BEING IN RESEARCH AND DOING RESEARCH ON HISTORY AND MATHEMATICS EDUCATION IN A DIALOGICAL PERSPECTIVE

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ABSTRACT

At the beginning of the 20th century, Mikhail Bakhtin developed a critical mode of literary analysis based on the dialogic principle. This dialogical principle emphasizes that each utterance of a discourse is necessarily "in dialogue" in a given sphere of speech communication. For Bakhtin, these utterances are following one another according to laws of appreciative convergence in a close dependence on historical conditions. Bakhtin's philosophy has inspired few researchers to establish or develop theoretical foundations in mathematics education, but also, closer to us, in history and mathematics education. In this paper, we will try to show how Bakhtin's thought can support reflection, not on history, mathematics or mathematics education *per se*, as it has often been discussed, but on our ways of being in research and doing research within or field.

1 Introduction

This paper is a development of the oral communication that took place during ESU-8. The objective, here, is to introduce some elements of the philosophy of Mikhail Bakhtin (1895-1975) in order to bring new ways of thinking about being and doing research within the field of research on history of mathematics in mathematics education.

As discussed by Fried, Guillemette and Jahnke (2016), the field has moved, for the past decade, toward a reflection on didactical and pedagogical foundations concerning the role of history of mathematics in teaching and learning, as well as the development of theoretical and conceptual frameworks. They argue that theoretical or conceptual element that could support research concerning history of mathematics in mathematics education should at least address the question of why we ought to learn history or historical elements related to mathematics? What part of pupils and students intellectual lives is touched by history? The historical nature of mathematics must at least be put on the table, as something to question. In other words, the nature of mathematics itself must be problematized. Our own view of mathematics of the past should also be problematized, asking what it means to stand facing the past? Our own posture towards the past should be explored.

In this quest, this paper tries to draw attention on the dialogical principle that has been developed by Bakhtin in order to bring theoretical and conceptual elements to support teachers' and researchers' reflection. We would like to argue, in this paper, that elements of Bakhtin's philosophy could support reflection, not only upon history, mathematics or mathematics education *per se*, as it has sometimes been discussed, but also upon our ways of being in research and doing research within or field. The main argument that will be discussed is that this perspective could help particularly the researchers by making possible and visible the dialogical interactions between researcher, participants and history

of mathematics, but also the teachers trying to deepen their comprehension ofthe relation between their interventions, the learners, the mathematics and the history of mathematics in their classrooms.

2 A Bakhtinian perspective

In this section, we will try to introduce key elements of the Bakhtinian philosophy (Bakhtin 1929/1977, 1979/1982, 1978/1997, 1963/1998). This quick introduction will focus on two major elements: the *dialogical principle* and the concept of *polyphony*.

The philosophical works of Mikhail Bakhtin, born in Russia, can be included in the dialectical tradition inaugurated by Hegel, developed by Marx and continued among Russian thinkers such as Ilienkov, Mikhailov and Vygotsky. At the beginning of the 20th century, after the Russian revolution, Marx's philosophy and the development of phenomenology in Europe have motivated and influenced a group of thinkers called *The Bakhtin Circle* constituted for instance by the linguist Valentin Voloshinov (1895-1936), the literary scholar Pavel Medvedev (1891-1938) and the philosopher Matvei Kagan (1889-1937). The circle was animated by profound insights around literature, linguistic, psychology and philosophy, at a time where Marx was discussed in its very philosophical potential. It was a very productive Marxist intellectual group. Unfortunately, the group didn't survive to the Stalinian purges.

2.1   The dialogical principle

From his analyses of Freud, de Saussure and linguistic theories conducted within the group, Bakhtin has developed a critical mode of analysis based on what he called the dialogical principle. He then applied it in different areas such as literary analysis and more generally to the analysis of ideology (see Sabo and Nielsen, 1984). This said, this reflection goes far beyond literary criticism, to the point of disrupting the foundations of the human sciences.

Succinctly, the dialogical principle emphasizes that each utterance of a speech is necessarily an answer to another utterance in a given sphere of speech communication. This concept of sphere of speech communication is important because it determines the very condition of the dialogue in which the speech inscribes itself inherently. Indeed, for Bakhtin, at every moment of the dialogical analysis, one must understand and utterance as an answer of another utterance, but, ate the same, time as a condition to other utterance to emerge. In other words, Bakhtin would say that any movement of consciousness is itself dialogical, penetrated by and in dialogues with other movements of consciousness, and thus, cannot be approached without consideration for other movements of consciousness to which it answers, and which it allows as an answer.

A speech is then perceived here as a dialogue, but this dialogue is not understood as a simple sequence of statements constituting a form of exchange or conversation. These aspects would be only the superficial manifestation of dialogism which "goes far beyond the relations between the replicas of a formally constructed dialogue, because it is almost universal and crosses the whole human discourse [...] in a general way, all that has meaning and value" (Bakhtin, 1963/1998, p. 77, our translation).

Indeed, for Bakhtin, there is an ongoing dialogue both at the level of language and at the level of ideas, and these levels are intimately linked. He emphasises that our inner or

outer speeches are constituted by whole monologues, analogous to paragraphs, or by whole utterances. But these monologues and dialogue hardly lends itself to an analysis of grammatical constituents. In this sense, Bakhtin would say that: "These units of speech, which could be called the global impression of enunciation, are related to each other and succeed one another not according to the rules of logic or grammar, but according to laws of appreciative convergence, of dialogical concatenation, and in a close dependence on the historical conditions of the social situation and the whole pragmatic course of existence. There is between the *psyche* and ideology an indissoluble dialectical interaction: the *psyche* dismantle itself, destroy itself to become ideology, and reciprocally" (*id*., pp. 63-64, our translation).

But what are those "laws of appreciative convergence" and "of dialogical concatenation"? What is happening concretely? For Bakhtin, "when the listener perceives and understands the meaning of speech, he simultaneously takes an active-responsive attitude toward it. Either he agrees or he disagrees with it (completely or partially), augments it, applies it, prepares for its execution, and so on. And the listener adopts this responsive attitude for the entire duration of the process of listening and understanding, from the very beginning - sometimes literally from the speaker's first word" (1986, p. 68). On the other side, the speaker is also expecting such an active-responsive relation. Bakhtin add that "[The speaker] does not expect passive understanding that, so to speak, only duplicates his or her own idea in someone else's mind. Rather, the speaker talks with an expectation of a response, agreement, sympathy, objection, execution, and so forth... Therefore, each kind of utterance is filled with various kinds of responsive-reactions to other utterances of the given sphere of speech" (*id*., p. 91). This attitude of the speaker and the listener as active-responsive communication lead inherently to the observation of different speech genres regarding the given sphere of speech communication. Indeed, the speaker will adapt is speech and perform a certain style when situated himself in relation with the listener, in the ways he perceived the listener, in a close relation to history, ideology, social determinations, but also to direct relations such as subordinated, fraternal or enmity relations for instance.

To sum up, Bakhtin suggests understanding the utterance as something produced within the dialectical relation between the self and the ideology. In other words, he simply means that the words that I used are necessarily the words of someone else. This simple observation will lead to deep insights regarding human sciences, making the possibility to bring new ways of thinking human subjectivity and human cultural production.

Indeed, for Bakthin, the dualism between the world of objects and the I who thinks it, isolated atom without history, immutability in prey to the laws of the phenomenon, is everywhere present in the philosophy of his time. In contrast, Bakhtin (1986/2003) argues that the impoverishing dualism of Cartesian rationalism must be fought by repudiating the abstractions of idealist philosophy, in order to better grasp the nature of the concrete action or "act" constituting the "value-center" of human existence. The ego is here a dynamic, corporeal, creative and moving entity. Bakhtin strives to formulate a phenomenology of "the practical act", a phenomenology that focuses on our activities as bodily beings in a world that pre-exists abstract constructions.

As we will see history of mathematics and mathematics education do not escape from the scope of this dialogical principle.

2.2 The concept of polyphony

Taking through the lens of this dialogical principle, a scientific, literary or philosophical work is called "polyphonic" when, setting the scene in a given sphere of communication; it offers a large plurality of discourses and understandings of the world.

For Bakhtin, Dostoyevsky's novel *The Brothers Karamazov* would be the archetype of the polyphonic work. Indeed, the author depicts many characters inhabited by singular personalities who have finely established roles (the bourgeois, the liberal, the scientist, the atheist, etc.). These characters act as "spokespersons of worldviews" (Sabo and Nielsen, 1984, p. 80), and Dostoevsky strives to make them talk together. The confrontation of these individuals, endowed with a strong ipseity, highlights the existential, ideological and socio-historical texture of that time.

For Bakhtin, the polyphonic aspect of the novel of Dostoyevsky is the more objective and effective way to describe the reality of the author, in this case, Russia after the reforms of 1860. In the dialogical perspective, the work of the author is to perceive "great ideas" and "representations of men who speak of their ideological universe" (Bakhtin, 1978/1997, p. 182). The concept of polyphony is central to the philosophy of Bakhtin. Indeed, for Bakhtin, it is the meeting of discourses, "acts" and social horizons that allows us to say something about reality. It is in the illustration of tensions and reconciliations between different positions that brings clarifications.

2.3 A critical standpoint

By promoting the polyphonic aspect of his work, the author ensures that "reality loses its statism and naturalism [...] the future begins to penetrate in the form of trends, possibilities, anticipations" (Bakhtin, 1970/1982, p. 129, our translation). Such cultural production, for Bakhtin, "has essential views on freedom, overcomes determinism and strict mechanisms" (*ibid.*). Indeed, for Bakhtin and is collaborators, there is an important link between human struggle for social justice and more broadly with the struggle for meaning that occurs within all sectors of human cultural production, such as sciences, literature, art and philosophy.

3 Bakhtin and mathematics education

Bakhtin's philosophy has often been summoned in human sciences. For instance, it has inspired many researchers in mathematics education to support theoretical and empirical research. We will present in this section some examples (of course, we will refer to work to whom we are familiar and will inevitably omit number of cases), and we hope that these examples could help to understand more precisely the Bakhtinian perspective and how it could be convoked in research.

In the emergent historico-cultural perspective on mathematics education (Radford, 2011, 2018; Roth and Radford, 2011), element of the Bakhtinian philosophy are discussed in order to think about the subjectivity of the learners and the very concept of sociability within the classroom. Inspired by Vygotsky, this point of view pleads for a non-mentalist conception of the mind. Opposed to rationalism and idealism, it proposes a sensitive and historical conception. On the one hand, it is sensitive, rooted in the body, senses and affect. Body, perception, gestures and signs are considered as constituent parts of the

mind. On the other hand, it is historical, rooted in culture, history and language. We shall then speak about the mind as a *praxis cogitans* (Radford, 2011).

Regarding mathematical objects, this perspective suggests that objects are "historically generated during the mathematical activity of individuals" and constitute "fixed patterns of reflective activities anchored in the changing world of social practice" (*id.*, p. 7, free translation). In other words, individual activities constitute the genetic root of the abstract object, which contains varied expressive dimensions; rational, aesthetic and functional aspects related to culture.

From this perspective, learning cannot be understood as merely a personal process of knowledge construction or reconstruction. Rather, learning results from our contact with our environment's cultural artefacts and social interactions. It is "the perceived which comes to light in the intention, which expresses itself in the sign or in the action mediated by the artefact during the sensorial practical activity […] something likely to be converted in a reproducible action, which meaning aims at this cultural eidetic pattern which is the abstract object itself" (*ibid.*).

Thus, learning mathematics, as it could be called cultural objectivation, is not simply learning "to do" mathematics (even less to solve mathematical problems), but rather "to be-in-mathematics", the mathematical activity being nothing else than a way "to be-with-others" (Radford, 2012). This is where the important ethic thematic of the theory settles down. As such, Radford (2008, 2012), through Bakhtin (and Levinas), insists on the fact that subjectivity "begins" in the ethical relation to the Other, "is" as a responsibility to the Other. Ethic here is not taken in as "satellite" elements of human existence; it is rather the central and the determinant field of reflections. As Bakhtin would say, "extracted from the interactive context which puts in relation the I, the Other, and the World, the subject succumbs to solipsism. At this moment, the subject loses its grip, becomes empty, arrogant, degenerates and dies" (1978/1997, p. 40).

From this perspective, history of mathematics in the context of mathematics education can take special meanings (see Guillemette (2015) for more elements of reflection about that). Indeed, history of mathematics offers meeting opportunities with ways to do and to be radically different in mathematics. Articulating the problem of learning with the question of Alterity, History could bring particular experiences of otherness. Thus, attention is not on an individual with personal possibilities of emancipation, but rather on the possibility for learners to discover new ways of being-in-mathematics, to open, with others, the realm of possibilities in mathematics.

Sfard's sociocultural approach in mathematics education and the concept of "commognition" (2001, 2008) also convoke elements of Bakhtin's philosophy. The idea here is to define "communication" which will become the central core of this perspective. For instance, Sfard and Keiren (2001) argue, with references to Bakhtin, that communication takes place between people, but it can also be an interaction between a person and herself, more often than not our thoughts take the form of an inner dialogue. This perspective claim, drawing on Bakhtin that communication is a process in which "any particular action always means addressing somebody or reacting to somebody's former utterances, or both" (*id.*, p. 58).

The authors speak about reactive and proactive (response-inviting) utterances, thus distinguishing between the two types of speaker's meta-discursive intentions: the wish to react to a previous speech or the wish to evoke a response in another interlocutor.

Therefore, when performing, for instance, data analyses, the idea is to perceive consecutive utterances in a discourse as endowed with invisible arrows that relate them to other utterances—those which have already been pronounced and those which are yet to come. As Sfard and Keiren explain, "these arrows are our metaphor for a speaker's meta-level intentions, communicated indirectly. By addressing her partners, the speaker lets them understand that she is interested in an interaction. The organization of these invisible arrows in a conversation often reveals certain regularities […] interaction analysis is performed with the help of a diagram in which the imaginary arrows […] are made visible" (*ibid.*).

In order to think about the process of learning mathematics, this discursive approach claim that there is a dialogical aspect of the mind, and that, at both individual and society levels, thinking, like a conversation between two people, "involves turn-taking, asking questions and giving answers, and building each new utterance—whether audible or silent, whether in words or in other symbols—on previous ones in such a manner that all are interconnected in an essential way" (Sfard, 2000, p. 299). These are the basic arguments deployed here to understand "thinking as communicating" (Sfard, 2008).

A third example can be found with the work of Barwell (2014, 2016) who has convoked a Bakhtinian perspective when analysing multilinguistic classrooms in mathematics. Attention to the dialogical interactions within the classroom has permitted to observe different speeches genre when teachers engage learners in mathematical activities. For instance, Barwell show importance of what he calls formal and informal speeches within this context, and how teachers answer (or sometimes didn't notice) to informal demands perceived from a dialogical perspective. These informal demands have, particularly here within the context multilinguistic classroom, important consequence concerning the pedagogical intervention of the teachers and classroom interactions.

A last example is the work of Gerovsky (2010, 2012) who tackles gender issues and problems related to social justice in mathematics education. Thinking with Bakthin about speech genre in the classroom, she observes that there are dominant voices and also marginal voices that have a hard time to be heard. The perspective developed by Gerovsky carries critical aspects by bringing into focus fragile, marginal or in-minority ways of being-in-mathematics, often suggesting social and political demands. This perspective also suggests that there is no ideologically neutral knowledge and that all acts of knowing are embedded in an ethical problem for which we need to develop our sensitivity.

4 Bakhtin and the field of history of mathematics in mathematics education

Closer to our concerns, Bakhtin has also been summoned more precisely in our field of research. Specifically, a Bakhtinian perspective has helped to think about what it means for the students or the pupils to meet history of mathematics or elements related to history of mathematics, but also helped to understand history of mathematics itself by giving insights about what to look for and to discuss about texts within history and to develop way to discuss about a historical text with the learners. Two examples will be quickly discussed in this section.

For Radford, Furinghetti and Katz (2007), the particular meaning attributed to mathematical objects is circumscribed to the limits of our own experience. This limit can

only be crossed by the encounter with a foreign form of understanding, as Bakhtin would say "A meaning only reveals its depths once it has encountered and come into contact with another, foreign meaning: they engage in a kind of dialogue, which surmounts the closeness and one-sidedness of these particular meanings" (Bakhtin, 1986, cited in Radford, Furinghetti and Katz, 2007, p. 108). In this sense, history of mathematics is a possible place where it is possible to overcome the peculiarity of our own understanding of mathematical objects limited to our personal experiences. It "history erects itself as the place where we can surmount the one-sidedness of our particular meanings; it is a place to enter into a dialogue with others, and with the historical conceptual products produced by the cognitive activity of those who have preceded us in the always-changing life of cultures." (*ibid.*, p. 109).

The story here appears as the background or a place that could help to stimulate introspection, or what Bakhtin would call our inner dialogue, and also to bring confrontation and critical reflection around our own conceptions and knowledge in mathematics. In this sense, Radford et al. (2000) pointed out that the history of mathematics is "a wonderful place where it is possible to reconstruct and reinterpret the past in order to open up new possibilities for future teachers" (p. 165).

It should be noted that the focus here is not on an individual experiencing personal emancipation possibilities, in a more or less sustained movement of self-reliance and self-reference, but towards the possibility for the learners to discover new ways-of-being-in-mathematics, to open, with others, the space of possibilities of the mathematical activity. Indeed, the mathematics class is perceived here as a community space, political and ethical, open to novelty and subversion (see Radford, 2006, 2008, 2011).

A second example can be found in the work of Barbin, who thinks both history itself and the work of historians in terms of a Bakthinian perspective (see Barbin, 2014), but also the interaction between voices from history of mathematics and the mathematics classroom (see Barbin, 2011).

For Barbin, history has a certain subversive potential that can be linked to the Bakhtinian critical standpoint mentioned above. One of the major roles that history could play in the scientific and educational world is that it can dethrone styles and worldviews that enjoy a traditional or official status. The idea here is to look at the place of the Other within historical text and dialogical interaction in the writings of mathematicians from the past. She emphasises the necessity to read the author as somebody explaining something to somebody else, and that both are holding a specific and to-be-described position in the ongoing dialogue.

In this sense, she argues that: "An original source has to be read as a rejoinder in a dialogue. What dialogue? Firstly, it is a dialogue between author and his or her contemporaries. To take dialogism into account is a good means for pupils to understand that mathematics is not a 'long quiet way', but that mathematics is a struggle for spirit. We have to read the author as somebody explaining something to somebody else. So, it is also a means to establish a second dialogue, a dialogue between the teacher and his or her students. In this case, an original source could be filled also with utterances between the teacher and the students" (Barbin, 2011, p. 15).

The purpose, in the perspective developed by Barbin, is especially not to separate the dialogue taken place within history and the one taken place within the mathematics classroom. On the contrary, the idea is to put forward the passage between the two spheres

of speech communication. This, in order to understand better the role and the implication of introducing history in the classroom, but also, for the classroom itself, to maintain the polyphonic aspect of the interactions that are taken place in order to achieve pedagogical goals such as reorientation (*dépaysement* in French) (see Barbin, 1997).

5 Bakhtin and research itself within the field of history of mathematics in mathematics education

In this section, we will try to show how Bakhtin's thought can support reflection, not on history, mathematics or mathematics education *per se*, as it has been discussed above, but on our very ways of being in research and doing research within or field.

Through our reading of Bakhtin's works, we will propose new ways of thinking about the role and position of the researcher and the participants, but also that of history of mathematics, in such a context of research. This will include ways to appreciate and account for the dialogical interaction between researchers, participants (teachers and students) and history of mathematics (understood as a third-party interlocutor) that this perspective suggests. To support our point, and to reach more "applied" or "practical" issues, some examples of interaction between researchers and participants from our own research concerning mathematics teachers' education will be discussed within this dialogical perspective.

5.1 A reflection around research and history

Research in our field, or a certain important part of it, is characterised by the objective of understanding better the role and the potential of history of mathematics within mathematics education.

What could it mean to think about research itself in a dialogical way? The idea is to think about research itself as an opportunity to get ourselves in a dialogue with the participants and elements of the history of mathematics. In this sense, researchers are creators of events, of course, related to specific objects of research. The objective here is to give back to the community the voices of the participants that are confronted, and in dialogue, with elements of history of mathematics.

This said, for Bakhtin, any utterance of a speech cannot be understood without considering the sphere of communication in which it inscribes itself. It means that each voice that is presented and analysed in research cannot be presented and analysed without any reference to order voices that have made it possible and to other voices that are possible because of it. This is why a proper Bakhtinian perspective in research carries this injunction to give not only the voices to participants, but to present these voices in their dialogical interaction.

This said, what is interesting in the Bakhtinian perspective regarding our own field of research is that this perspective can include voices that cannot manifest themselves, voices from the past. Indeed, theses voices coming from the history of mathematics cannot manifest themselves in the classroom, but they can be summoned by the means of interpretation. Interpretation is from the Latin *interpretatio*; itself built on *inter*, which is between, and *pretare*, which is close, *praesto*, what is present. Here, voices from the past have to join voices in the present by the means of something in-between. But, again, the movements of consciousness that bring these voices in the present have themselves to be

understood here in their dialogical interaction. This is a very delicate aspect to take in consideration, because the pedagogical implications are very important.

As Barbin put it many times, having in head what has been brought in section 4 above), history could bring a "culture shock" in "immediately immersing the history of mathematics in history itself" (2012, p. 552, our translation). Therefore, the objective is not to read historical texts simply related to our (modern) knowledge, but rather in the context of the one who wrote them. This is where history becomes a source of "epistemological astonishment" by questioning knowledge and procedures typically taken as "self-evident" (*ibid.*).

We join here the position of Jahnke (1994; 2014) around the idea of learning-to-listen with the history of mathematics. For Jahnke, starting from a hermeneutic approach, the reading of a historical text in mathematics brings two interrelated forms of reflections. Firstly, there is the experience of "dissonance" or "alienation", just like the feeling of being in a foreign country. The students learn something about their own mathematics by experiencing and "reflecting on the contrast between modern concepts and their historical counterparts" (Fried et al., p. 218). This reflection goes in both directions, so that the students deepen both their understanding of history and of their own set of modern conceptualizations regarding mathematics and mathematical objects. Secondly, the task is now to think about the situation of the mathematicians living in the past. This task requires being able to argue from the assumptions of these persons, to use their symbols and methods. This poses completely new demands on the students' abilities in their mathematical activities.

According to the hermeneutic perspective, a text consists in the merging of different horizons, the horizon of the reader and the horizon of the author. This means, of courses, that different readers embedded in their different backgrounds arrive at different interpretations. As Jahnke would say, the texts and historical artefacts are here the problems and the things that students are confronted with.

Yet, we would like to say with Bakhtin that these interpretations arise in the classroom within an already ongoing dialogue on mathematics, and that this dialogue could include the teacher and also the researcher. The concepts of dialogism and polyphony, borrowed from Bakhtin, can here provide the necessary means to think about the elaboration of a description of what happened objectively and that includes us as a teacher/researcher in its description, and also voices from the past.

5.2 The example of *Direct cinema*

Within the oral communication that took place during ESU-8, we brought the example of *Direct Cinema* in order to give some insights about our understanding of this Bakhtinian perspective on research. This example seems for us particularly relevant as one can retrieve an investigation that is going on, researchers (here the filmmakers) that try to understand a community (here the protagonists) trying to make sense of their past. In this context, the dialogical principle and the concept of polyphony can be illustrated concretely.

In every encyclopaedia related to cinema, there is a small part of it dedicated to *Direct Cinema*, a way of doing documentaries that arise during the1960s in the province of Québec in Canada. Filmmakers from this movement had the idea to produce a very special kind of documentary. They were searching to present some kind of fiction related to real

events that could include themselves. Paradoxically, they were looking for a more authentic way of investigating phenomena with this medium. More concretely, the idea was about to go within the community that is concerned with the phenomena that interest the filmmakers. The film proposes to show this meeting and how the interaction with the members of the community has created a certain event that could have brought new ways of thinking about the phenomena. There is no "God's view" or contemplative perspective, but a real engagement by the filmmakers in their object of investigation.

The movie *Pour la suite du monde* by Michel Brault and Pierre Perrault (1962) is a perfect example of a movie from this *Direct Cinema* movement. In this film, the filmmakers are interested in a community that are living on an isolated island on the St-Laurent river in the province of Québec in Canada, called *Ile aux Coudres*. This more or less isolated community has conserved a kind of ancient ways of living and traditions from the people of Québec and are confronted, by that time, with modern ways of living. They went there with their camera and tried to create something with the community. As the spectator understands it when seeing the movie, the filmmakers proposed to the inhabitants of the island to go fishing for the beluga whale. At that time, this particular fishing activity has been abandoned 40 years ago. The camera then follows the members of the community in his quest to retrieve the way to organise this fishing for the beluga whale that requires special technique and competencies.

What the film provides is a series of conversation between the members of the community, as well as decisions and actions that they are taking and doing together. The camera is present during these truly realised events, but the filmmakers do not interact explicitly. The result is that we have a tissue of dialogical interactions that include the members of the community, the filmmakers (that implicitly influencing the events and by controlling the camera) and the voices from the past that arisen from artefacts related to the ancient fishing activity. Soon, in the film, tensions emerged from these interactions, a very polyphonic dimension appears as progressive, conservative or pragmatic, for instance, speeches and ways-of-being reveals themselves. It is by revealing these tensions and by finding a way to make accessible the dialogical interaction that the filmmakers succeeded in their quest to describe the reality of this special community.

5.3 To be and to do research in a Bakhtinian perspective

This little example of *Direct Cinema* can help to understand more concretely the Bakhtinian perspective. We will now describe how the perspective was deployed our research recently. From this description, we will try to highlights some final reflections related to the pertinence of it in our field and ways to pragmatically conduct research in this sense.

In this study (see Guillemette 2017, 2018), we were searching to describe the *dépaysement épistémologique* lived by prospective teachers engaged in the reading of historical texts during a history of mathematics courses. Six participants were recruited in this study. Seven activities consisting in the readings of historical texts were experienced:
  - A'hmosè: Rhind Papyrus, problem 24
  - Euclid: Elements, proposition 14, book 2
  - Archimedes: The Quadrature of the Parabola
  - Al-Khwarizmi: The Compendious Book on Calculation by Completion and Balancing (Al-kitāb al-mukhtaṣarfīḥisāb al-ğabrwa'l-muqābala), types 4-5

- Chuquet: Tripartys en sciences des nombres, problem 166
- Roberval: Observations sur la composition des mouvements et sur le moyen de trouver les touchantes des lignes courbes, problem 1
- Fermat: Méthode pour la recherche du minimum et du maximum, problems 1 to 5

Phenomenology, the dialogical principle and the concept of polyphony help us to develop our methodological framework. Inhabited by the comprehensive and critical perspective that carried these elements in human sciences, the study proposes a description of the lived experience of *dépaysement épistémologique* that takes the form of a polyphonic narration.

These reading activities were conducted following Fried's (2007, 2008) recommendations. For this author, just like many others, the reading of historical texts appears to be the preferred approach when using history of mathematics in order to create this *dépaysement épistémologique*, the very meeting with mathematicians from the past.

Video recordings of classroom activities, individual interviews and a group interview were conducted and provide the data. For video recordings, analysis allowed us to describe the learning process that took place in the classroom. The individual interviews dealt with the experience of the course, the experience of the readings and the experience of *dépaysement épistémologique*.

The polyphonic novel was then constructed from extracts of the interview group and enhanced by video recordings and individual interviews previous analysis phases.

More precisely, in order to obtain this polyphonic novel, the first step was to construct the transcript of the group interview with care. Then, several attentive readings of the transcript were made. These readings have revealed some extracts of dialogue containing rich and profound reflections in relation to the lived experience of the participants. Twelve extracts of the transcript were selected. Thereafter, a careful reading of each of these extracts was made again and a list of various topics, thematics, reflections or statements were created for each of these extracts. The twelve extracts were then systematically treated individually. For each of them, four writing phases succeeded each other.

The first step of writing was to rework the raw extract from the transcription of the dialogue. The dialogue was then shaped so as to make it more readable with the addition of paragraphs and spacing.

The second writing step was to complete the extract, with the addition of information on the participants. These additions allowed to "defend" each participant in the dialogue and to refine and highlight their thoughts and appreciative orientations. Taking the form of paragraphs inserted into the dialogue, these additions allow us to position ourselves author/researcher as the agent of the participants, as their spokesman. These intercessions were both fuelled and justified by the descriptions of reading activities and the specific descriptions of the experience of the participants obtained during previous phases of analysis.

In the third step of writing, personal reflections were added. It was to be heard more as an author/researcher in the narrative. Usually at the beginning of the extract, one or more paragraphs were added. These provided space to express our thoughts that were emerging at the time of writing.

The fourth and final step of writing was to refine the narrative by emphasizing the theme of the extract and the polyphonic style exercised.

These four writing steps were repeated for each of the twelve extracts released initially. These were then combined to form the final polyphonic novel describing the *dépaysement épistémologique* experienced by future teachers of mathematics. This narration of the collective experience takes its density from fine description of each character/participants from previous analyses. It has led to the emergence of tensions, viewpoints moving away and approaching each other, viewpoints that overlap and influence each other.

The description provides multiple looks, which, in tension, carries fruitful discourses on the lived experience of participants. As Bakhtin put it in its dialogical critic explained above, it is in the tension between discourses coming from different spheres of communication, different ideological horizons and different aesthetic spaces that one could grasp the reality of human life.

Globally, the form of a polyphonic narration for this description is a methodological response to an epistemological challenge that underpinned this research. Indeed, this discursive strategy allows the production of a description that, first, can respect the phenomenological requirement and stringency to keep alive the subjectivity of the participants without objectivizing it in any manner and, second, embrace a conception of teaching and learning in mathematics education that claims that learning is necessarily "learning-with-others" (Radford 2011, 2013).

Yet, this study cannot provide any clue concerning the way one could provoke "systematically" *dépaysement épistémologique* in his classroom, and above all, in the same way that happened in this particular study or in any "positive" way. This study had much humbler objectives, by trying, from an empirical position, not to "confirm" or "infirm" theoretical considerations around the introduction of history of mathematics in the classroom, but to enrich and deepen them by a reflection that is emerging from the contact with the participants.

In the next excerpts from video analysis, we can see how participants' mathematical activity interacts with Fermat's minima and maxima method, and how it is interpreted. The excerpt concern Fermat's general description of his method and the first example given. He finds the maximum or minimum of a given term $f(x)$ by "adequating" (which means approximately equal) the two expressions $f(x)$ and $f(x + e)$, reducing and clearing remaining "e-terms". The first example (divide a line AC at a point E such that rectangle ACE area is maximized) involve a term in the form of $f(x) = bx - x^2$.

A team of three students (Martha, Aliocha and Ninotchka) are engaged in this reading:
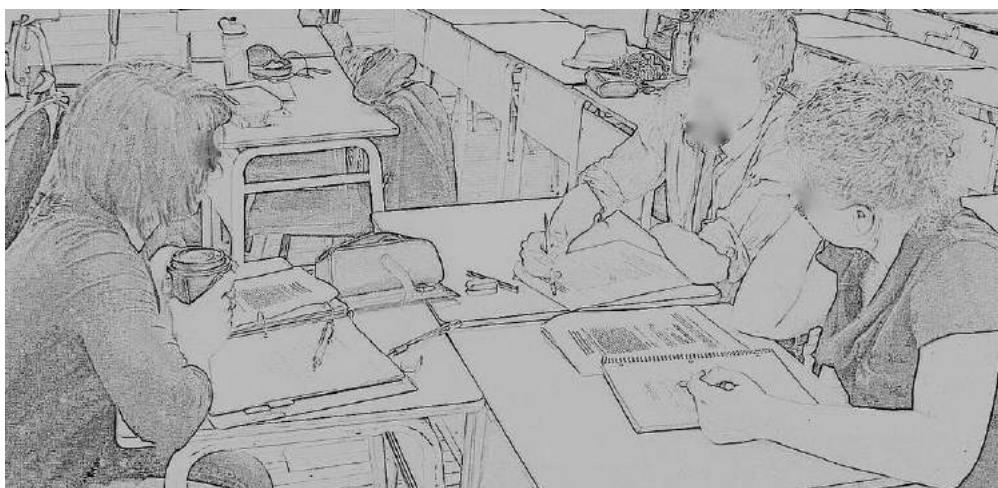

Fig. 5.1: Martha, Aliocha and Ninotchka reading Fermat.

Martha is saying that e is a very small value.

Aliocha is trying to reconciliate Fermat's method and the basic elements of modern calculus. He asks, "if adequating means to subtract the terms?". Ninotchka answers that "adequating means simply to equalise".

Aliocha then asks how she relates Fermat to modern calculus. Ninotchka shows his calculations and Aliocha concludes that their reasoning is equivalent.

After few moments, Martha points out that Fermat removes e. Aliocha indicates that "e is almost 0, so the multiplication by e also gives almost 0". Martha asks herself whether the reader should "decide on the value of e". Aliocha replied that yes. Martha emphasizes that there is something missing in the reasoning. Aliocha asks why Fermat is using symbol of inequality, and concludes that adequating means to reduce to the minimum.

With Fermat's method, which participates of the beginnings of calculus formalization, participants are confronted with an exploratory reasoning showing genuine and foreign way of dealing with these mathematical objects and procedures. The encounter with this fragmented and emerging mathematical discourse brings an impression of distance to the participants. They cannot do nothing else but to convoke their own modern modalities of expression (especially here representation of algebraic quantities) in order to enter in dialogue with Fermat's utterances (reflecting on geometrical magnitudes), responding themselves to other utterances (especially here to Diophantus around the notion of *adaequalitas*). This distance, which is here a temporal one, as well as the polyphonic aspect of the text itself and of the classroom organisation, emphasis for the participants how individual activities, mediated by the sociocultural context, constitute the genetic root of the mathematical activity, containing rational, aesthetic and functional expressive dimensions. In our reading, this is where history of mathematics, with the experience that it provides, and the dialogue that it forces, seems to bring the most for pre-service teacher's reflection on mathematics and mathematics education.

The methodological framework inspired here by elements of Bakhtin's philosophy has helped us to get in a dialogue with the participants and the mathematicians of the past. It has also helped to describe how together the researcher, the participants, the voices from the past and the actual mathematical culture come into dialogical interactions.

On the one hand, from a research perspective, the idea is to give access to this share meaning, to grasp the world in common that emerges from the introduction of history into the mathematics classroom. From this perspective there is a need to report the multiplicity of experiences. This doesn't mean to report, side-by-side, each of the participants' experiences, but to really provide the "common world". This common world has nothing to do with the consensus that could emerge around a certain understanding of history or mathematics or their relations, but is constituted of tensions emerging from dialogical interactions between participants researchers and voices from the history of mathematics.

It is by revealing these tensions and by finding a way to make accessible this dialogical interaction that a study could eventually succeed in its objective to describe what it means for the students or the pupils to meet history of mathematics or elements related to history of mathematics. Again, the challenge here is to find ways to "write" and "present" a description that could introduce these dialogical interactions to the research community.

On the other hand, from a more pedagogical perspective, the idea could be to think about the sphere of speech communication related to Fermat and his contemporaries. Without referring explicitly to Bakhtin with the students (but why not?), it could be to

interesting to investigate more profoundly the link between the participants of this sphere of speech communication in which Fermat is inscribed. Asking to whom Fermat is writing? In response to what? How Fermat's methods, concepts and idea have been received? Which attitude does he enact and which attitudes are expected by Fermat from his reader? The reading of a historical text such as Fermat's text on minima and maxima method could then become an exploratory and interpretative activity within mathematics that could generate element of a debate (such as Barbin 2011 suggests) both at the level of Fermat and his contemporaries, but also at the level of the classroom when students enter in a dialogue with Fermat having themselves an active-responsive attitude related to the mathematics that are proposed by Fermat. Indeed, from a Bakhtinian perspective, students have expectations from a mathematical text, could be in terms of rigor, organisation, tone, clarity, utility, generalizability, notation, etc. and those expectation could lead to disillusion, agreement, disagreement, consideration, reconsideration, etc.

From this Bakhtinian perspective on the mathematics classroom and on this particular type of activity referring to history, one could perceive an ongoing dialogue in the classroom. Pedagogical goals could emerge from this perspective and be pursued. For instance, reflection on meta-issues in mathematics such as the historicity of concepts, methods, definitions and notion, historicity of notation and rigor, mechanisms underlying the discovery or development of mathematical objects or procedures, intrinsic and extrinsic forces that drive mathematicians, links between the development of these concepts and the development of societies and cultures, etc. Moreover, these pedagogical goals could be pursued by emphasising, promoting and maintaining the polyphonic aspect of the context in which the students are subsumed, both at the level of history and at the level of the classroom.

6 Conclusion

We hope that his paper could help researchers and teachers to reflect upon ways of conducting research in our field and to think about the very meaning of introducing history in the mathematics' classroom. In our quest to develop reflection around conceptual and theoretical elements related to history in mathematics education, this paper has tried to draw attention on the dialogical principle and the concept of polyphony that has been developed by Bakhtin and his collaborators. We have argued that elements of Bakhtin philosophy could support reflection, not only on history, mathematics or mathematics education, but on our very ways of being in research and doing research within or field and to think about the classrooms in such context.

We believe that the Bakhtinian perspective deserves to be explored and discussed within our field. The work of Bakhtin presents an important radicalness regarding its way of thinking about the human subjectivity and the way in which it is inserted in the historical, social and cultural world. We think that the elucidation and the development of these positions could, in many ways, help investigations and intervention related to the introduction of history of mathematics in mathematics education.

REFERENCES

Bakhtin, M. (1977) [Published by the name of V. N. Volochinov]. *Le marxisme et la philosophie du langage* [Marxism and the philosophy of language]. Paris: Minuit. (Originally published in 1929)

Bakhtin, M. (1982). *L'œuvre de François Rabelais et la culture populaire au Moyen-Âge et sous la Renaissance* [Rabelais and his world]. Paris: Gallimard. (Originally published in 1970)

Bakhtin, M. (1986). *Speech genres and other late essays*. Austin: University of Texas Press.

Bakhtin, M. (1997). *Esthétique et théorie du roman* [Questions of literature and aesthetics]. Paris: Gallimard. (Originally published in 1978)

Bakhtin, M. (1998). *La poétique de Dostoïevski* [Problems of Dostoevsky's poetics]. Paris: Seuil. (Originally published in 1963)

Bakhtin, M. (2003). *Pour unephilosophie de l'acte* [Toward a philosophy of the act]. Lausanne: L'Âge d'Homme. (Originally published in 1986)

Barbin, É. (1997). Histoire et enseignement des mathématiques: pourquoi? comment? *Bulletin de l'AMQ (Association Mathématique du Québec)*, *37*(1) , 20-25.

Barbin, E. (2011). Dialogism in mathematical writing: historical, philosophical and pedagogical issues. In V. Katz & C. Tzanakis (Eds.), *MAA notes, vol. 78* (pp. 9-16). Washington: MAA.

Barbin, É. (2012). L'histoire des mathématiques dans la formation: une perspective historique. In J.-L. Dorier & S. Coutat (Eds.), *Acte du congrès de l'EMF 2012* (pp. 546-554). Geneva: EMF.

Barbin, É. (2014). Une approche bakhtinienne des textes d'histoire des sciences. In A. L. Rey (Ed.), *Méthode et histoire: Quelle histoire font les historiens des sciences et des techniques ?* (pp. 217-232). Paris: Classiques Garnier.

Barwell, R. (2014) Centripetal and centrifugal language forces in one elementary school second language mathematics classroom. *ZDM*, *46*(6) 911-922

Barwell, R. (2016). Formal and informal mathematical discourses: Bakhtin and Vygotsky, dialogue and dialectic. *Educational Studies in Mathematics*, *92*(3), 331-345.

Brault, M., & Perrault, P. (1962). *Pour la suite du monde*. Canada: Office National du Film du Canada. [available on line at: http://www.onf.ca/film/pour_la_suite_du_monde/]

Fried, M. N., Guillemette, D., & Jahnke, H. N. (2016). Theoretical and/or conceptual frameworks for integrating history in mathematics education. In L.Radford, F. Furinghetti & T. Hausberger (Eds.), *Proceedings of the 2016 ICME Satellite Meeting of the International Study Group on the Relations Between the History and Pedagogy of Mathematics* (pp. 211-230), Montpellier: IREM de Montpellier.

Gerofsky, S. (2010). The impossibility of "real-life" word problems (according to Bakhtin, Lacan, Zizek and Baudrillard). *Discourse*, *31*(1), 61-74.

Gerofsky, S. (2012). Democratizing "big ideas" of mathematics through multimodality: using gesture, movement, sound and narrative as non-algebraic modalities for learning about functions. *Hellenic Mathematical Society: International Journal for Mathematics in Education*, *4*, 145-150.

Guillemette, D. (2015). Rôle de l'histoire des mathématiques dans l'enseignement-apprentissage des mathématiques : le point de vue socioculturel. In C. Sabena & B. Di Paola (Eds.), *Actes de la 67ᵉ réunion de la Commission internationale pour l'étude et l'amélioration de l'enseignement des mathématiques (CIEAEM 67) - Enseigner et apprendre les mathématiques :ressources et obstacles* (pp. 607-618), Aosta, Italie: CIEAEM.

Guillemette, D. (2017). History of mathematics in secondary school teachers' training: towards a nonviolent mathematics education. *Educational Studies in Mathematics*, *96*(3), 349-365.

Jahnke H. N. (1994). The historical dimension of mathematical understanding: Objectifying the Subjective. In J. P. da Ponte & J. F. Mato (Eds.), *Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education (vol. 1)* (pp. 139-156). Lisbon: University of Lisbon.

Jahnke H. N. (2014). History in mathematics education: a hermeneutic approach. In M. N. Fried & T. Dreyfus (Eds.), *Mathematics & mathematics education: searching for common ground* (pp. 75-88), Dordrecht: Springer.

Radford, L. (2006). Communication, apprentissage et formation du je-communautaire. In B. D'Amore & S. Sbaragli (Eds.), *Proceedings of the 20th National Italien Conference "Incontri con la Matematica"* (pp. 65-72). Bologna: University of Bologna.

Radford, L. (2008). The ethics of being and knowing: towards a cultural theory of learning. In L. Radford, G. Schubring& F. Seeger (Eds.), *Semiotics in mathematics education: epistemology, history, classroom and culture* (pp. 215-234). Rotterdam: Sense Publishers.

Radford, L. (2011). Vers une théorie socioculturelle de l'enseignement-apprentissage: la théorie de l'Objectivation. *Éléments*, *1*, 1-27.

Radford, L., Bartolini Bussi, M. G., Bekken, O., Boero, P., Dorier, J.-L., Katz, V. & Vasco, C. (2000). Historical formation and student understanding of mathematics. In J. Fauvel & J. van Maanen (Eds.), *History in mathematics education: the ICMI study* (pp. 143-167). Dordrecht: Kluwer Academic Publishers.

Radford, L., Furinghetti, F., & Katz, V. (2007). Introduction: the topos of meaning or the encounter between past and present. *Educational Studies in Mathematics*, *66*(1), 107-110.

Roth, W.-M., & Radford, L. (2011). *A cultural historical perspective on teaching and learning*. Rotterdam: Sense.

Sabo, K., & Nielsen, G. M. (1984). Critique dialogique et postmodernisme. *Études françaises*, *20*(1), 74-86.

Sfard, A. (2000). Steering (dis)course between metaphors and rigor: using focal analysis to investigate an emergence of mathematical objects. *Journal for Research in Mathematics Education*, *31*(3), 296-327.

Sfard, A. (2001). There is more to discourse than meets the ears: looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics*, *46*(1-3), 13-47.

Sfard, A. (2008). *Thinking as Communicating*. Cambridge: Cambridge University Press.

Sfard, A., & Kieran, C. (2001). Cognition as communication: rethinking learning-by-talking through multi-faceted analysis of students' mathematical interaction. *Mind, culture, and activity*, *8*(1), 42-76.