NIELS ABEL: '*SO MANY IDEAS …*'

A workshop on using theatre to bring episodes in the history of mathematics to life in the classroom

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ABSTRACT

This workshop explored the creative re-construction of historical events in theatrical form, and practical ways of using such plays in mathematics classroom contexts with minimal rehearsal. Participants were invited to stage cooperatively a 30-minute pre-scripted play featuring Niels Abel and contemporaries, fronted by a historian and an educator, aiming to place Abel's mathematical work in the context of his personal life. The dialogue is designed to display his courage and audacity, and the frustrations and pathos of his brief life. Poignant speeches drawn from his correspondence give insight into personal hopes and fears, publication challenges, and the intellectual excitement of creating new mathematics. Afterwards, participants were invited to critique the play, share ideas or experiences of using such theatrical tools in classroom settings, and then discuss in groups how they might go about weaving highlights from primary sources and biographical materials (distributed to groups) into dialogues for classroom use.

1 Why are mathematical plays important? Why the Abel play?

Niels Henrik Abel (1802-1829) was a Norwegian mathematician educated in Christiania (Oslo). During his short life Abel made ground-breaking contributions in many fields of mathematics. This workshop had a dual aim: to pay tribute to Abel and celebrate his life and work, but also to experience and reflect on ways that the devices of dialogue and theatre can bring mathematical history to life in the classroom. Participants were challenged to construct their own dialogues, drawn from primary sources and biographical materials, and encouraged to use theatre in the classroom to contextualise and enliven the teaching of mathematics, thus engaging learners in the excitement of mathematics-making and celebration of the people and stories behind the symbols, concepts and theorems. The story of Abel's personal life, linked with his extraordinary mathematical achievements, makes for excellent and unforgettable drama, constituting a good example of how such a theatrical experience can be both engaging and mathematically inspiring.

Framing the challenge for this workshop

This subsection contains my introduction to the workshop at ESU8:

My talk on Saturday introduced the idea of using dialogue or theatre as a communication tool. This is not a new idea – it is as old as humanity, and has long been used for communicating mathematical, scientific and philosophical ideas, by Plato, Galileo, and many more. But live dialogue form is sadly neglected in current curriculum-driven, time-constrained educational systems. Films with mathematical, historical & biographical themes are great, but I am convinced, having observed the effect of spontaneous drama and improvisation on young people of various ages, that positively *involving them in the re-enactment* of historical episodes is the key to getting their attention, and engaging their hearts and their minds. This workshop will make the case and (I hope) be inspirational too, by demonstrating theatre in action, involving all of you

in the production and enactment. I will then invite you to share ideas and any similar experiences, and also to discuss in smaller groups how to use primary source and biographical material to generate different plays and dialogues for different levels, and to use the power of theatre in your own teaching. In summary, our workshop aim is: *observe, enact, critique, create*!

Now I am not primarily a playwright or a historian, but a mathematician. I gradually stopped going to topology conferences and started going to mathematical history and education conferences. Since then I have had plays enacted in whole or in part at a number of ESUs, HPM meetings, and other conferences, in Toronto, Braga, Stockholm, Oxford, Uppsala, Dublin, Leeds, Washington, Montpellier, Oslo, as well as in some African countries: Zimbabwe, Lesotho and South Africa. Some of these involved teacher trainees, and some of my plays have been used in classrooms or mathematics-camps elsewhere by others. But I am an academic, and though passionate about using history and mathematical competitions in mathematical enrichment in schools, I have had limited opportunity to mount plays under real classroom constraints, and so I am not the expert! I am hoping some of you may be, or may become, experts.

I challenge you to become co-creators of an exciting educational tool and art-form! I want you to run with the ball, catch the vision, and translate your own love of mathematics and its history into magic theatrical moments in your own classrooms. It's not so daunting – to convince you of this is a major goal of this workshop. We'll put this play on now with minimal fuss, minimal props and rehearsal, and (I expect) have lots of fun! Here's the plan for the workshop: [*displayed on a slide*]

Welcome & introduction – 5 minutes
Allocation of parts & distribution of scripts – 15 minutes
Rehearsing in corners – 15 minutes
Performance of play – 30 minutes
Reactions & critique – 15 minutes
Discussion in groups with source material – 20 minutes
General feedback – 15 minutes
Conclusion and thanks – 5 minutes

First we will allocate parts. Please don't be shy to volunteer or to nominate somebody: 2 stage-crew, 1 sound person, 3 directors, 3 major parts, 4 smaller parts, 6 one-speech parts, 6 mime/impro parts; others may help with direction or staging.

2 Casting, rehearsing & stage setting

Colour-coded name cards were hung around the participants' necks as they were cast; actors and directors of each mini-scene were given the same colour, so they could find each other quickly. A few props were handed to appropriate people with their name cards: manuscripts, envelopes, etc. Full scripts were given to participants with major parts, and to directors so they could coach their actors in entries and exits. Partial scripts were given to others, as necessary. Each script had the relevant part highlighted. The parts were listed in order on-screen as below, with colour coding to match the name cards.

Stage crew & Sound person

Directors of mini-scenes

- *Abel and friends in Copenhagen*
- *Abel & Cauchy in Paris*
- *Abel, Crelle & Crelle's Secretary in Berlin*
- *Abel & Christine Kemp & Maid in Frolen*

Major parts:

- MARIA (narrator, historian of mathematics)
- EMMY (co-narrator, mathematics teacher)
- NIELS HENRIK ABEL (young Norwegian mathematician)

Smaller parts:

- AUGUST CRELLE (German mathematician and journal editor)
- AUGUSTIN-LOUIS CAUCHY (renowned French mathematician)
- CHRISTINE KEMP (Abel's young Danish fiancée)
- BERNT HOLMBOE (Norwegian mathematics teacher & textbook writer)

One speech:

- LEONHARD EULER (German-Swiss mathematician)
- JOSEPH-LOUIS LAGRANGE (Italian-French mathematician)
- CARL FRIEDRICH GAUSS (German mathematician)
- GIFFEN WILSON (English mathematician)
- FERDINAND DEGEN (Danish mathematician)
- CHRISTOPHER HANSTEEN (Norwegian mathematician)
- CRELLE'S SECRETARY (German)

Improvisation / mime

- FRU HANSTEEN (Hansteen's wife)
- FOUR FELLOW STUDENTS & FRIENDS OF ABEL
- CHRISTINE'S MAID (Danish/Norwegian)

The stage crew co-operated with directors in placing chairs, tables, desk and couch in place for the mini-scenes. The scripted 'curtain' was simply imagined. The 'sound' was an optional feature and was dispensed with to minimise technical problems.

Casting was done within the allocated 15 minutes, with very little pressure. The use of colour-coded name cards on string necklaces was important in achieving this. Rehearsal and stage-setting were completed in the next 15 minutes. The staging and delivery suggestions in the scripts were kept to a minimum to emphasise the informal, light-hearted and impromptu nature of the production. No suggestion was made (even for short speeches) that people should try to memorise lines. This, together with the minimal guidelines, helped to allay any anxiety and allow freedom for creative improvisation. Participants responded well, and there were sounds of much enjoyment and hilarity in the

mini-scene rehearsing; people with short speeches took the time to run through their parts, often reading to each other.

Figure 2.1: A mini-scene in rehearsal. From left to right:
Marcela Chiorescu (USA), Peter Ransom (UK), Fàtima Romero-Vallhonesta (Spain)

3 THE PLAY

[*Maria & Emmy appear front stage, seated to one side*]

MARIA: Hello everyone! Welcome to our celebration of one of Norway's greatest mathematicians, Niels Abel! [*points to slide showing portrait of Niels Abel*]

Figure 3.1: Niels Henrik Abel, *Lithograph after a drawing by Johan Gorbitz, 1826*
From Encyclopaedia Britannica,
Courtesy of the Royal Norwegian Embassy, Washington, DC

My name is Maria, and I am a historian – I am very interested in the story behind mathematics. And this is Emmy, who is a mathematician and teacher.

EMMY: Hello! [*smiles and waves*]

MARIA: Emmy, tell us – what is Niels Abel most famous for?

EMMY: Well, he did *many* amazing things during his short working life in that second decade of the nineteenth century, but perhaps it's his resolution of the problem of the quintic!

MARIA: Ah yes, conquering the quintic equation! Let's set the scene, mathematically. One of the great achievements of the sixteenth century was the cracking by Italian mathematicians of the long-standing problem of finding general algebraic rules for solving cubic equations, and also quartic equations – fourth degree equations. But to find a rule for quintics – fifth degree equations – taxed the ingenuity of the *greatest* mathematicians. Here is Leonhard Euler writing in 1767, expressing his disappointment:

[*cameo appearance of Euler, aged 60, frowning and shaking his head*]

EULER: All the pains that have been taken in order to resolve equations of the fifth degree, and those of higher dimensions, ... or, at least to reduce them to inferior degrees, have been unsuccessful; so we *cannot* give any general rules for finding the roots of equations which exceed the fourth degree.[1]
[*EXIT*]

MARIA: Four years later, Joseph-Louis Lagrange surveyed and analysed the efforts of mathematicians over the three centuries before his time, and reported pessimistically:

[*cameo appearance of Lagrange, aged 35, sighing*]

LAGRANGE: The result of these reflections is that it is *very* doubtful that the methods of which we have just spoken can give the complete solution of equations of the fifth degree. And, for even stronger reasons, those of higher degree. This uncertainty, together with the length of the calculations that the methods display, must put off in advance all those who might be tempted to make use of them to solve one of the most celebrated and important problems in Algebra.[2]
[*EXIT*]

MARIA: The great Carl Gauss himself, as the nineteenth century began, was pretty sure that the thing was impossible, and said so in the final chapter of his *Disquisitiones*. First he gave a masterly analysis of solutions of what he called pure equations – in particular what we now call cyclotomic equations [*slide appears*], and then he continued:

Gauss's cyclotomic equations: $x^n - 1 = 0$
Gauss's 'pure equations': $x^m - A = 0$

[*cameo appearance of Gauss, aged 23*]

[1] Based on (Euler, 1770, p. 286), quoted in (Katz, 2009, p. 665).
[2] Based on (Lagrange, 1771, p. 140); English translation in (Stedall, 2008, p. 348).

GAUSS: Everyone knows that the most eminent geometers have been unsuccessful in the search for a general solution of equations higher than the fourth degree, or (to define the search more accurately) for the reduction of mixed equations to pure equations. And there is little doubt that this problem is not merely beyond the powers of contemporary analysis but proposes the impossible ... [3]
[*shrugs and EXITS*]

MARIA: During the eighteenth century, British mathematicians, too, had been battling with this problem. Listen now to the English mathematician, Giffen Wilson, reading a paper to the Royal Society of London, at the close of the century. You can feel the frustration and puzzlement caused by prolonged failure to resolve the problem of equations of higher degree.

[*cameo appearance of Wilson, showing his frustration; he uses a formal, even pompous style, suitable for the Royal Society; italics are mine*]

WILSON: The difficulties under which the higher branches of algebra still labour are generally known. […] *[pause]* No degree of equations beyond the second is yet perfectly resolved. Cubics present frequently an irreducible case; biquadratics have, by several methods, been reduced to cubics; but no formula exhibiting to the eye the actual resolution of a biquadratic has yet appeared. And, for the fifth degree, and all upwards, not even a *clue* to general resolution has been found, by the combined labour and ingenuity of mathematicians for several centuries. This failure has been *very puzzling* and mortifying to the cultivators of algebra […] *[pause]*

Whether any method will ever be discovered, it is not easy to say. Whoever may be fortunate enough to discern a method for fifth degree and higher, will have the honour of removing an important and inexplicable barrier, which has so long obstructed the further improvement of algebra.[4]
[*EXIT*]

MARIA: That honour would go, twenty-seven years later, to the young Niels Henrik Abel.

EMMY: Except that his insight led, not to the algebraic solution of quintic equations, but to a proof that a general method did not exist! However, his work opened up wonderful new directions of research and discovery.

MARIA: Yes, indeed! Though I should mention that the Italian mathematician, Paolo Ruffini, produced a slightly earlier impossibility proof that was obscure and flawed but probably influenced Abel.
EMMY: We call it the Abel-Ruffini theorem today!

MARIA: And so the long quest to solve the quintic ended, as mathematical quests usually do, in many more questions. But what questions they were!

[3] Based on (Gauss, 1801 & 1966, 445), quoted in (Katz, 2009, 722).
[4] Based on (Wilson, 1799, 265).

EMMY: Did the young Niels show his talent early in life?

MARIA: Oh yes, I think so! He was born in Norway in 1802, the year after Gauss published his great *Disquisitiones*. His father, with a degree in theology and philology, taught him until he was thirteen, then he attended the Cathedral School in Christiania – now called Oslo. In his mid-teens, he was recognised as a potentially great mathematician by a new teacher, Bernt Holmboe.

EMMY: [*smiles*] Once again, the teacher is key to the unlocking of gifts in the child! Did nobody recognise his brilliance earlier?

MARIA: Sadly, no. Maybe because the opening of the new University of Christiania a few years earlier had drained the Cathedral School of its best teachers.

EMMY: [*sighs*] If only the best teachers were encouraged, and content, to be where they can be most influential, instead of wasting their gifts becoming deputy heads, heads, and academics!

MARIA: Well, a good head can be very influential! And so can an academic, for students and for creative research –

EMMY: [*emphatically*] Good *heads*, good *academics* and good *teachers* all have distinct combinations of gifts. Helping people find their true vocation must be one of the most under-rated professions in the world.

MARIA: Here is Bernt Holmboe:

[*cameo appearance of Holmboe*]

HOLMBOE: [*beams*] What a privilege it was to have this splendid young genius Niels under my charge! I not only helped him win a scholarship to continue at school after his father died, but encouraged him to read works by the greats: Euler, Newton, Lalande, d'Alembert, Lagrange and Laplace, even before he went to the University of Christiania at the age of 19.[5]
[*EXIT*]

EMMY: So the teenage Abel would have been aware of Lagrange's and even Cauchy's work, and the outstanding problem in the theory of equations – the quintic equation!

MARIA: He even thought he had found a general method of solution of the quintic, and hoped that his paper might be published by the Royal Society of Copenhagen. Fortunately, his paper *was* read carefully by a Danish mathematician, Ferdinand Degen, who demanded numerical examples. In looking for these, Abel realised his proof was faulty.

[5] This speech is my own invention, based on sources such as the biography in the MacTutor archive.

EMMY: Ah – it's good to see an instance of helpful reviewing! Sometimes new and ground-breaking work can be damned out of hand! But a good referee can make all the difference! So, what was Abel's reaction?

MARIA: Undaunted, he put the theory of equations on a back-burner, while he launched into a study of elliptic functions and other things.

EMMY: How did he pick up that elliptic functions and elliptic integrals would be critically important fields for research?

MARIA: It was that reviewer, Ferdinand Degen!

[*cameo appearance of Degen, reading slowly and carefully from a draft of his letter*]

DEGEN: The development of elliptic integrals would have the greatest consequences for analysis and mechanics. ... A serious investigator with suitable qualifications for research of this kind would by no means be restricted to the many beautiful properties of these most remarkable functions, but could discover a Strait of Magellan leading into wide expanses of a tremendous analytic ocean.[6]
[*EXIT*]

EMMY: Abel would certainly have discovered the work of his fellow sailor, the German mathematician Carl Jacobi!

MARIA: Yes, and found that Jacobi's results followed from his own. But he was very curious to know how Jacobi arrived at his.

EMMY: Did Abel publish anything at that stage?

MARIA: A scientific journal had just been launched at the University of Christiania by the professor of astronomy, Christopher Hansteen. From 1823, Abel published in Hansteen's journal papers on functional equations and integral equations.

EMMY: Ah, so he found another mentor at the University!

MARIA: Yes! Here is Hansteen:

[*cameo appearance of Hansteen with his wife Fru Hansteen*]

HANSTEEN: *[looking pleased with himself]* Ah, never was there another student of mine to compare with the young Niels Abel! I encouraged him in his work, and I also worked hard to find him a stipend. And my wife, Fru Hansteen, treated him like her own son! [*he acknowledges her and she smiles, nods, ad libs agreement*][7]
[*EXIT*]

[6] From the biography of Abel in the online *Dictionary of Scientific Biography*.
[7] This speech is my own invention, based on standard biographical sources.

EMMY: Meanwhile, I expect, Niels would have kept struggling with the elusive problem of quintic equations.

MARIA: And longing to visit the great European centres of mathematics, like Paris and Berlin! Here he is, a year later:

[*cameo appearance of Abel, holding manuscript*]

ABEL: I am convinced I have proved the impossibility of general solution of the quintic! I have written it out, but the printing costs are so expensive. I have had to leave out most of the details. Now I am told nobody will understand it or believe it. [*pause*] I am going to have to write a better, longer version in German … [*surveys his manuscript, sadly*]

But how can I get the right people to look at it?
[*EXIT*]

MARIA: This paper would finally be published in a new Berlin journal, edited by a man who gave Abel much-needed support, August Crelle. The journal became known simply as *Crelle's Journal*. It played a major role in the development of effective mathematical communication in the nineteenth century.

EMMY: Isolation can kill genius – so can poverty. It's wonderful that Abel had perceptive and supportive teachers and mentors at school and University, and then found a man of such influence in the mathematical establishment!

MARIA: Yes indeed – many poor, young geniuses are never recognised and encouraged!

EMMY: Did Abel ever meet August Crelle?

MARIA: It happened like this. In 1826, having worked at learning French and German, and raised some funds, he went off, aged 24, on what we might call a mathematical pilgrimage, with a group of friends...

[*Abel and four friends cross the stage, talking (in mime) and laughing*]

First, they visited mathematicians in Norway and in Denmark. Once in Copenhagen, they had to decide whether to go to Paris – Abel's heart's desire – or to Berlin, where his friends wanted to go ...

[*Abel and four friends return to mid-stage and mime an argument, pointing in different directions*]

Professor Hansteen had urged Abel to make for Paris, but he stayed with his friends and went to Berlin...

[t*hey go off-stage together, but Abel stops briefly and confides to audience, shrugging*]

ABEL: ... I am so constituted that I cannot endure solitude. Alone, I am depressed, I get cantankerous, and I have little inclination to work.[8] I will go with my friends to Berlin! Then to Paris, I hope!
[*EXIT*]

MARIA: Before leaving Copenhagen, Abel secured a letter of introduction to August Crelle, from a Danish mathematician. Going to Berlin turned out to be the best thing he could have done, for he managed to see his epic paper on the impossibility of solution of the quintic safely into Crelle's trusted hands, earmarked for Crelle's Journal, and also six other papers.

CURTAIN RISES
[*A Secretary shows the nervous Abel into the office of August Crelle*]

SECRETARY: Excuse me, Herr Crelle, that Norwegian mathematician has arrived to see you.

CRELLE: Ah, Herr … er … Abel … Niels Abel? Is that how I pronounce it? I do not often meet mathematicians from Norway! Welcome to Berlin!

ABEL: *Guten Morgen, mein Herr*! I am deeply honoured that you have consented to see me.

CRELLE: *Ach*, young man, your letter of introduction from the Danish mathematician was glowing, to say the least; but I suspected him of going over the top! However, once I had spent some time with the packet of papers you delivered last week, I had no doubt at all that I wished to meet the author! I did not expect him to be so youthful! I can see that already you are able to penetrate to the very foundations of problems, attacking them with extraordinary energy. You seem to see things in new and higher ways that your seniors have not seen.

ABEL: I am deeply grateful, Herr Crelle, for your encouragement –

CRELLE: *Nein*, I suspect the benefit will be mutual, my friend, for your work might well make my new journal famous! Come, let us discuss what needs to be done to get your work into a form I can publish...

[*Abel sits down at Crelle's invitation, and they pore over the manuscript*]

CURTAIN FALLS
EMMY: Did Abel stay long in Berlin? He might have been able to meet other important mathematicians.

[8] From the MacTutor archive, quoting E T Bell, *Men of Mathematics* (New York, 1986), 307-326.

MARIA: It was probably very exciting but quite stressful having to win his way into the corridors of mathematical power, and survive in a big German city. He was certainly inspired, but longed to have peaceful thinking time! From Berlin, he writes a letter home in March:

[*Abel sits at a café table in Berlin, pen in hand, reading his scribbled letter*]

ABEL: I look very much forward to my return home to obtain the possibility to work in tranquillity. I have material enough for many years, and just now so many ideas are wandering around in my head.[9]

EMMY: He must have been starting to do that great work in analysis for which we celebrate him today in a number of theorems! He was a pioneer of true rigour in analysis, wasn't he?

ABEL: All my efforts I will now apply to bring more light into the tremendous obscurity which undoubtedly there is in the Analysis.
[*shakes his head in amazement*] It is quite amazing that it can be studied by so many, and yet is not at all strictly treated. There are exceedingly few theorems in the Higher Analysis which are being demonstrated with convincing rigour. [*sighs*] Everywhere one finds this unfortunate habit of reasoning from the particular to the general.

EMMY: Yes! He really shared the vision of Cauchy!

MARIA: And, like Cauchy, he was appalled at the careless manner in which mathematicians had been treating infinite series. He wrote to his old teacher and friend, Bernt Holmboe:

ABEL: My eyes have been opened in the most surprising manner. If you disregard the very simplest cases, there is in all of mathematics not a single infinite series whose sum had been rigorously determined! In other words, the *most important parts of mathematics* stand without foundation. It is true that most of it is valid, but that is very surprising. I struggle to find a reason for it, an exceedingly interesting problem …

[*Abel gets up from chair and makes EXIT*]

MARIA: Before going back to Norway to work out all the ideas beginning to simmer in his mind, Abel continued to travel through Europe, intending to meet and talk with some of the mathematicians whose works he had read. He arrived in Paris in July, hoping especially to meet Augustin-Louis Cauchy, the great proponent of rigour in analysis.

From Paris, in August, he writes to Professor Hansteen:

[9] This and the following speeches to Holmboe from Berlin are from Abel's biography in *Encyclopaedia Britannica.*

[*Abel sits at a café table in Paris, reading his scribbled letter*]

ABEL: Finally I have arrived at the Focus of all my mathematical wishes, to Paris! [...] Above all I would like to have my memoir completed [...] to be presented to the Institute. I have the hope that the Academy will print it.
[*EXIT*]

MARIA: Young and unknown, poor Abel was too optimistic.

CURTAIN RISES
[*Abel excitedly runs after Cauchy as he walks by, and attempts to speak to him*]

ABEL: Euh, *excusez moi*, Professeur Docteur Cauchy!

CAUCHY: [*brusquely, over his shoulder*] Oui, monsieur, qu'est-ce que tu veux?

ABEL*:* [*trying to keep up with Cauchy*] Bonjour, Monsieur Professeur! I would be greatly honoured if I could have a talk about mathematics with you at your convenience.

CAUCHY: [*stops*] I am very busy, young man! Do you not purchase my *Exercises des Mathematiques*?

ABEL: *Oui*, Monsieur, I am very impressed, but I was hoping to discuss some work of my own –

CAUCHY: [*glares at Abel*] Your work? With me? Indeed! Young man, I have far too much of great importance to do, I cannot spare time to listen to your speculations.

ABEL: [*thrusting a packet at Cauchy*] But, Monsieur Cauchy, may I give you my paper to read? I can come back and discuss it later.

CAUCHY: [*hesitating, and then reluctantly taking the packet and glancing at the contents*] Hmm, *oui*, if I have time perhaps I may look at it. But no – if you are serious, you should submit it through the proper channels to the Académie. [*hands the pile of papers back and walks off*] Adieu, Monsieur.

ABEL: Adieu, Professeur, merci!
[*sighs, stuffs the papers back in the packet, then turns and walks away, looking sadly over his shoulder as the great Cauchy strides off*]

CURTAIN FALLS
MARIA: Two months later Bernt Holmboe receives a letter:

[*cameo appearance of Holmboe*]

HOLMBOE: [*holding an envelope, looking excited*] A letter – from Paris! It must be from young Niels!

[*taking out and reading letter – which can be this script*] ... Hmmm ... The French are much more reserved with strangers than the Germans. It is extremely difficult to gain their intimacy, and I do not dare to urge my pretensions as far as that; any beginner struggles to get noticed.
[*aside to audience*] Ah, poor Niels – a little fish in a big pond! But what a very talented fish! I wonder if he has managed to meet the biggest fish of all – Monsieur Cauchy...

ABEL*:* [*voice off-stage, while Holmboe reads on silently, pulling appropriate faces*] I have just finished an extensive treatise on a certain class of transcendental functions to present it to the Institute which will be done next Monday. I showed it to Mr Cauchy, but he scarcely deigned to glance at it. Without bragging I dare say it is good. I am curious to hear the verdict of the Institute. Cauchy is '*fou*' [*proud, aloof*], and he is unapproachable. But he is the mathematician who these days best knows how to present mathematics. He is now publishing a series of memoirs entitled '*Exercises des Mathematiques'*. I buy them and read them diligently.[10]

HOLMBOE: [*shaking his head*] Poor, poor, Niels! How did he find the money to buy those pamphlets? I expect he is starving himself...
[*EXIT*]

MARIA: This paper that the great Cauchy would scarcely glance at is known to historians today simply as Abel's Paris Memoir. Dealing with the transcendental functions, this was one of the most important mathematical achievements of the early 19th century. Cauchy was supposed to referee it, but *[speaking with sarcasm]* it presumably stayed buried on his desk, while poor Abel worried about it for the rest of his short life. In the winter of the year 1826, Abel returned home to Oslo.

EMMY: His head must have been buzzing with new ideas! Did his home University give him a post?

MARIA: He was desperately hoping so, but the one available post went to his old teacher, Bernt Holmboe, who would be famous for writing many of the first Norwegian mathematics textbooks. The University did give Abel a small grant, but he had to scratch a living as a supply-teacher and tutor.

He was deeply in love with a girl called Christine Kemp. They became engaged and hoped to be married as soon as he secured some means of supporting a family. Meanwhile he researched and wrote with all the energy he could muster, while battling recurrent illness. Much of his time was spent trying to unwrap the mysteries of the general theory of solvability of equations:

[*cameo appearance of Abel, writing at table in between bouts of coughing*]

[10] Partially in MacTutor archive and in E. T. Bell, *Men of Mathematics* (New York, 1986), 307-326.

ABEL: I aim to investigate the following problems: One – To find all equations of any given degree that are algebraically solvable. Two – To decide whether a given equation is algebraically solvable or not.

MARIA: These questions would not be fully resolved in his short life. What he did show, and published in Crelle's Journal, in Berlin in 1829, is this result:

ABEL: Gauss's method for finding the roots of the equation $x^n - 1 = 0$ can be generalised – I have to find a criterion for algebraic solvability of a general equation. His method depends on the fact that there is one of the *n* roots such that each root is a power of that one.

[*slide*]

Gauss's method for: $x^n - 1 = 0$,
depending on the fact that there is one of the *n* roots
such that each root is a power of that one,
can be generalised to find a criterion
for algebraic solvability of a general equation.

ABEL: Now, at last, I believe I have proved a wonderful theorem: If the roots of an equation ... [*scribbles while theorem is displayed on another slide*]

If the roots of an equation of any degree are related so that
all of them are rationally expressible in terms of one, designated as x,
and if, furthermore, for any two of the roots, θx and $\theta_1 x$,
where θ and θ_1 are rational functions, we have

$$\theta\,\theta_1 x = \theta_1\,\theta x,$$

then the equation is algebraically solvable.

In an **Abelian group**, composition is **commutative**: for any elements g, g_1 of the group

$$g g_1 = g_1 g.$$

[*EXIT*]

MARIA: Today Abel's name is celebrated in the name 'Abelian group' – a group in which the composition operation is commutative: multiplying any two elements of the group can be done in any order, the answer will be the same. You can see the connection with his theorem, so it is a fitting way to remember him.

EMMY: Whatever became of that paper Abel left with the Paris Académie?

MARIA: He wrote a two-page note to Crelle with a brief summary, which was published in Crelle's Journal. The original manuscript was only found by Cauchy after a search

prompted by Jacobi! It was finally published in 1841. That ill-fated original manuscript was mysteriously lost again, but over a century later there was great excitement among historians when it was discovered in Florence, in 1953.[11]
But let's go back to September 1828 ….[12]

CURTAIN RISES
[*Abel lying on a couch holding manuscript and pen; he coughs periodically. His* fiancée *Christine Kemp is seated beside him*]

ABEL: Ah, Christine, you must help me reply to Herr Crelle! It's over two weeks since I received his kind letter.

CHRISTINE: I remember how happy you were when you received it!

ABEL: [*smiling*] Yes, I had not expected such a quick answer. And he also kindly took the trouble to copy and send me excerpts from letters of Jacobi and Legendre! I really must thank him. I should have replied sooner, but I wanted to finish this short manuscript on elliptic functions and send it too.

CHRISTINE: It's wonderful, isn't it, that he holds out good hope of an appointment for you in Berlin? Maybe we can get married at last and live in Berlin!

ABEL: I will implore him to let me know as soon as possible when anything has been decided, favourable or not, for if it does not work out as I hope, I must be prepared to improve on my conditions here. I can do nothing until I know what will happen.

CHRISTINE: He will certainly grant you this request – he knows your abilities, Niels!

ABEL: You must help me draft this letter! I want to say how much pleasure I have taken in the remarks of Legendre and Jacobi! I can see that Jacobi has arrived at my transformation theory for elliptic functions by taking a different path, and I am very curious to learn about his method.

CHRISTINE: Are you going to ask him to publish in the journal this new paper of yours?

ABEL: Yes, perhaps he will have a few pages to spare in the upcoming fourth issue. I want to ask him to print this paper first. Then the paper on equations can follow afterwards.

CHRISTINE: Why? He is very keen on the equations paper isn't he?

ABEL: I know, but I believe that the elliptic functions will be of greater interest to mathematicians. And also, you know, my health will hardly permit me to occupy myself with the equations for a while – it's exceedingly difficult stuff, and will take too much

[11] See full story in (Del Centina, 2002).
[12] The following scene is based on a letter from Abel to Crelle (Abel, 1828).

effort. Shall I confide in him that I have been ill for a considerable period of time, and compelled to stay in bed?

CHRISTINE: Oh, Niels, you must be careful! [*she puts arms around him*] Even if you are now recovered, the physician has warned you against any strong exertion!

CURTAIN FALLS

MARIA: Christine's fears were justified. Six months later, Niels Abel died, on the 6th of April 1829.

[*cameo appearance of Christine, sobbing. A maid brings a letter in on a tray*]

CHRISTINE: Oh! A letter ... from Berlin! It must be from Herr Crelle ... *[tears the envelope open, then gasps]* Niels has been appointed to a post in Berlin! ... Too late, too late, oh my poor Niels, two days ago you would have been so, so proud! ...

[*she breaks down, crumpling the letter and hurling it on the floor, then she runs off stage covering her face*]

MARIA: Niels Abel's body was buried in the churchyard at Froland. August Crelle writes a glowing tribute to his young friend, in his journal:

[*cameo appearance of Crelle with pen in hand, writing obituary, and reading from his draft*]

CRELLE: All of Abel's works carry the imprint of an *ingenuity* and *force of thought* which is unusual and sometimes amazing, even if the youth of the author is not taken into consideration! … [*reflective pause*]

One may say that he was able to penetrate *all* obstacles down to the very foundations, with a force which appeared irresistible. He attacked the problems with an extraordinary energy. He regarded them from above and was able to soar so high over their present state that all the difficulties seemed to vanish. … [*pause*]

But it was not only his great talent which created the respect for Abel and made his loss *infinitely* regrettable. He distinguished himself equally by the *purity* and *nobility* of his character … and by a rare modesty equal by his genius – what a combination!

[*walks off slowly, speaking to himself and looking very sad*]

Those of us who knew him cherished him. If only he'd lived and come to Berlin …

[*EXIT*]

EMMY: Looking back, it seems that in spite of the efforts on his behalf by Crelle, Abel was relatively unknown when he died – so sad!

MARIA: Well, both Legendre and Jacobi recognised his brilliance. But Cauchy mislaid his monumental Paris manuscript, and Gauss seems never to have read the copy Abel sent him of his 1826 paper on the quintic!

EMMY: Maybe because Gauss still believed the thing was impossible! [*snorts*]

MARIA: But maybe because Gauss minimised the problem, imagining he himself had more or less solved it earlier. Nobody can be sure with Gauss! Some say Gauss was loathe to give the glory to another. It has even been suggested that, because Abel omitted to slit the pages before posting it, Gauss couldn't be *bothered* to cut it open and read it.

EMMY: [*groaning, and covering her face with her hands*] Oh, no! I feel like I could slit his throat!

THE END

Figure 3.2: Abel monument, Oslo, designed by Gustav Vigeland (1908)

4 Review, critique & reflection

Following the play, open discussion took place, with guiding questions: What were the best moments? What didn't work well? What improvements (for this script) can you suggest, in staging, action, emotion, voice projection? How could you re-script Abel's story for different groups, levels of communication, different balances of the personal, the historical, and the mathematical? In particular, how much mathematics to include? Did the nature and magnitude of Abel's mathematical achievements get through? The problem of authenticity versus accessibility – how close should such a play stay to the primary source material? Can any of you share experiences of using theatre?

In the glow of achievement, few participants seemed in the mood to criticise, and the discussion was largely positive, with different people particularly liking different moments of the play. There was little experience of using anything similar in teaching. Some feedback is collected in the final section below.

During the discussion some hardcopy materials were circulated, and then participants were invited to form smaller groups, and reflect on how they might go about weaving highlights from such primary sources and biographical materials into dialogues suitable for classroom use at various levels. The materials were selected and extracted according to

a number of criteria: relevance to Abel's life and work, reasonably short, potential for dramatization, easy availability (to promote the idea that source material for creating reasonably authentic dialogues is not hard to find), illustration of the richness of further resources, insight into the historiographical challenges of achieving authentic dialogues. The materials included extracts from the *MacTutor History of Mathematics archive*; one of Abel's letters in full; biographical material on Holmboe and Hansteen from the *Dictionary of Scientific Biography* and *Encyclopaedia Britannica*; a collection of quotes from Abel; a paper on the fortunes of Abel's famous 'Paris Memoir' which features in the play; and references for further reading and possible play-writing.

5 General feedback & conclusions

The workshop concluded with a short plenary discussion, inviting feedback from the small groups. This section collects some responses and retrospective feedback.

Length of play

The scheduling was generally agreed to be fine for such a workshop, but would naturally have to be adjusted for a particular classroom at a particular level. Some felt plays of 30 minutes would be the maximum to aim for. "This workshop worked well as people were there because they wanted to attend. In a classroom there are often students present who would rather be elsewhere, so in my opinion shorter plays may be a way forward." "It was amazing how well the actors played their role to bring the persons to life with such short notice." "I think it is a good idea to write such pieces for schools. They would have to be short like this one or even shorter, but they can awaken feeling for the human side of these scientific persons."

Practicalities for classroom use

It was important for this workshop to get the casting and rehearsing done in a very short time, and to demonstrate how to achieve this. Useful strategies were the use of large colour-coded name-labels for the characters, casting directors to lead the rehearsing of mini-scenes, highlighting the relevant lines in everybody's script, encouraging a relaxed attitude to reading the lines, and keeping the staging simple. The enthusiasm and enjoyment shown by the participants was palpable. "What made this work well, was the limited time we had to prepare ourselves. It would not have worked if we had a longer time because some people would not have been engaged." "I was surprised how well the event ran practically without any preparation." "Having a large cast was important: everyone had a part to play." "Everybody was required to participate and it was extremely enjoyable." "It is good to give opportunity for everyone to participate in some way, but to balance this with each person having space to watch some of the action." "It helped not to have to worry about memorizing lines. This freed up some people to improvise."

Coverage and general content

While the play was felt to do justice to the main events of Abel's life, some discussion took place on omitted episodes with dramatic potential, e.g. the story of how Holmboe became Abel's teacher when the previous teacher was removed from his position after beating a student who died of the injuries. In any attempt to portray in a classroom some facets and moments of a mathematician's life, there will be a rich menu of possibilities,

and selection will depend on the class and level. It is not necessary to be comprehensive; one small dramatic episode can spark interest in the person's life and give colour to the mathematics. Humour was seen to be an important ingredient.

The mathematics

In this workshop "there was a good balance between personal and mathematical", "good balance between maths and biographical information", but the question of how much mathematics to bring into a play, and how to frame it, is heavily dependent on the class and the purpose. This play presents just three glimpses of mathematics (on slides). Equations should be kept to a minimum, adjusted according to the background of the participants. At higher levels it may be worth weaving in some information on the development of the formula for third and fourth degree equations. To avoid the mathematics entirely is to miss the chance to signal that such apparently dry symbols are tied up with human drama. At appropriate levels teachers could follow up with work in the classroom. "… it engages students who might not have taken an active participation in mathematics in the past as well as introducing them to their heritage." "[While the] level of maths in the play is too advanced for high school, yet because it involves solving of equations, which is a very familiar subject to students, it shouldn't be a problem [to interest them]"; "[Such theatre] enriches the mathematics curriculum and gives students an appreciation of the thoughts behind the evolution of mathematics … it shows them that everyone has struggled with new concepts, so they should not feel bad about themselves when they do not understand the mathematics they are being taught the first time they meet it." "In the average classroom, there will be a few students who are mathematically talented, others who have dramatic talents, and such a joint activity can open up lines of peer communication."

Can such theatrical experience enliven mathematics education?

Participants commented as follows: "Some people who were generally shy and quiet came out of themselves and so seemed to grow in stature." "In general, it was a fun experience and very memorable." "It was great! I really enjoyed to be part of the play." "What made this memorable for me was working with such a lovely group of people and discussing how we would present it. It threw light on Abel and Cauchy as people and portrayed events in a vivid way that was not possible reading about them in a book." "The play brings you very close to a different perspective of mathematics, the appreciation of mathematics as a cultural-human endeavour. I hope one day I will incorporate it in one of my classes." "This play is a great way to expose students to lesser-known aspects of the human side of Niels Abel." "It was the first time I had attended a theatrical workshop based on mathematics, so I didn't know what to expect. I came because I'm interested in exploring different ways to present the mathematical concepts to students. I found it interesting, amusing and original." "I love the concept of a play as a means to incorporate history of maths in education."

Future use and new directions

We close with some ideas from participants: "… we were presented with the text as a finished article, but for it to be an enriching experience for the students; they will need to be … part of the creative process." "It would be worthwhile to look at the social context of the time, the geographical location and other events of the time leading up to the

development of the concepts involved." "Teachers should try something different and outside their comfort zone sometimes!" "[When I incorporate this in one of my classes] I will first assign the roles to my students, then give them time to practice on their own and then play together. It will be very similar to what we did in Oslo, but … I will probably not do it in one class period. I would want them to have more time to think about this play and the role Niels Abel played in the history of mathematics." "I intend to use the play on either Abel or Cauchy/Gauss at a conference in 2020; have to translate it, to make it really work with my students (Dutch teacher training, ages 17-20) … I will find a student to write a play in Dutch or on Dutch mathematicians."

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