USING ARTIFACTS AND DYNAMIC GEOMETRY SOFTWARE IN PRIMARY SCHOOL INSPIRED BY MONTESSORI METHOD

Roberto CAPONE[1,3], Maria Rosaria DEL SORBO[2,4], Virginia NINNI[1,5]

[1]University of Salerno, Fisciano (SA), Italy,
[2]IS “Leonardo da Vinci”, Poggiomarino (NA), Italy,
[3]rcapone@unisa.it, [4]marodel@gmail.com, [5]vninni@gmail.com

ABSTRACT

Latest interactive and dynamic geometry software introduces new practices in Mathematics education, adding virtual 2D or 3D animated graphs to traditional artifacts and compass-and-straightedge constructions. This work presents the results of a teaching experiment in primary school, realized using Montessori method and setting math education through perceptual-sensory inputs.

Cheap material was exploited to introduce pupils to one of the most important deductive process: the geometric proof. An all brand-new artifact was realized in order to explain Pythagorean Theorem using a hydro-mechanical system. The proof was performed starting from practice, direct observation by real and virtual tools. Pupils used GeoGebra, a dynamic geometry freeware: they built shapes, determined areas and verified the equivalence. Pupils repeated the proof using crayons, producing pictures full of creativity. Finally, we show a comparison between new technological tools and traditional real tools. Teacher’s role was to involve students in an educational project with real and virtual tools.

1 Introduction

Traditional mathematics education throughout the world, especially in the early historical phases, was based on a set of tools borrowed from the world of practical experience (Lave, 1988; Gravemeijer et al, 2013). More so nowadays, in the era of fastest and most pervasive technological advance, we can observe more and more complex relationships between mathematics as a purely conceptual discipline and mathematics as an applicative activity (Blum and Niss, 1991). This ambiguity emerges in expressions like “mathematical models”, referred to physical models of mathematical concepts, e.g., reproductions of plane and solid geometric figures made of cardboard, wood, plastic and models of surfaces of higher order that make it possible to materialize abstract concepts of mathematics (Ahmed ed al. 2004; Bartolini Bussi, 1996; Hershkowitz et al., 1990).At the beginning of the last century this trend in mathematics education was well in tune with the development of active methods in education, supported in the same period by John Dewey (Dewey, 1938). Afterwards, the active involvement of students was supported within a laboratory setting, with hands-on approach and exploiting Information and Communications Technology (ICT). This experimental approach, where exploration plays a major role, seems appealing for students who quite often find the evidence offered by experiments much more compelling than a rigorous proof and are bored by the request to produce mathematical arguments. On one side, the experimental approach is suspected to obstacle the development of mathematical styles of reasoning. On the other side, the use of technology suggests many motivations to sustain the experimental approach, as seen in many experiences throughout history. Moreover, there are many researches producing formal results inspired by trialing, conjectures suggested by experiments, descriptions of algorithms, and software for mathematical exploration (Laborde et al. 2006; Capone et al.

2018).

Latest interactive and dynamic geometry software are nowadays introducing new practices in math's education, supporting traditional artifacts (Verillon and Rabardell, 1995) and compass-and-straightedge construction with virtual but very effective 2D or 3D graph, geometric shapes and animations (Arzarello et al., 2014). Nevertheless, as a result of recent researches in cognitive neurosciences (Kosslyn, 1991; Cheng and Newcombe, 2005), learning processes of mathematics and geometry happen in the same brain regions and circuits which people exploit for intuitions about space, time, approximate amount and number awareness. Language is ordinarily used while learning mathematics, but mathematical reasoning itself seems to happen in its own parts of the brain (Dehaene, 2011). On the other side, among the main features of Montessori method, there are respect for individuality of each child, hands-on experiences, skills in cooperative and manual work. Therefore, this method is one of the possible instances of Pedagogical Activism: each pupil, if properly taught, may become an active constructor of his own knowledge.

In our work, we show the results of a teaching experiment in a primary school, realized using Montessori method and setting math education through perceptual-sensory inputs, especially by touch sense and visual representations. Everyday and cheap material, e.g. colored frames, tiles, golden pearls, and many other classical tools, as “regoli”, have been exploited to create geometric shapes and to introduce pupils to one of the most important deductive process: the geometric proof.

In order to achieve this goal, an artifact was realized in order to explain Pythagorean Theorem using a hydro-mechanical system, relying on a fluid contained in three squared boxes, two built on the two perpendicular of a right-angled triangle and the third built on the hypotenuse. In this way, the artifact gives a visible proof of the equivalence of the areas. Therefore, a backtrack path was treading: the theorem was not demonstrated as in traditional proofs but starting from the opposite point of view - practice, direct observation by real and virtual tools, then, only at the end, thorough formulation of the theorem.

In addition, a specific user friendly and intuitive dynamic geometry freeware, GeoGebra, was used, eliciting enthusiasm and curiosity in the pupils: they built the shapes, determined the areas and so it was very straightforward to verify the equivalence between the squares. Furthermore, they repeated the proof of the theorem using real graphics tools, producing pictures full of creativity. Finally, we present a short comparison between technologically innovative virtual tools and traditional real tools, such as ruler and compass. Teacher's task was to motivate the students, involving them in an education project that refers to an effective integration of both real and virtual tools.

2 Principles of Montessori's Method

Maria Montessori was an Italian physician and educator who lived in Italy at the turn of the 19^{th} and 20^{th} century. She was a great innovator: she founded a new pedagogical thinking about children with mental illness and disability, up to the formulation of a method suitable for all children. Montessori's approach is founded on original principles: teachers must start from "things", concrete representations of geometrical objects. But, above all, teachers are facilitators and they must let the things themselves speak to the students.

Essential Montessori education principles include teaching child-centered, early scholarization, pupil's freedom, centrality of environment, senses education through material objects, the school is considered as a small society and teachers play the role of guides.

According to the principles of Pedagogical Activism, the synthesis of the Montessori method is the importance attributed to direct experiences. There is a revaluation of the realistic exercise, concreteness, primacy of activity and learning by doing: practice and technology are media and not targets. There is respect for individuality, hands-on experiences and skills in cooperation are considered very relevant. Each pupil, if properly taught, may become an active constructor of his own knowledge.

3 The Pythagorean Theorem

As in precious Montessori's book "Psicogeometria" (Montessori, 1934/2012), the target of our experiment was to study the potential benefit of artifacts and dynamic geometry software to get to the concept of one of the best-known topics in elementary Geometry, the Pythagorean theorem.

3.1 Project outlines

Our experiment was set in a primary school, for fifth year's pupils and in the lower level of a secondary school. Teachers worked, at first, in homogeneous groups, with parallel classes, with the same objectives. Subsequently, teachers of primary and secondary school worked in heterogeneous groups, to try to build vertical curricula.

The organization of the vertical curriculum has stimulated innovations both on the methodological and managing level of the disciplines also to facilitate connections, relationships and awareness.

3.2 Lab activities

The terms of the so-called Pythagorean Theorem were already known by Babylonian a thousand years before Pythagoras, but the famous philosopher was the first to prove it. Over 371 different proofs of the Pythagorean theorem were found and proposed in the history of Mathematics, as collected in a book in 1927 (Loomis, 1927). Our project is aimed to develop a practical approach to this fundamental theorem to investigate the impact on learning of this methodology, inspired by Montessori's ideas: it is fundamental to teach mathematics first of all through perceptive-sensorial stimuli, especially through the hands, because the cerebral areas that allow us movements are very close to those that make us perceive the geometric shapes and the approximate quantities.

The explanation of the Pythagorean theorem usually starts with the statement, continues with the demonstration and then the applications follow. Montessori's concept, on the other hand, is different and very simple: we need to start from "things", that is, from the concrete representations of geometrical objects. Here is her quote: "Was it not from the things that the first surveyors drew their knowledge? Were not correspondences and relations between things, which stimulated some active and interested mind to formulate axioms and therefore theorems?" "The way a concept has been understood for the first time by human beings is the natural way to present that concept to children".

In the following subsections we will describe 3 lab activities, through which we conducted our experiment in Montessori's spirit.

3.2.1 The artifact

Theoretical reflections just described inspired our educational experimentation in primary school, in a fifth-grade class, aiming to study the potential benefits of artifacts, in particular a hydraulic device, for helping students to conceptualize Pythagorean Theorem through water containers properly shaped.

The artifact is realized starting from the construction of a box shaped as a right triangle, on whose sides squares were built; the square built on the hypotenuse has a hole on the outer side, to fill the device with a liquid. All the boxes are communicating with each other through pipes, so that the liquid can flow from the big square to the small ones at the same time and vice versa (figure 3.1). The entire device is built on a circular plane with a pin that allows the rotation of the artifact. The smallest perpendicular side measures 15 cm, the biggest one 20 cm and the hypotenuse 25 cm. Then, it is possible to show how all the liquid contained in the square box built on the hypotenuse can be contained exactly in the square boxes built on the perpendicular sides.

Figure 3.1: Hydraulic artifact to prove the Pythagorean Theorem

Artifact can help children to have a different view of mathematics, which is often distressing and negative; so, mathematics appears "colored" and touchable, improving their attitude towards mathematics and their mathematical skills. In this case all the channels are involved to receive the information: you learn exploiting visual memory, (therefore non-verbal visual channel), by listening (auditory channel), by reading (verbal channel) and by doing (kinesthetic channel). While using an artifact, a teacher observes how students use it and their cognitive patterns, as well as their special ways of thinking/knowing. Here is the importance of mental images in mathematics: mental images are not only passive figures inside the head, but productive mental representations that allow us to imagine something, even in the absence of perceptive stimuli and which therefore allow us to construct forms of creative thought in order to realize new forms of knowledge.

3.2.2 Crayons

After formalizing the theorem, children reproduced the construction in the maths notebook, using the squares as minimum units. This activity allowed them to have an immediate, visual and practical approach to the equivalence between the squares, as in figure 3.2.

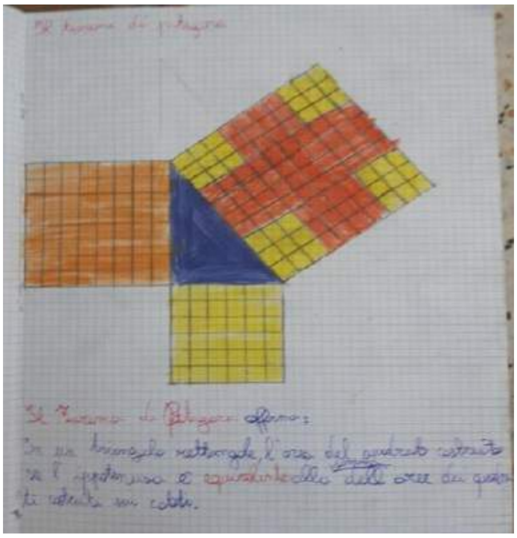


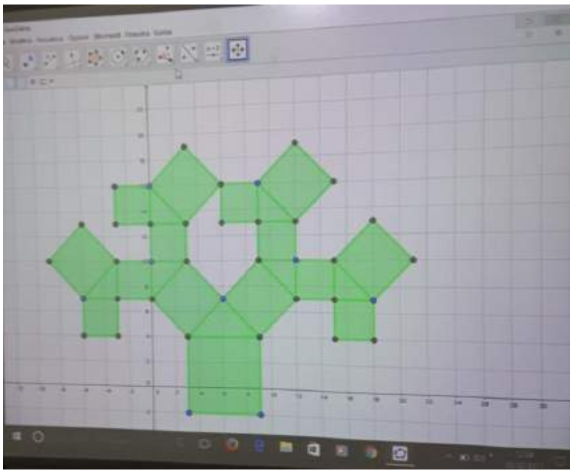
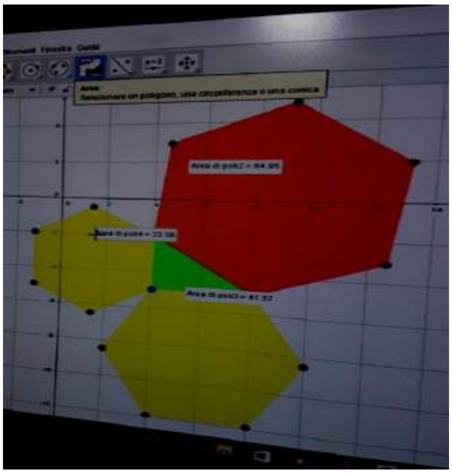
Figure 3.2: Construction of squares on the sides of a right triangle with crayons

The exercise was carried out with the construction of squares; we can certainlysay that there were no difficulties in the construction of the squares on the perpendicular sides, but it was different for the construction of the square on the hypotenuse. The students showed uncertainties about the correct position of the lines; in this sense, there was the necessity to talk about technical expertise that they will have to acquire in the following of their learning process. Thus, once the sketch was drawn and colored, the children counted the unit squares and verified the Pythagorean theorem. Moreover, this activity further simplified and facilitated the understanding of the concept of area as the measure of a surface.

3.2.3 Geogebra

Pupils, after few and short preliminary lessons inspired by practical learning by doing education methodology, got mastery of basic features and functions of GeoGebra (Zenging, 2018) so that they could begin to draw geometric shapes and right triangles and squares. Subsequently, the theorem was depicted, calculating the areas of the surfaces of the squares by GeoGebra’s specific command and directly proving the evidence. In addition, they spontaneously tried to extend the Pythagorean theorem by iterating the construction indefinitely, giving life to creative images and to apply Pythagoras’s theorem also to polygons with 3, 5 or more sides, as shown in figures 3.3 and 3.4.

Nevertheless, it was very interesting to let pupils experiment the difference between GeoGebra and compass-and-straightedge construction of geometric figures and besides, to exploit GeoGebra’s toolbox and functions for step by step constructions, which melts old- and new-fashioned geometry learning methods.

Figures 3.3 and 3.4: Example of development of ideas in Geogebra activity

3.3 Rethink and discovery

After GeoGebra experience, we draw many considerations: for example, we can assert that we detected an increasing students' attention about the possibilities of exploiting this software to design, analyze, calculate, proof. We tried to summarize and list our observations, classifying them in pros and cons.

3.3.1 Pros of Virtual

We registered a significant reduction of teaching time, because, first, pupils are very much attracted by computer's features and quickly and intuitively become used to employ active software graphical instruments. This educational process helped students to achieve a better awareness of the demonstration in much shorter time and an improvement of learning effort perception.

Thus, there was also an optimal integration between class teaching and preparation of lessons and materials: teacher essentially played the role of a guide, but pupils impressed their personal fingerprint in the development of the lessons and the results were finally original and sometimes unexpected.

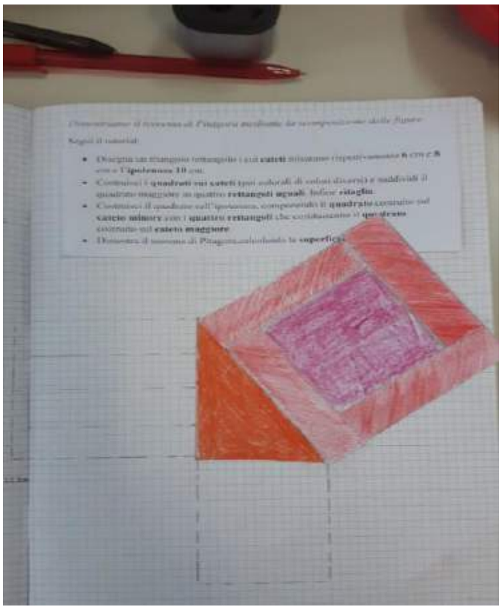
Lastly, GeoGebra's dynamical geometry tools, with its easy way to draw lines and shapes, which does not require manual ability to get perfect graphical outcomes, helped students to directly concentrate on the process and then they had the opportunity to better understand the deep meaning of geometrical constructions and relationships.

3.3.2 Cons of Virtual

Even though the overall outcomes of virtual applications were positive, as described above, students' trend is towards action without pre-thinking: it does not happen rarely that pupils fail to understand the reason for some results, typically those that are an exception to the rules, while they tend to accept uncritically the output produced by the app they are using. "Even if confidence and motivation could be enhanced by using software, the question still remains if this translates into better overall performance in the classroom. This may be a question as to how the software is implemented as to how well it enhances or deteriorates a student's learning." (Formaneck, 2013). Generally speaking, software use could generate a superficial attitude in the learner, whose confidence with virtual tools risks to make him to surrogate the necessity of learning with the ease of quickly getting a result, without minimally submitting it to an aware checking process.

3.4 Evaluation

In order to test the effectiveness of our teaching activities, we administered to pupils several worksheets, basically centered on two different strategic approaches: equidecomposability and application of inverse formula, as respectively shown in figures 3.5 and 3.6.



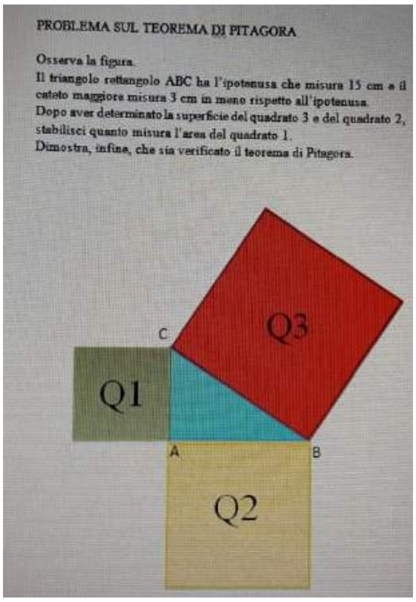


Figure 3.5 and 3.6: two examples of worksheet administered to test pupils after activities about Pythagorean theorem.

The text in figure 3.5 is: Let's demonstrate Pythagorean theorem using decomposition. Follow the tutorial: Draw a right triangle of dimensions 6, 8, 10 cm. Build the squares on the perpendicular sides and colour them with two different colours and then divide the greater sqaure in four equal rectangles. Finally cut these shapes. Build the square on the hypotenuse, composing the shapes just cut. Calculate the areas as a proof of Pytagorean theorem.

The text in figure 3.6: Problem on Pythagorean theorem. Observe the figure. The hypotenuse in a right triangle ABC is 15 cm and the greater perpendicular side is 3 cm less than hypotenuse. After have calculated the area of Q3 and Q2, calculate the area of Q1. Finally, confirm Pythagorean theorem.

The first worksheet aims to prove the Pythagorean theorem by decomposing figures. The worksheet allows the student to reflect on the statement of the theorem, to demonstrate it practically by decomposing the squares built on the perpendicular sides in two different ways, and then constructing the square on the hypotenuse using the pieces obtained cutting the smaller squares on the perpendicular sides. Despite all previous multiple activities, yet some pupils hesitated about how to combine the four rectangles and the small square (see figure 3.5). The second worksheets a more traditional geometry problem with an illustration and then it also *required reading, reasoning and calculation skills, as well as logic and the capability to find an inverse formula (figure 3.6). Often pupils read the text and the questions too quickly and with a low attention level; this is often the origin of multiple errors. However, in our strategy, pupils were pulled to analyze the text and also reasoned about the inverse formulas and the triangles with sides multiple of 3, 4 and 5 were used only for simplicity's sake, to facilitate the task of decomposing and reasoning at the beginning. Later pupils were made aware of the general case, with right triangle with sides measures different from Pythagorean triples, more difficult to draw but equally compliant the theorem.*

The evaluation table, depicted in figure 3.7, is related to the activities carried out in the learning unit and it was directly conceived according to new teaching guidelines, oriented towards an accurate evaluation of the competences achieved by the pupils. As in figure 3.7, there are descriptors that explain the different levels of competence achieved in the different tasks.

Dimensions	**Startinglevel**	**Basic level**	**Intermediate level**	**Expert level**
To know how to calculate area of the main polygons	Students don't know formulas and cannot calculate areas	Students know formula but they don't know how to apply it	Students can calculate areas of polygons	Students can calculate areas even in different situations
To know how to apply Pytagorean theorem	Students cannot apply the theorem	Students apply the theorem only if they are assisted	Students can apply the theorem autonomously	Students can apply theorem even using inverse formulas
To know how to use geometrical software	Students cannot use geometrical software	Students can use geometrical software with the help of a tutorial	Students can use the software with mastery	Students can use the software also to experiment in new contexts

Figure 3.7: Evaluation table

4 Conclusions and future work

At the end of this experiment, we stress our final observations about multiple ways to represent and prove the Pythagorean theorem using artifacts and dynamic geometry software. It was possible, first, to realize a peer education experience, with much enthusiasm and curiosity in learners. The pupils ran across arithmetic and geometry paths at the same time, enriching their knowledge and skills through a variety of activities and methodologies.

The artifact allowed them to get an immediate understanding of the theorem, thanks to the visual and original aspect given by the presence of the water. Alongside the purely practical activities, experiments have been carried out with the GeoGebra software to develop technological skill as in National Guidelines.

Students' motivation led to new discoveries, as with GeoGebra it was shown that the theorem is valid for all regular polygons and in a iterative application; specifically, this work has come across the demonstration of the validity of the theorem also with equilateral triangles and hexagons. Besides, simulations technologies were considered with new awareness, moving forward to more critical and conscious use of virtual tools. Test worksheets were indeed a way to compare all the activities done previously and draw indications about the effectiveness of teaching and learning.

Though, the time available did not allow us to investigate other important aspects, such as a study of a wider range of topics and the evolution in time and the impact on long term memorization of the concepts learned by this approach; in this regard, we hope that this work can be a starting point for future reflections and studies.

From the results achieved up to now, we had the opportunity to verify how the proposed method led to a concrete and more aware learning, to be regarded as acquisition of skills by primary and secondary students, along with the construction of new knowledge through repeated real and virtual lab experiences.

REFERENCES

Ahmed, A., Clark-Jeavons, A., & Oldknow, A. (2004). How can teaching aids improve the quality of mathematics education. *Educational Studies in Mathematics*, *56*(2-3), 313-328.

Arzarello, F., Bairral, M. A., & Danè, C. (2014). Moving from dragging to touchscreen: geometrical learning with geometric dynamic software. *Teaching Mathematics and Its Applications: International Journal of the IMA*, *33*(1), 39-51.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modeling, applications, and links to other subjects—State, trends and issues in mathematics instruction. *Educational Studies in Mathematics*, *22*(1), 37-68.

Bussi, M. G. B. (1996). Mathematical discussion and perspective drawing in primary school. *Educational studies in mathematics*, *31*(1-2), 11-41.

Capone, R., Del Regno, F., & Tortoriello, F. (2018). E-Teaching in Mathematics Education: The Teacher's Role in Online Discussion. *Journal of e-Learning and Knowledge Society, 14*(3).

Cheng, K., & Newcombe, N. S. (2005). Is there a geometric module for spatial orientation? Squaring theory and evidence. *Psychonomic bulletin & review*, *12*(1), 1-23.

Construction (Measurement and Geometry: Module 13) For teachers of Primary and Secondary Mathematics - http://www.amsi.org.au/teacher_modules/pdfs/Construction.pdf

Dehaene, S. (2011). *The number sense: How the mind creates mathematics*. New York: Oxford University Press.

Dewey J.,(1938). *Experience and Education,* Kappa Delta Pi.

Formaneck, S.D. (2013) Math Software in the Classroom: Pros, Cons and Tips for Implementation, *International Journal of Pedagogical Innovations*, *209*(1178), 1-4.

Geogebra https://www.geogebra.org/

Gravemeijer, K. P., Lehrer, R., van Oers, H. J., & Verschaffel, L. (Eds.) (2013). *Symbolizing, modeling and tool use in mathematics education*. Vol. 30. Springer Science & Business Media.

Hershkowitz R., Ben-Chaim, D., Lappan, C., Mithelmore, M., & Vinner, S. (1990). Psychological aspects of learning geometry. In P. Nesher and J. Kilpatrick (Eds.), *Mathematics and Cognition* (pp. 70–95). Cambridge: Cambridge University Press.

Kosslyn, S. M. (1991). A cognitive neuroscience of visual cognition: Further developments. In *Advances in psychology* (Vol. 80, pp. 351-381). Amsterdam:North-Holland.

Laborde, C., Kynigos, C., Hollebrands, K., & Strässer, R. (2006). Teaching and learning geometry with technology. In A. Gutiérrez, & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future*, 275-304. Rotterdam: Brill Sense

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life*. Cambridge: Cambridge University Press.

Montessori, M. (1934/2012). *Psicogeometria.* Roma: Edizioni Opera Nazionale Montessori.

Loomis E. S. (1927). *The Pythagorean Proposition*. Washington DC: The National Council of Teachers of

Mathematics.

Verillon, P., & Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of though in relation to instrumented activity. *European Journal of Psychology of Education*, *10*(1), 77-10.

Zengin, Y. (2018). Incorporating the dynamic mathematics software GeoGebra into a history of mathematics course. *International Journal of Mathematical Education in Science and Technology*, *49*(7), 1083-1098.