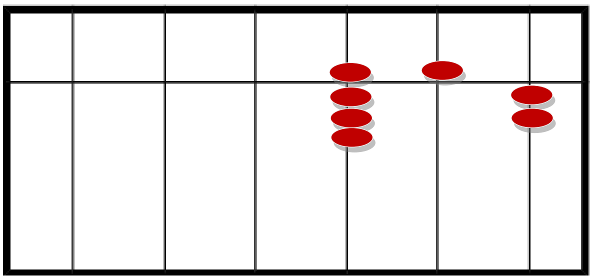
Then, they correctly formed nine three-digit numbers and one four-digit number on empty abaci by drawing beads. Previously, they verified their answers on the actual abacus.

9. *Can you draw beads on each abacus to represent the number written above it?*

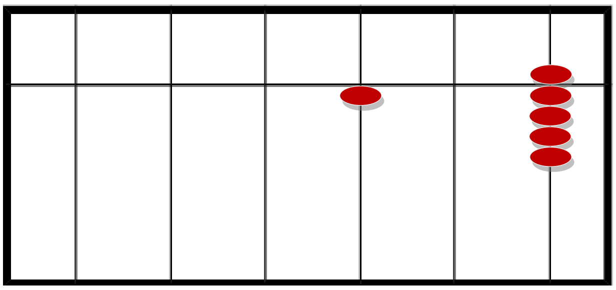
300



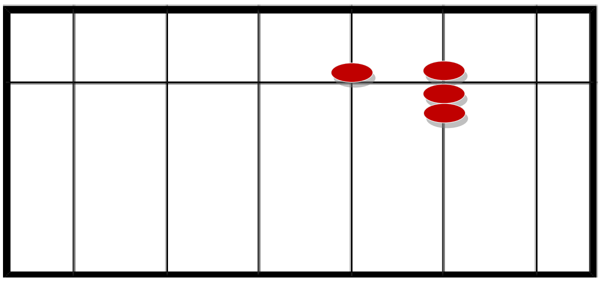
852



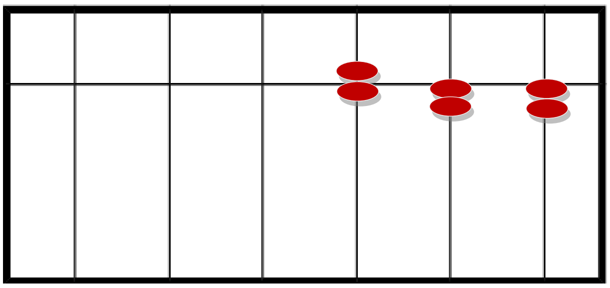
109



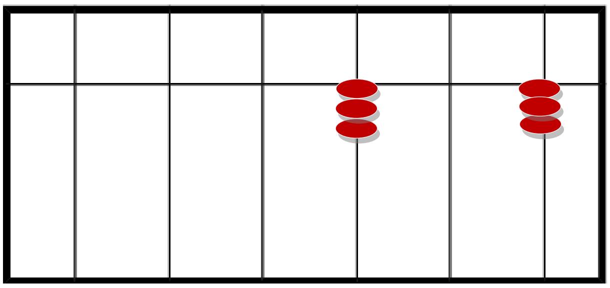
570

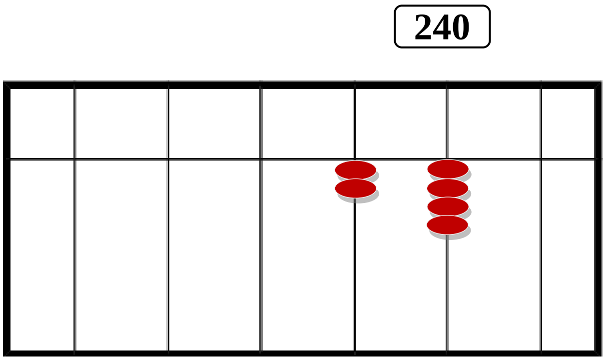


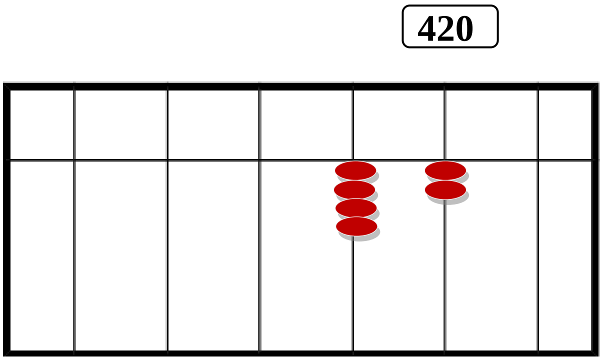
622

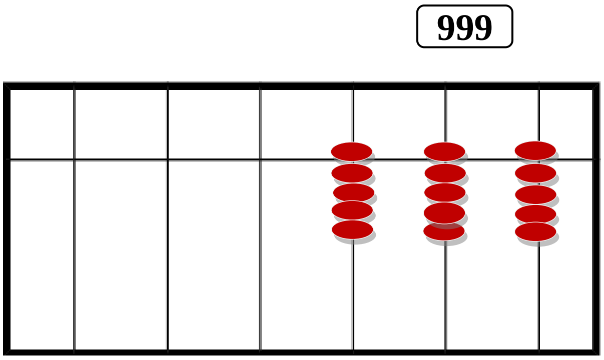


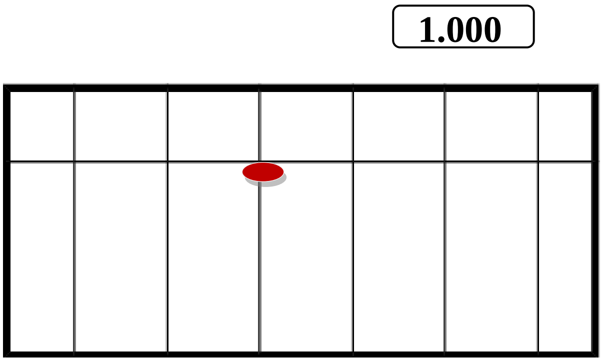
303











4.3 Third part

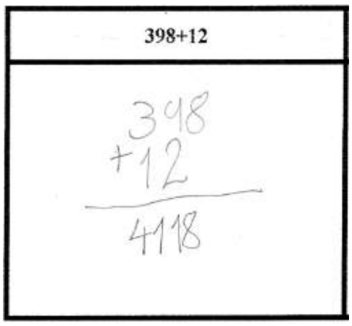
The third part of the workshop activities concerned the addition algorithm. In the plenary discussion, the participants used their experience to share common students' mistakes in the implementation of the addition algorithm.

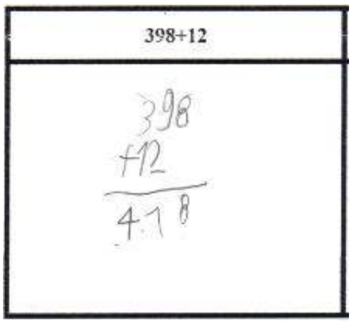
10. *What kind of mistakes do students usually make in the addition algorithm?*

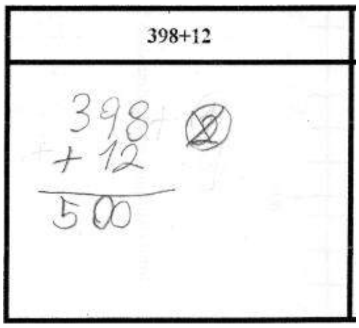
They mentioned mistakes in the placement of the addends, wrong sums at place values, non-creation of carried number, etc.

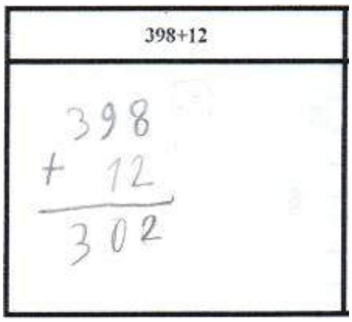
Afterwards, they worked on an activity that presented five different attempts to execute the addition 398+12. These were the answers that were given by five different students at the pre-test questionnaire of our research study. The participants were asked to identify the students' mistakes and to guess their way of thinking that led to these mistakes.

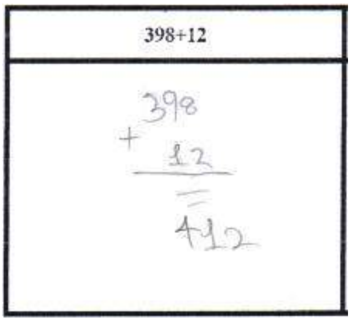
11. *Can you identify the students' mistakes while they executed vertically the addition 398+12? Can you guess their way of thinking that led to these mistakes?*











The participants were very interested in this activity. They easily identified the mistakes and they announced their remarks in the plenary discussion that followed the activity.

Then, they were asked to explain what carried number is.

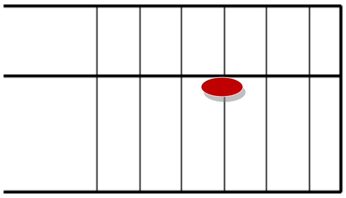
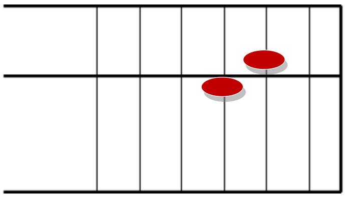
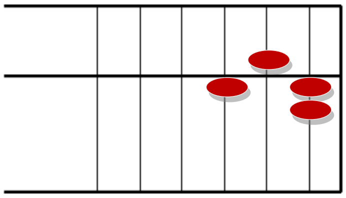
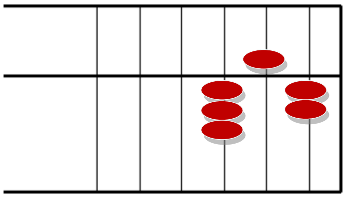
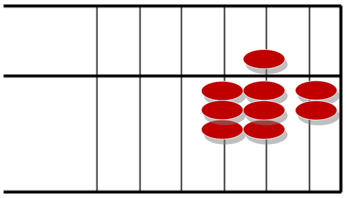
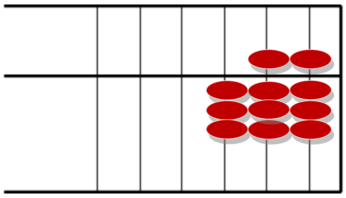
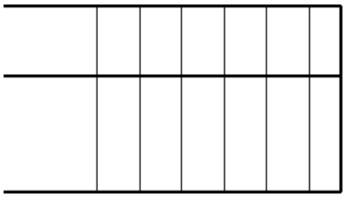
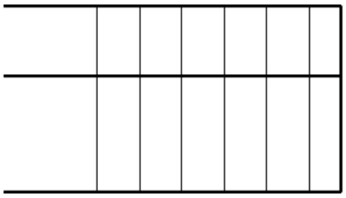
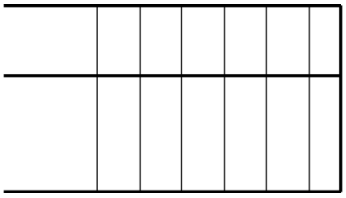
12. *Can you explain what carried number is?*

Poisard (2006), in order to explain the concept of carried number, states that in each rank of the decimal number system (units, tens, hundreds, etc.) only one digit from 0 to 9 can be written. In the addition, as soon as ten is reached in any place value, there is a transfer of numbers between ranks (always from right to left). One transfers 1 ten for every 10 units, 1hundred for every 10 tens, 1 thousand for every 10 hundreds, and so on. “The carried number enables us to manage the change of the place-value by making a transfer of numbers between ranks” (Poisard, 2006, p. 417). That is the definition she provides for carried number. Moreover, the understanding of the meaning of carried number requires a profound comprehension of the place value system.

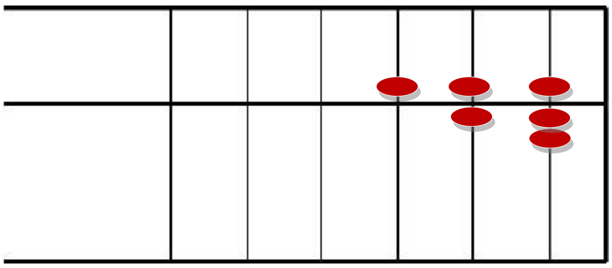
Next, the participants were asked to perform step-by-step both on sketched abaci on the worksheet and on the actual abacus an addition without carried number and an addition with carried number, following instructions. Thus the concept of carried number was visualized. At the same time, by using the concrete abacus, the visual and the tactile perception of the concept were combined (Nagaoka et al, 2000).

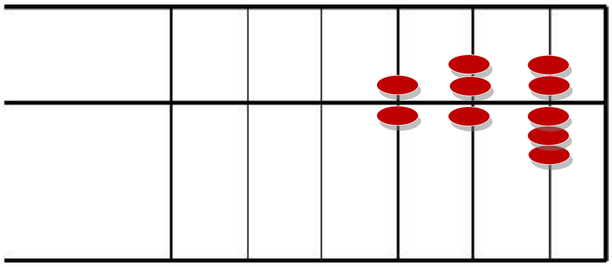
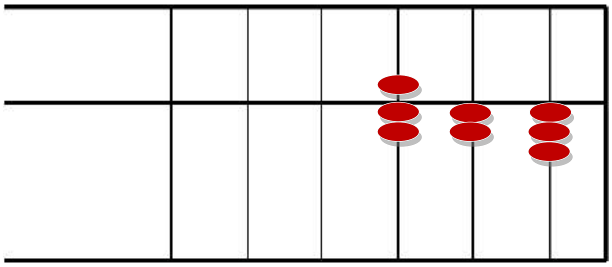
13. *Can you draw beads on the following sketched abaci to show step-by-step how you would perform the addition 152+236 on the Chinese abacus? Draw only the activated beads. On each line explain what you are doing. Use only the abaci you need. Finally, write the sum in the box.*

152 + 236 = **388**

1st abacus **I activate 1 hundred.** 	2nd abacus **I activate 5 tens.** 	3rd abacus **I activate 2 units.** 
4th abacus **I activate 2 more hundreds.** 	5th abacus **I activate 3 more tens.** 	6th abacus **I activate 6 more units.** 
7th abacus ……………………………… 	8th abacus ……………………………… 	9th abacus ……………………………… 

14. *Can you write vertically the following horizontal addition and find the sum? Verify the result by using your abacus. Then, follow the instructions to show, by drawing beads, the way of executing an addition on the Chinese abacus.*

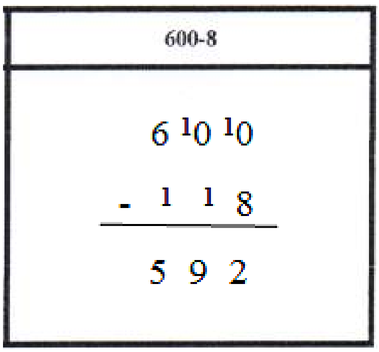
Horizontal addition	**vertical addition**	**Abacus**
		Place the number 567 on the abacus. Then, write under each place value how many Units, Tens and Hundreds are activated.  **5 6 7**

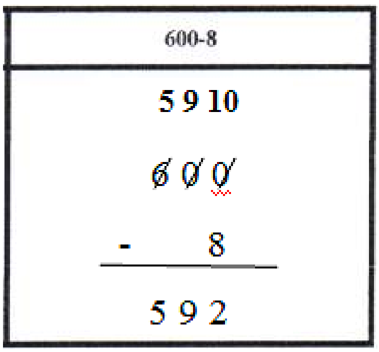
567+156= **723**	**1 1** 5 6 7 + 1 5 6 **7 2 3**	Add the number 156 by drawing beads. Then, write under each place value how many Units, Tens and Hundreds are activated. Paint the carried numbers with red color.  **6 11 13** Make the necessary exchanges between place values.  **7 2 3** Can you explain what carried number is?

4.4 Fourth part

In the fourth part of the workshop the participants dealt with subtraction. Initially, they were asked to perform the subtraction 600-8 with two different algorithms that students learn in primary school.

15. *Can you carry out the subtraction 600-8 using two different algorithms that students are taught in primary school?*





In the plenary discussion, the participants mentioned two methods: “regrouping the minuend” and “adding equal amounts”.

Then, they were asked to think about which one of the two ways could be applied to the Chinese abacus.

16. *Which algorithm do you think the Chinese abacus can support?*

They explained that the first method can be applied on the Chinese abacus, while a participant tried to explain that the “adding equal amounts” method could be applied, too.

Later, the participants were asked to mention common mistakes that students usually make in the subtraction algorithm.

17. *What kind of mistakes do students usually make in the subtraction algorithm?*

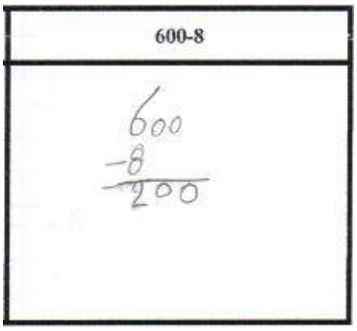
They mentioned that usually students place wrongly the subtrahend, they don’t know from which place value to borrow and they also make mistakes in the calculations.

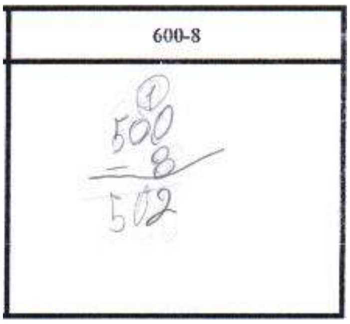
Next they were asked what borrowed number is.

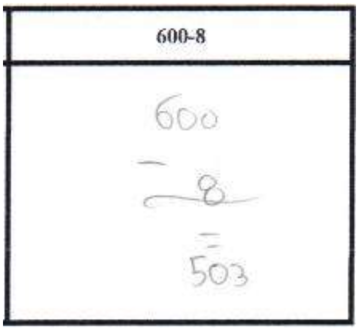
18. *Can you explain what borrowed number is?*

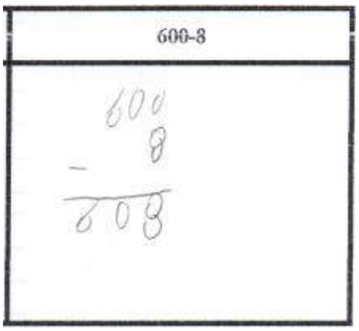
Then, as in the addition, five different attempts to perform the subtraction 600-8 were presented. Once again the participants were asked to identify the students’ mistakes and to think what led them to these mistakes.

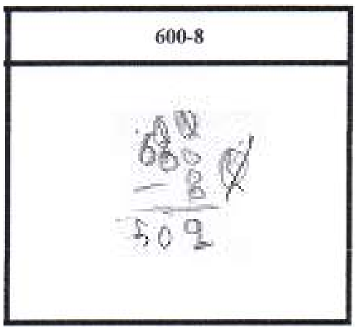
19. *Can you identify the students’ mistakes while they carried out the subtraction 600-8? Can you guess their way of thinking that led to these mistakes?*









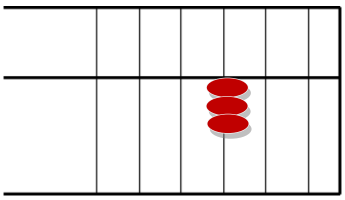
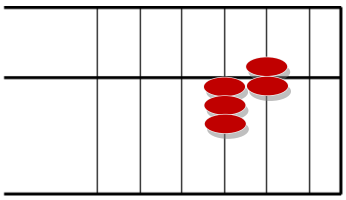
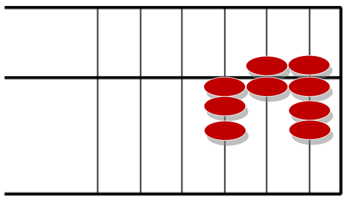


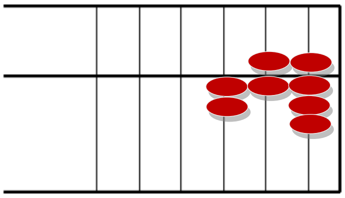
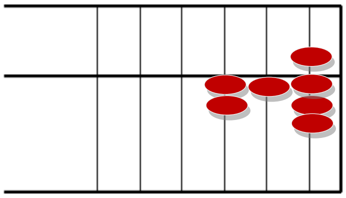
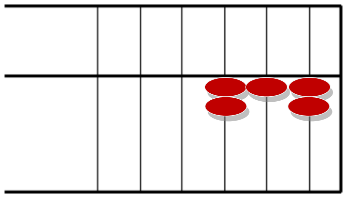
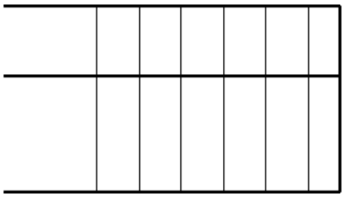
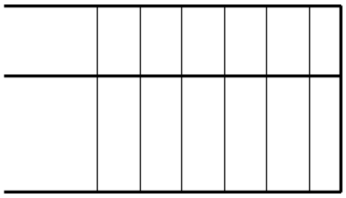
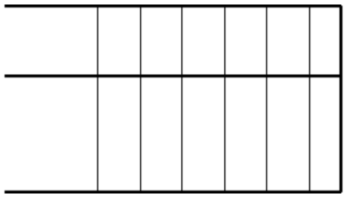
It is another activity that the participants said that it was interesting and they announced their thoughts in the plenary discussion that followed.

Finally, both in sketched and actual abaci, the participants were asked to perform a subtraction without borrowed number and a subtraction with two borrowed numbers, following the worksheet's instructions.

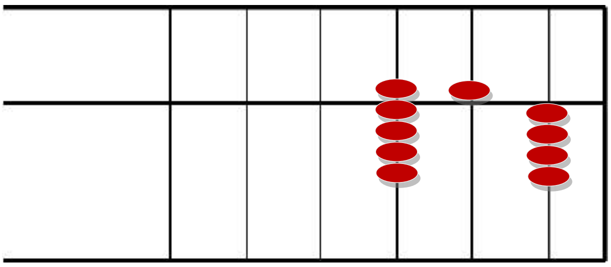
20. *Can you draw beads in the following abaci to show step-by-step how you would perform the subtraction 368-156 on the Chinese abacus? Draw only the activated beads. On each line explain what you are doing. Use only the abaci you need. Finally, write the difference in the box.*

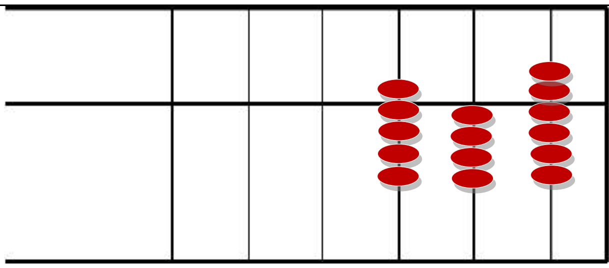
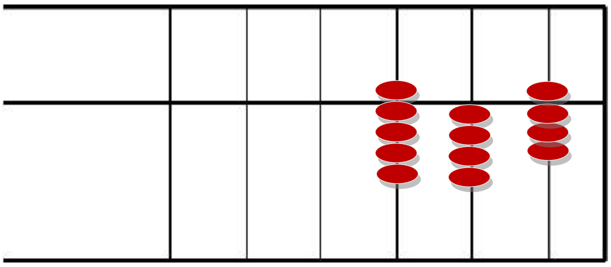
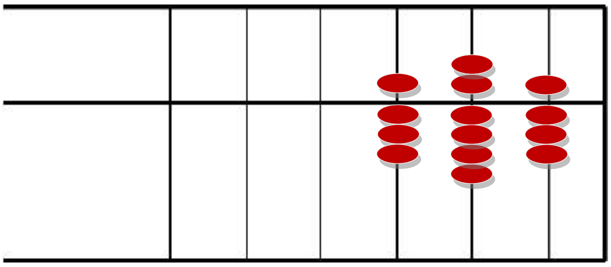
368 - 156 = **212**

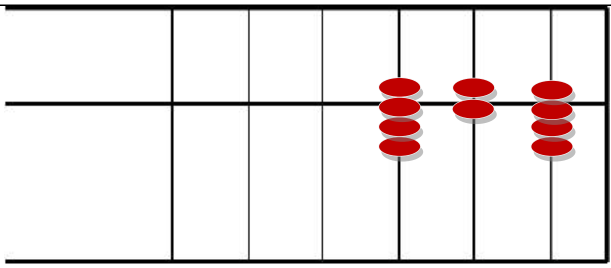
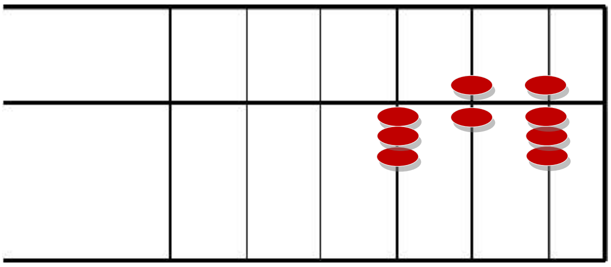
1st abacus	2nd abacus	3rd abacus
I activate 3 hundreds.	**I activate 6 tens.**	**I activate 8 units.**
		

4^{th} abacus **I deactivate 1 hundred.** 	5^{th} abacus **I deactivate 5 tens.** 	6^{th} abacus **I deactivate 6 units.** 
7^{th} abacus .... 	8^{th} abacus .... 	9^{th} abacus .... 

21. *Can you write vertically the following horizontal subtraction and find the difference? Verify the result by using your abacus. Afterwards, follow the instructions to show, by drawing beads, the way of executing a subtraction on the Chinese abacus.*

horizontal subtraction	vertical subtraction	Abacus
		Place with beads the number 954 on the abacus. Then, write under each place value how many Units, Tens and Hundreds are activated.  **9 5 4** Start subtracting. You cannot subtract 6 Units from 4 Units. Borrow 1 Ten (=10 Units) from the Tens' place and make the necessary exchanges between place values. Then, draw the beads which are activated on the place values and write under each place value how many Units, Tens and Hundreds are activated. Color in red the borrowed number.

<table>
<tr>
<td>954-586= 368</td>
<td>14
8 $\cancel{4}$ 14
$\cancel{9}$ $\cancel{5}$ $\cancel{4}$
- 5 8 6
3 6 8</td>
<td>
9 4 14
Subtract the Units in the Units’ place and draw what remains. Then, draw beads on the Tens’ and Hundreds’ place. Write under each place value how many Units, Tens and Hundreds are activated and go to the Tens’ place.

9 4 8
You cannot subtract 8 Tens from 4 Tens. Borrow 1 Hundred (=10 Tens) from the Hundreds’ place and make the necessary exchanges between place values. Then, draw the beads which are activated on the place values and write under each one how many Units, Tens and Hundreds are activated. Color in red the borrowed number.

8 14 8
Subtract the Tens in the Tens’ place and draw what remains. Then, draw beads on the Units’ and Hundreds’ place. Write under each place value how many Units, Tens and Hundreds are activated and go to the Hundreds’ place.</td>
</tr>
</table>

		 **8 6 8** Subtract the Hundreds in the Hundreds' place and draw what remains. Then, draw beads on the Tens' and Units' place and write under each place value how many Units, Tens and Hundreds are activated.  **3 6 8** Can you explain what borrowed number is?

At the end of the workshop we presented to the participants an outline of the theory of instrumental genesis and semiotic mediation that we used in our instructional intervention, and several examples of the implementation of the two theories.

5 Discussion

Poisard (2006) argues that for many students and teachers the notion of carried number appears not to be developed mathematically. Usually the addition is taught in the form of learning the algorithm, which means the execution of a series of steps that represent the syntactic rules of the algorithm. However, the performance of an algorithm does not require deep awareness of the meaning of the successive steps. For example, when the student sums up the units of the addends and finds a result equal to or greater than ten units, he says "one is the carried number". Then, he marks it with the digit 1 either above the tens' place that will carry it over or on the right side of the addition. However, it is not certain that the student understands that this 1 is 1 ten, which means 10 units, and not 1 unit (Fuson, 1990). This misconception is clarified by the Chinese abacus. The fact that in each column of the abacus can be represented numbers up to 15 (i.e. 15 units, 15 tens, etc.) and someone can make exchanges between the columns with his hand, enhances the understanding of the carried number concept (Poisard, 2006). Thus carried number acquires a material substance, which did not have until now, since the addition algorithm is usually taught without the use of physical manipulatives. Therefore, it is understood through the visual and tactile perception of the Chinese abacus. At the same time, the knowledge of carried number also contributes to the reduction of algorithmic errors.

This visual and tactile perception of the Chinese abacus also applies to the case of the borrowed number. As it has already been mentioned, the Chinese abacus can support the visualization of the subtraction algorithm by the type of “regrouping the minuend”. This regrouping requires exchanges between place values in the opposite direction from addition. Thus, the intangible borrowing, which so far has been taught to children in the context of the subtraction algorithm, becomes tangible, and it is obvious now what is being borrowed and from where. In this way errors can be avoided when performing the subtraction algorithm, especially when the student is asked to borrow from a place value that has a zero digit.

Therefore, the Chinese abacus is a resource for the teaching and learning of the place value system in primary school. It benefits both students’ learning and teachers’ practice in the mathematics classroom.

REFERENCES

Fuson, K. C. (1990). Conceptual structures for multiunit numbers: Implications for learning and teaching multidigit addition, subtraction, and place value. *Cognition and instruction*, *7*(4), 343-403.

Korros, F. (2012). *Mathematics of flat Earth-The history of Mathematics in China until the 15th century*. Athens: Bookstars-Free Publishing. (in Greek) [Κόρρος, Φ. (2012). *Τα μαθηματικά της επίπεδης γης-Η ιστορία των Μαθηματικών στην Κίνα μέχρι τον 15ο αιώνα.* Αθήνα: Bookstars-Free Publishing.]

Li, S. T. (1958). The origin of the abacus and its development. In *ACM Annual Conference/Annual Meeting: Preprints of papers presented at the 13th national meeting of the Association for Computing Machinery* (Vol. 11, No. 13, pp.102-110). Urbana, Illinois.

Nagaoka, R., Barrow-Green, J., Bartoloni Bussi, M. G., Isoda, M., van Maanen, J., Michalowicz, K. D., Ponza, M. V., & van Brummelen, G. (2000). Non-standard media and other resources. In J. Fauvel, & J. van Maanen (Eds.), *History in mathematics education: the ICMI study* (pp. 329-370). Dordrecht: Kluwer.

Poisard, C. (2006). The notion of carried-number, between the history of calculating instruments and arithmetic. In P. Groontenboer, R. Zevenbergen, & M. Chinnapan (Eds.), *Identities, cultures and learning spaces*. *Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia, Canberra* (Vol 2, pp. 416-423). Adelaide: MERGA.

Poisard, C. (2017). Introducing an old calculating instrument in a new technologies environment: a praxeological analysis of students’ tasks using different registers. *Re SMITECH Review of Science, Mathematics and ICT Education, 11*(2), 47-67.

Price, P. (2002). Children’s difficulties with base-ten numbers: “Face-Value” and “Independent-Place” Constructs. In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.), *Mathematics Education in the South Pacific*. *Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia, Auckland* (pp. 592–599). Sydney: MERGA.

Spitzer, H. F. (1942). The abacus in the teaching of arithmetic. The Elementary School Journal, *42*(6), 448-451.

Stigler, J. W. (1984). “Mental abacus”: The effect of abacus training on Chinese children’s mental calculation. *Cognitive Psychology, 16*(2), 145-176.

Tsiapou, V., & Nikolantonakis, K. (2013). The Development of Place Value Concepts to Sixth Grade Students via the Study of the Chinese Abacus. In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *Proceedings of the Eight Congress of the European Society for Research in Mathematics Education* (pp. 2058-2067). Ankara: Middle East Technical University.

Tsiapou, V., & Nikolantonakis, K. (2016). The development of place value concepts and the notion of carried number among sixth grade students via the study of the Chinese abacus. *Menon: Journal of Educational Research*, 2nd thematic issue, 51-65.

Tzekaki, M. (2007). *Little children big mathematical meanings*. Athens: Gutenberg. (in Greek) [Τζεκάκη, Μ. (2007). *Μικρά παιδιά μεγάλα μαθηματικά νοήματα.* Αθήνα: Gutenberg.]

Tzekaki, M. (2016). Educational material as an element of mathematical activity. In Skoumios, M. & Skoumpourdi, C. (Eds.), *Proceedings of the 2nd Pan-Hellenic Conference "The educational material in Mathematics and the educational material in Physical Sciences: lonely trajectories or interactions?"* (pp. 380-390). Rhodes, Greece: University of the Aegean. (in Greek) [Τζεκάκη, Μ. (2016). Εκπαιδευτικό υλικό ως στοιχείο της μαθηματικής δραστηριότητας. Στο Σκουμιός, Μ. & Σκουμπουρδή, Χ. (επιμ.), *Πρακτικά 2ου Πανελληνίου Συνεδρίου "Το εκπαιδευτικό υλικό στα Μαθηματικά και το εκπαιδευτικό υλικό στις Φυσικές Επιστήμες: μοναχικές πορείες ή αλληλεπιδράσεις;"* (σ. 380-390). Ρόδος: ΤΕΠΑΕΣ, ΠΤΔΕ, Πανεπιστήμιο Αιγαίου.]

Zhou, Z., & Peverly, S. T. (2005). Teaching addition and subtraction to first graders: A Chinese perspective. *Psychology in the Schools, 42*(3), 259-272.