

THINKING WITH LEVINAS ABOUT HISTORY OF MATHEMATICS IN MATHEMATICS EDUCATION

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ABSTRACT

For the French philosopher Emmanuel Levinas, human beings reveal themselves in relation to the Other (ethical perspective). In mathematics education, Levinas' philosophy helps to constitute theoretical frameworks in which one could avoid drawing on a private, self-regulated and autonomous (in a rationalist and dualist perspective) subject. The phenomenological reflections of Levinas on Alterity support our reflection on history and mathematics education, particularly with respect to the history of mathematics for a nonviolent mathematics education. Data from empirical studies and from our own research with secondary school students (15-18 years old) and prospective teachers have been provided in order to stimulate in vivo reflection and feedback of the group. A dialog took place during the Workshop. Excerpts from the written, short essays produced by participants and their oral remarks are presented and commented in the present article.

1 Introduction

The importance of Levinas' thought in the field of education is widely known and confirmed by several publications (*see* Atweh & Brady, 2009; Ernest, 2012; Radford, 2008; 2012). Indeed, Levinas has helped us, as teachers (and researchers, as we will see), to develop a useful rethinking of our relation with our students and mathematics. We became aware of the presence of the "third party" (Levinas, 2010, p.213) - persons outside the I - Other relations who require us to perform a list of duties. That fact tends to overcome our being available to accept the infinity in the student. For instance, the students have to pass national exams; therefore, my professional duty is to make them store quite a lot of notions. Moreover, the acceptance of that infinity, we believe, can for example imply that I, as a teacher, do not fall either into the temptation of a definitive judgment about the students' performance or into not admitting that they could get another chance in case of failure. In connection with that, Levinas helped us to develop the concept of violence as the simplistic solution to reduce the student to our criteria of judgment.

On a larger perspective, Levinassian ethics suggested us a new line of reflections about some aspects of education that are inherent both to the problem of motivation, engagement, willingness of deepening the subjects, etc. and to the question, 'How should a student get involved in the study of mathematics?'

To be honest, each of these points recalls some points of Levinassian philosophy but not all, indeed. That philosophy refers to our subjective experience, and suggests us how to use part of it in the attempt of contextualizing it in our teaching.

Moreover, Levinassian ethics helped to realize that students live in the “postmodern ethics”, after the title of (Bauman, 1993), which is radically different from the traditional ethics dealing with virtues, duties and principles by means of which to take decisions in actual situations. Levinassian ethics avoids “the temptation and the illusion that would consist of finding again by philosophy the empirical data of positive religions” (*Trace of the Other*). On the contrary, the presence of the Other is “meeting and friendship” (*Alterity and Transcendence*, p. 56).

We had the idea to discuss the work of Emmanuel Levinas within a workshop. The point was that this workshop might lead us to share with the participants how Levinas' phenomenological reflection on otherness supports our respective research on history and teaching-learning of mathematics.

We thought that this workshop could engage the participants to think with us about the learner's subjectivity, our relation to history, our experience of history and the role of history in mathematics education having in mind this Levinassian ethics. Even if there was certainly a didactical and educational dimension in this workshop, our approach and the discussions that took place were more theoretical and philosophical.

2 A workshop: setting the scene

This paper is a resume of the presentations, activities and discussion that took place during the two-hour workshop entitled *Thinking with Levinas about history of mathematics in mathematics education* that took place during ESU-8.

The workshop involved about 30 participants. The present report aims to describe the activities and considers the outcomes as contributions of/for the participants. We are aware that the limited number of hours did not permit to take into account the possibility to improve the inquiry by qualitative research, for instance by interviewing the participants who wrote their opinions following the question form we provided them with. In any case, various techniques (dialogue, short oral interviews, written answers, informal exchange of opinions) were used.

Therefore, final conclusions can consider the specific outcomes about Levinas' philosophy with respect to educational use of the history of mathematics compared with the limited time at disposal. Considering the involvement of the teachers and researchers taking part in ESU, we think that it is of great importance to have their opinions taken into account. Therefore, we believe that a useful discussion should be started on how to stimulate and collect the participants' contributions either in the one and a half or in the two hours ESU workshops.

Our idea of a workshop led us to exclude a long introductory talk and to introduce the activities right away. The proposal to the participants was to discuss educational problems, not only to examine excerpts from Levinas' works, and that seemed to complicate the task by extending the range of questions. Anyway, one must also consider that an almost mandatory choice, in view of both ESU8 topics and the need to stick the aspects of which the participants are specialized in.

We chose to propose the workshop as an opportunity for a dialogue on Levinas' philosophy. The importance of this author stands on the idea of alterity in mathematics education. The seminal character coexisted with the need to get reflections and contributions from the participants, establishing a dialogue in which

the participants who already knew Levinas could get involved successfully with those who approached the French philosopher for the first time. Mathematical content in historical documents was not the main focus but it was a resource to introduce some important educational problems inherent to its use. Levinas's philosophy, in fact, could help us to explain the importance of history in mathematics education, with some questions in the background such as, 'Why history?', 'For what purposes', 'What can history add to mathematics teaching?'

We considered the actual behavior of the participants as one of the aims of the workshop and in particular we observed both the discussion of some of the topics included in the trace form we gave them and the participants' request of explanations about materials and excerpts from Levinas' works and their suggestions for ways to compare Levinas' philosophy with school education and classroom activities. The aim of the dialogue, however, made the outcomes of the workshop hardly predictable.

Our aim was not to get a critical point of view of Levinas' works, but on the contrary, it was that of finding out the main ideas that his thought can offer. Even if his philosophy is today encountering a wider and wider acceptance, we believe that it contains a lot of aspects that could be perceived as paradoxical and should be clarified: the idea of face, for example, considered not (only) as a part of the body but as the way to transcendence. These aspects, we believe, can be the subject of discussions. The workshop activities were planned looking to establishing a dialogue between conductors and participants as well as among the participants themselves.

Before the workshop started, it was indeed impossible to know who would be the participants, and also which should be their expectations about the proposed activities. We did not know if they had previously studied Levinas' works or made any reference to him in their works, etc. A priori, we could expect that, for some of the participants, the workshop could be an introduction to Levinas' philosophy, while for others the deepening of aspects of his thought, and for others a useful exchange of ideas with participants who already knew Levinas, and finally for others just a rhetorical exercise. All considered, the outcome was not predictable in view of the participants' prior ideas and knowledge of the problems. It would have been impossible for us to write the present paper before the workshop had been held!

The workshop, which focused on theoretical concerns, proposed an introduction to Emmanuel Levinas' philosophy (1989, 2010, 2011a, 2011b) by highlighting how the phenomenological reflections of Levinas on Otherness support our (the authors') own respective research activities on history and mathematics education. Several elements were expected to be discussed with regard to Levinas' perspective, more specifically:

- History of mathematics for a nonviolent mathematics education
- History of mathematics as an experience of radical otherness
- Reflections on humanism and "antihumanism" with regard to the history of mathematics
- Reconsiderations of our ground pedagogical objectives related to the introduction of history of mathematics in the classroom
- Reflections on how to operationalize these elements in research.

As we will explain in more detail below, data from empirical studies and from our own research with secondary school students and prospective teachers were expected

to be analysed and discussed by taking the Levinassian perspective. More concretely, we planned:

- An introduction at a theoretical level to Levinas's philosophy
- Readings of carefully chosen short excerpts from Levinas' texts
- Discussions in dialogical form aimed to formulate questions and stimulate the reflections of the participants
- Data analysis of empirical studies with the aim to reflect on Levinas' perspective in a more "applied" or "practical" way
- Reflections upon a list of questions aiming at collecting the participants' ideas that could lead to short writing activities performed in small groups.

The following sections explain how those activities were carried out and a number of objectives were reached.

3 The Levinassian perspective

Our first objective was to introduce the group to Levinas's philosophy by defining both the main philosophical projects of Levinas and our own understanding of the main concepts that was introduced by Levinas in his philosophical investigation.

A short PowerPoint presentation was made in order to introduce Levinas and some major concepts in his work. Then, the readings of some excerpts of Levinas's main texts were organised. Finally, a group discussion of those readings took place.

3.1 The presentation of Levinas' philosophy

The presentation of elements of Levinas' philosophy took about 25 minutes. It was pointed at some key elements such as: 1) the phenomenological positioning of the work of Levinas; 2) his project of a phenomenological investigation of otherness; 3) major concepts arisen from this project; 4) and some reflections on the meaning of the encounter with the past.

During the presentation, the philosophy of Levinas was situated in the continental tradition and more precisely in phenomenology. Phenomenology was succinctly presented as a way to conduct philosophical investigation that is characterized by a certain rehabilitation of the mind sensitivity through the work of Edmund Husserl (1859-1938). In a bold move against metaphysics, phenomena are here considered as objects of consciousness, things toward which we orient ourselves, observe and reflect upon. This capacity of the mind to be "affected" (in the deep sense of a capacity to be influenced bodily, emotionally, cognitively, etc) leads to a philosophy that finds its way through idealism and empiricism. In a way, phenomenology was presented as a method for those who want to orient themselves in the field of thinking or a way to develop a philosophical thinking aimed at the description of how things give themselves (im)mediately. Phenomenology, at the time of Husserl, was taking a strong stance against positivism and psychologism.

The work of Levinas (who was a student of Husserl and Heidegger, himself a student of Husserl) was presented as situated in that phenomenological tradition. That has permitted scholars to view the philosophical project of Levinas as the phenomenological investigation of otherness in its fundamentals.

Otherness is taken by Levinas not exactly as a simple object of phenomenology in the continuity of tradition, but as a particular thematic enabling him both to enter into discussions with his masters Husserl and Heidegger and to criticize some of their results and to renew the phenomenological tradition.

As Levinas put it in his several phenomenological essays, the notion of Otherness is taken as the central part and the core of the human being. In his work, Levinas overturned the traditional (from Plato to Heidegger) ontological way of thinking of the human being. For him, the philosophical inquiry on the human being does not begin from his nature (the ontological perspective), but from his relation to the Other (the ethical perspective). In other words, ethics here is not understood as a “satellite” element of human existing, it is rather the central and the determinant field of reflections and research. Levinas opened a very new perspective on the human being which is no longer perceived as an isolated subject beset by phenomena, an *ipseity* thrown into reality, as Heidegger would say, but an ethical subject revealing himself in relation with the Other.

In this perspective, Levinas introduces the concept of Face (in French *visage*) which could be considered the authority of the Other, the injunction of the Other who imperatively asks for a careful relation. Levinas would say that Kant’s categorical imperatives are in the Face of the Other. That leads Levinas to see the other human being as something infinitely-other, something that cannot be totalised, that cannot be filled with intuition.

To introduce a reflection on historical images and documents, some quotations from (Levinas, 1989) were given. Levinas introduces the concept of Resemblance, referring to the fact that we have the opportunity to get to know the thing through its image. The image is not to be considered a mere simulacrum and resemblance, not even a vague link to reality: “We will say the thing is itself and is its image. And that this relationship between the thing and its image is resemblance [...] there is the simultaneity of a being and its reflection. [...] Resemblance is [...] the very structure of the sensible as such [...]”.

Fig. 3.1 shows a person who is doing an action or, better, who is going to do an action. We say that the person shows an intention, like in every figurative work of art: “[E]ternally Laocoon will be caught up in the grip of serpents; the Mona Lisa will smile eternally. Eternally the future announced in the strained muscles of Laocoon will be unable to become present. Eternally, the smile of the Mona Lisa about to broaden will not broaden”.

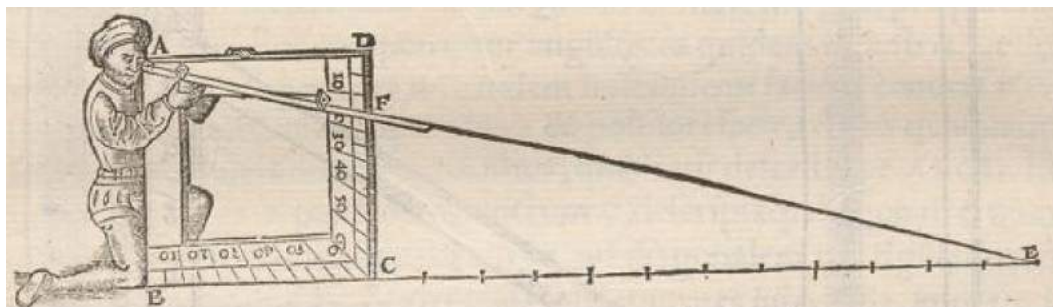


Figure 3.1: Oronce Finé, 1532, *Protomathesis*, Lib. II, fo. 66.

But “the immobile statue has to be put in movement and made to speak...”. That image requires the student, who analyses it, to carry out the action: mentally or practically. The students of the second class of the upper secondary school Liceo Rosmini of Trent – Italy used that image to be able to understand how to measure a distance of a point far away by means of the above instrument, the quadrant. It was used in the Middle Ages and in the Renaissance for topographic and astronomical measurements. It is made of a wooden square having sides of about one meter. The index can turn around the point A until reaching the direction AE. E is the point we want to know the distance of, which is calculated by considering that the triangles AFD and ABE are similar.

A few other Levinassian concepts were introduced in order to give further insights into his philosophy, above all in order to help the participants in the reading of the excerpts. These concepts will be introduced in the next section, where we present the excerpts that have been chosen for the reading activities.

3.2 Readings of some excerpts from Levinas’ main texts

After this short exploration of Levinas’ philosophy, we proposed a reading activity to the group. The objectives were not only to deepen the exploration of the Levinassian perspective, but also to give a hint of the style of analysis that is deployed in that work. It was also a way to “enter” Levinas’ work and to start a deeper discussion about it. We will try to highlight here how, more specifically, the selected excerpts refer to the following notions:

- Alterity and Violence (in a way, understanding the Other is to commit violence against him)
- Ethics and Hermeneutics (the Other and interpretation)
- Face of the Other (to be nude in front of the Other, not in an oblique standing)
- Metaphysical Desire (the total towardness to the Other, the dis(inter)ested relation with the Other)

Concerning the notion of Alterity and Violence, the introducing “In what sense [...] does the *absolutely other* concern me?” (Levinas, 1986) can be borrowed as the fundamental question with regard to the reading of a historical document considered as an experience of alterity. Reflections aimed at looking for some answers to that question can be found in the following reports of educational activities with students of different levels (secondary school students and prospective teachers, see section 4). Necessarily, each of those answers will be partial, even if we refuse to consider the psychological specificity of the students. In some lines ahead in the same work, Levinas insists on the relation with the other and puts forward the danger of “transmutation of the other into the same”. Levinas deepens the meaning of the previous sentences speaking of the reduction of “an alien world to a world whose alterity is converted into my idea”. On the contrary, he opposes the “thought which is [...] thought of itself [...] *a movement of the same unto the other which never returns to the same*”, that is to “the myth of Ulysses returning to Ithaca we wish to oppose the story of Abraham who leaves his fatherland forever for a yet unknown land”. That reduction of the other to a mine is what Levinas calls *violence* (Guillemette, 2017). In that we can also see the “ambiguous nature of knowledge” which

is “representation and movement [...], the dynamism of the infinite and the fullness of actuality” (Levinas, 1999, p. 58).

Concerning Ethics and Hermeneutics, “The manifestation of the other” (Levinas, 1986) is, in a sense, not different from every “signification”. The cultural context illuminates the other. Here Levinas makes a comparison with a text and its context. In philosophical hermeneutics, the context has a central role in understanding a text, specifically both in the fusion of the horizon with the author and, through the reference to specificity and whole, in the hermeneutic circle. Levinas points out that “the comprehension of the other is thus a hermeneutic and an exegesis”. The other appears in his corporeity, linguistic acts, artistic gestures. “The other is given in the concept of the totality to which he is immanent”. The use of the term “totality” produces the expectation that Levinas would introduce another level into his discourse; in fact, reminding the title of his masterpiece *Totality and Infinity*, we can expect that he goes beyond that term derived from ontology in order to introduce infinity. That is made with reference to the *face*.

Concerning the concept of the Face of the Other, Levinas emphasizes that “[T]he phenomenon which is the apparition of the other is also a *face* [...] the other does not only come to us out of a context, but comes without mediation” (Levinas, 1986). The face opens to infinity through his independent signification. But how can the infinity of the face appear in a phenomenon? The face, like “every entity, when it enters into immanence, that is, when it exposes itself as a theme, is already dissimulated” (Levinas, 1986). The Other (“the absolutely other”) can be friendship and can be the way through which the *I* can open to infinity. “The face-to-face is a relation in which the *I* frees itself from being limited to itself” (Levinas, 1999, p.56). In that lies the pedagogical value of that relation. The ethical relation depends on the fact that the approach to the other establishes “an experience different from that in which the other is transmuted into the same” (Levinas, 1986). The temptation to make the other a mine - to become his owner, to commit violence against him - is not possibly related to that situation.

Concerning the concept of Metaphysical Desire, Levinas emphasizes that “The metaphysical desire tends toward *something else entirely*, toward the *absolutely other*” (Levinas, 2010, p.33). It passes through the acceptance of the alterity of the other and also through the openness to walk on a path without end, without aim, through the impossibility to anticipate what is being desired. “It is a desire that cannot be satisfied”. When a student abandons any reference to the teacher’s requests and follows his personal interest for a topic, he desires to enter alterity without wondering where to go; it is the situation of somebody who does something without thinking of constraints or rewards; it is, in any case, what happens - also as the consequence of a request by the teacher - when a student has to understand a mathematical concept. Necessarily, he/she abandons, for a moment, any reference to the teacher’s requests, and begins an unpredictable process in order to get the desired acquisition. So, even if the student can outline the desired acquisition as a performance - for example as the solution of exercises – he/she must follow, at least for a moment, a blind desire. If the student completely lacks that desire, he renounces to enter the process of understanding.

Having a complementary role, the excerpts from page 275 to page 282 of (Levinas, 1997) aimed at discussing with the participants in what sense the abandoning of reference to the other in science can get start to negative consequences. In fact, Levinas problematizes the role of science (Levinas, 1999, p.71) by speaking of “its incompleteness” as regards the idea of infinity. The statement of the central place of “Man” and the “respect for the person, both in itself and in the Other” (Levinas, 1997, p.275) contrasts with the “anti-humanism” of the 20th century. We wanted to submit to the participants those reflections in order to recall the context in which Levinas’ thought originated (a Hebrew, a prisoner during World War II, his father and brothers killed by the SS...) and to focus on the importance of looking for the other in scientific documents as a way to meet “man” while speaking of mathematics. In a wider sense, we would underline that science is meant for mankind.

4 Levinas and the convocation of historical elements in the mathematics classroom

After the exploration of Levinas’ philosophy with the group, our objective was to highlight how the phenomenological reflections of Levinas on Otherness support our (of the authors) own respective research activities on history and mathematics education as well as to discuss it with the group. Two activities based on the data analysis from empirical studies were simultaneously organised. Participants had the possibility to work on the situations of their choice and were encouraged to reflect upon them. A few questions were given to the group in order to help the participants to start their reflection.

4.1 Data analysis concerning Luca Pacioli’s work

A data analysis activity with the group was organized on Luca Pacioli’s work. Some excerpts from Pacioli’s *Summa* (Fig. 4.1 and Fig. 4.2) were interpreted by students (17-18 years old).

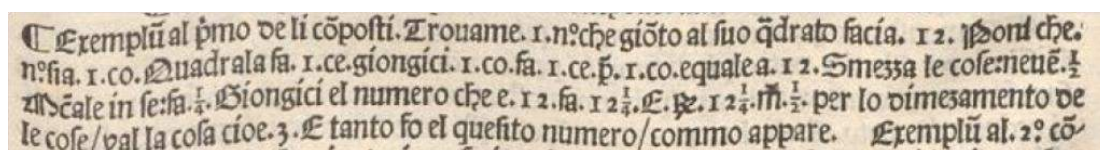


Figure 4.1: Pacioli 1.

[For the following English translations: “co” literally means “thing”; “ce” means “census”.]

“[...] Find me a number that, if joined to its square, makes 12. Imagine that the number be a thing. Square it. It makes 1 census. Join 1 thing. It makes 1 census plus 1 thing equals 12. Halve the things. It becomes $\frac{1}{2}$. Multiply by themselves. It makes $\frac{1}{4}$. Joint the number which is 12. It makes $12\frac{1}{4}$. And the square root of $12\frac{1}{4}$ minus $\frac{1}{2}$, because of the halving of the things, equals the thing that is 3. And the required number makes this amount, as it appears. [...]”

Luca Pacioli, 1494, *Summa*, folio 145.

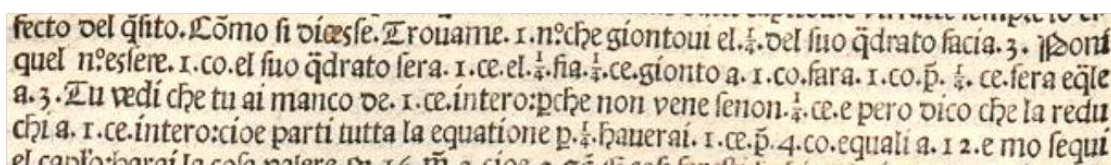


Figure 4.2: Pacioli 2

“[...] Find me a number such that, if $\frac{1}{4}$ of its square is added, makes 3. Let that number be 1 co; its square will be 1 ce. Its $\frac{1}{4}$ be $\frac{1}{4}$ ce which added to 1 co will make 1 co p[us] $\frac{1}{4}$ ce, it will be equal to 3. You see that you have less than 1 whole ce because it results $\frac{1}{4}$ ce, but I say that you [can] reduce it to 1 whole ce, that is divide all equation by $\frac{1}{4}$; you will have 1 ce p 4 co equals 12 [...]”

Luca Pacioli, 1494, Summa, folio 146.

Students synthesized their difficulties through questions or utterances, such as: “Where is the question? Where does the solution begin? No modern symbols!” This suggests their discouragement. The experiment is described in (Demattè, 2015). The same documents were presented to the group of participants.

In this study, we wanted to highlight just the students’ behavior in front of their difficulties. They renounced in planning actions in order to reconstruct the meaning of specific parts with reference to each other and to the whole text, neither to take into account the global meaning as a resource to understand single parts. In the Levinassian perspective, that suggests students’ incapability to see the other in the text and to consider that the comprehension of the other is hermeneutics. In the meanwhile, that suggests the importance to help students in creating the “cultural whole” (Levinas, 1986) in which the other is present. In classroom activities, the original text and a brief oral presentation by the teacher have not been enough to put the students in the trace of the other. In that presentation, the reference to mathematical aspects (quadratic equations) was prevalent. This, directly or indirectly, led – teacher and students - to violent behaviors, namely to neglect the richness the documents offered and, instead, to use it only for the specific purpose to train students with problems and equations. In this way, alterity has been converted into a narrow idea that came from the teacher. That situation could have hindered the students’ approach to the other in the document, considering also the question mark we can put on the possibility they share the usefulness of that purpose.

4.2 Data examination concerning Fermat’s work

A second data analysis activity was proposed to the group. It was around some of the data from a study (Guillemette, 2017) concerning the actual experience of prospective teachers engaged in the reading of historical texts.

Here is the material that was given to the participants:

In the next two excerpts from video analysis, we can see how future teachers’ mathematical activity interacts with Fermat’s minima and maxima method, and how it is interpreted. The activity concern Fermat’s general description of his method and the first example given. He finds the maximum or minimum of a given term $f(x)$ by “adequating” the two expressions $f(x)$ and $f(x + e)$, reducing and clearing remaining “e-terms”. The example analysed here (divide a line AC at a point E such that rectangle ACE area is maximized) involve a term in the form of $f(x) = bx - x^2$.

First excerpt (Katia and Mitia):

Mitia says he doesn't understand why his [Fermat's] approach works. Katia says the same.

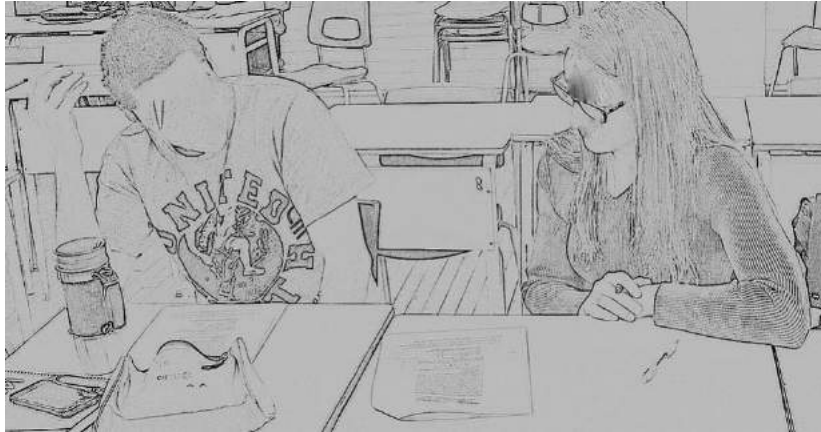


Figure 4.3: Katia and Mitia reading Fermat

Mitia then read the first paragraph aloud and tries to know the meaning of the unknown e .

Katia gives the hypothesis that e is a variable and that one must find for which value of e the area is maximum. Mitia doesn't see what it comes about. Katia then represents the function to be maximized and a second one whose side is increased by e . She emphasizes that it is difficult to represent the division by e with this geometrical representation.

Mitia notices that e is 0. Martha, from the other side of the classroom, indicates that Fermat previously divides by e . Mitia wonders, "How could he divide by 0?". He concludes that " e is not worth 0, but not far". Katia continually tries to illustrate the procedure geometrically, while Mitia invalidates her reasoning, claiming that the value of e is zero. Katia disagrees.

Second excerpt (Martha, Aliocha and Ninotchka):

Martha is saying that e is a very small value.



Figure 4.4: Martha, Aliocha and Ninotchka reading Fermat

Aliocha is trying to reconcile Fermat's method with the basic elements of modern calculus. He asks: "if adequating means to subtract the terms". Ninotchka answers that "adequating means simply to equalise".

Then Aliocha asks how she relates Fermat to modern calculus. Ninotchka shows his calculations and Aliocha concludes that their reasoning is equivalent.

After a few moments, Martha points out that Fermat removes e . Aliocha indicates that “ e is almost 0, so the multiplication by e also gives almost 0”. Martha asks herself whether the reader should “decide on the value of e ”. Aliocha replies, “Yes”. Martha emphasizes that there is something missing in the reasoning. Aliocha asks why Fermat is using symbol of inequality, and concludes that adequating means to reduce to the minimum.

In this study, the investigation points out two interrelated experiences lived by the prospective teachers: *otherness* and *empathy*. Participants have shown serious efforts to understand the historical texts without uprooting them from the context in which they were produced. Indeed, the associated experience of otherness in mathematics seems rough from a cognitive and affective viewpoint, and it may lead to violent responses. For Levinas, violence is a “thematization of the Other”, a reification of the Other, a way to make the Other a “Mine”. Empathy and violence have been observed. Indeed, the subjectivity of the authors is sometimes arduously preserved. The students not always maintain an empathic relation with the authors. This violence can result in the disappearance of the empathic relationship. The authors are then dispossessed of their peculiarities; they are translated, summarized and reified. For us, in a Levinassian perspective, there is a violence of synchronization (Levinas, 1987).

Looking to engage the participants of the workshop in a reflection around these elements, the group was asked the following questions aimed at suggesting a trace of reflection in order to prepare the written work (see section 5):

- Would you agree that their way to engage in the reading of Fermat’s text is related to ethics? In what sense?
- In what sense did they (or did not) actually show an ethical relationship regarding Fermat in the excerpts?
- How Levinas can help us to understand the encounter with Fermat (from the researcher’s or the educator’s viewpoint)?

Globally, the ethics to which we refer in these excerpts are that of the interaction I-Other. Indeed, some ethics that subvert the traditional principles! In a sense, the Levinassian approach goes back to the origins of the traditional ethics. We do not made reference to the ethical responsibility of the teacher as in Boylan (2016). The students showed their ethical involvement in reading the historical documents deepening their interpretation, not limiting to the comprehension of the procedural aspects. In that way, they started an infinite process, as Dilthey pointed out when speaking of the possibility to know the writer and considering “the limits of all interpretation, which is able to fulfil its task only up to a certain point. For all understanding always remains partial and can never be completed. Individuum est ineffabile” (1996, p.249).

5 Final writing activity

For the final writing activities, participants were invited to work together in small groups. Most of them accepted that proposal but some preferred to work individually. The suggested task was to answer, in a written form, the questions below. During the

work, we talked to the participants. Some participants preferred to only orally discuss their opinions with others and not to write down their answers. After the workshop, we (the authors) met and shared some oral remarks of the participants.

5.1 List of questions

The questions to the group were:

1. From the educational point of view, what are the differences between Pacioli's and Finé's documents? Does the visualization of people help in the Levinassian perspective?
2. How can a student find the "presence" of the Other in a historical document?
3. Describe your agreement about the following statement: 'For students, analysing historical documents is an experience of extreme alterity, positive *per se*. Students, though, are sometimes attracted by that experience and sometimes discouraged by it'.
4. Suppose that instead of debating about Fermat's *Methodus ad disquirendam maximam et minimam* and posing questions about the sense of the procedure, Katia and Mitia might, for instance, have only tried to use the method in another example. Do you agree that their behavior is related to Levinas' ethics?
5. What did you treasure of Levinas' philosophy in this workshop?
6. Which links can you make between Levinas' philosophy and the history of mathematics in mathematics education? What has ethics to do with history of mathematics in mathematics education?
7. Do you agree that not considering mathematical and, generally, scientific knowledge related to the Other's involvement leads to negative consequences? Do they regard mathematics education? (See excerpt from *Difficult Freedom*)

The rationales for each question are the following.

Question 1 aims at discussing the relationship between secondary school students and authentic historical documents. The question requires comparing the document of Pacioli with the one of Finé. After the presentation of Levinas' philosophy, we assumed that the participants could have submitted answers related to the fact that Pacioli's document is somewhat obscure so that students could hardly find *another* in it. On the contrary, Finé's document shows a person acting with an instrument that suggests the presence of mathematics.

Question 2 is connected with question 1 and proposes a more general reflection inherent in the hermeneutical task to comprehend the other. Any reference to Levinas is missed out so that the question does not have a rhetorical role. Instead it is proposed to the participants in order to gather their opinions.

Question 3 completes the previous two and focuses on the concrete problem of the class. On the whole, the three questions propose different levels of reflection: class, historical documents, Levinas' philosophy. Sometimes the focus is apparent and sometimes the problem remains open to different references and elaborations leading to the answer.

Question 4 compares both a real and a hypothetical situation in order to reflect on the meaning of ethical involvement of students who read a historical document. In that case, the final question can be considered rhetorical because it requires to relate

the educational situations with what we consider the first point of Levinas's philosophy.

Through question 5 we wanted to inquire to what extent our proposal, in its general aspects, passed on to the participants.

Question 6 ideally completes question 4 and question 5. After the previous referred both to the class (with prospective teachers, in that case) and to Levinas, the focus is now on history in mathematics education.

Question 7 has a complementary role with the main points of the Levinassian philosophy we presented. It broadens the reflection to the responsibility of science and to the role of mathematics education: a different way to consider the role of ethics.

During the workshop, we realized that some participants already knew Levinas' philosophy. Therefore, they were able not only to give their opinions about the questions and the situations we submitted to them, but also to establish connections with works of other authors and to come up with suggestions about the workshop activities.

5.2 Resume of the writings

The following points are the transcription of the answers written by the participants during the workshop (the numbers correspond to the previous List of questions; the alphabet letters identify each participant. Please note that some of them did not answer all the questions).

1.

A. I think it is the "appearance" of Infinity and Totality.

B. Arithmetic – Algebra vs. Geometry. Visualization. Has to do with Levinas perspective, is a way of seeing, understanding – cure of other.

C. Yes, absolutely the visualization helps.

D. Pacioli's task is described via words. However, Finé's document offers a picture to help understand. There is no doubt that visualization helps students to read.

E.---

2.

A. He finds a lot of strange and unknown methods, ideas and practices.

B. The other is in expression, symbolism, other ways of written a text which is different from ours.

C. Maybe to put in context, it is the representation in this time, the discussions about the infinity concept in these times...

D. Different persons offer diverse viewpoints.

E.---

3.

A.---

B. Of course, it is a "dépaysement", they are trained on that they prefer little text, clear question.

C. Yes, I agree. It is like try to understand other languages. So the visual representations can be useful in this task. For instance, to know what kind of artefacts have in these times helps to understand the kind of productions awarded by the mathematicians in determined times.

D. Students might be interested in how historical situations came about. Their curiosity can be served as a stimulus to solve the problem themselves. Meanwhile, due to the era differences, students can also be discouraged by the solution mathematicians figured out. They vary from different notation to ideas.

E. (*Translation of the original in French.*) I totally agree with the statement: for a student (a majority of them) analysing historical documents is an experience of extreme alterity (differently from one another). More generally, learning something new is related to an absolute (more or less high) alterity. The student or pupil is tackled within his convictions, ideas which he lived, heard, learned before. Ideas that don't fit with the teaching that has occurred. The evidence acquired is now in question. What to do? Either to say no to the past for an alterity absolutely disturbing or to submit ourselves at a reality that is not ours? Here the work of the teacher begins and goes beyond the simple learning of notions.

To confront with historical text, not because they are ancient, but because they go beyond a conceptual level, because they destroy barricades, because the barricades had to be destroyed, and they access to other barricades that have to be destroyed, and so on, a path that the student will have to take himself.

4.

A.---

B. No, because they are in procedural way and not conceptual.

C. ---

D. Their behaviours are related to the Levinas's ethics as students (also us teachers) learn more from different approaches. In the process of listening and discussing with others, we learn.

E.---

5.

A. I was attracted very much and I have to mention that this was my first contact with his work. I leave from the workshop full of questions, quite interesting.

B. Alterity – otherness, subjectivity, resemblance

C.---

D. Otherness brings us with fresh ideas, approaches to activate our thinking.

E. The quest for the absolute: tiring, useless and alienating. No thanks! Indeed, the statement that teaching is, in a way, violence upon the other seems to me an idea more than interesting, more than judicious.

6.

A. In an aspect, we face several forms of relationships, the notion of power and how we use it.

B. Understanding the others (different period texts, ideals, goals)

C.---

D. Levinas' philosophy puts emphasis on otherness, which promotes students with abundant opportunities to get familiar with problems and inspire them to think more as alternatives are provided with.

E.---

7.

A. In some way I agree.

B. Yes.

C. Absolutely yes! The students tend to think that the mathematics algorithms or procedures are impersonal and disconnected with humanity problems. Other bad beliefs which are consequence of this “not considering” is, for instance, the mathematicians never wrong and all problems have a only one solution way...

D.---

E.---

F. (Not a particular question, just thoughts.)

Students usually want a method they can follow, which can be violence. But from the excerpts they are m??? wondering, trying to understand, finding obstacles while doing it. Then they adjust and renew their own understanding.

They need to respect the others understanding by not just taking it over.

They learn by making their own understanding and keeping the other alive with respect.

When you get to know an other you need to care about the other person and not make the other yours by thinking you understand them.

5.3 Resume of the oral commentaries

Some participants who chose not to write their answers preferred to give oral commentaries. Just after the workshop, we took note in a debriefing.

Participant G. He posed the problem of identifying the Other and highlighted the difficulties to “choose” proper manifestations of the Other for ourselves during a history-based activity. He pointed out the example of the hat of the person depicted in the image from Finé, underlining that it probably had nothing to do with mathematics. From that, he raised the pedagogical problem of how to lead students to approach a historical document, and to help them to focus on mathematical concepts.

H. She noted that Levinas can help to think about mathematics itself as something that has inherently to be done with that experience of otherness. This helps one thinking of mathematics as a way to be-with-the-others.

I. He insisted on the question “What/Who is the Other?” From a more strictly philosophical point of view, he spoke of the necessity of a “mutual transformation” of identities.

L. He made a remark about the notion of trace. Presence and absence are modalities of the manifestation of the Other. This implies the responsibility for the teacher to make the otherness of the Other to appear.

M. She insisted on a non-violent relation with the other as something necessary for learning. There is the necessity of making room to the Other within the act of learning, by letting the Other show himself up.

N. She referred to Arthur Rimbaud’s “Je est un autre” [I is anOther], as a way to think about human subjectivity as something multiple, permeable, etc.

O. He referred to Jean-Luc Godard “Si vous m’avez bien compris, c’est que je me suis mal exprimé” [If you understood me well, it means that I didn’t express myself well], as a way to think about the violence of the act of understanding.

P. She considered multicultural class issues, where history is a way to think about different possibilities of teaching mathematics and different capabilities of students in learning it.

6 Discussion about the activities

Just after the workshop, we reflected on the fact that participants' answers could surely have been influenced by the reference to the Levinas' thought: we feared that it might even have happened in contrast with their real convictions. To be honest, in some sense the questions have to be considered a sort of exercises with respect to Levinas's works. In such perspective, it would have been better to express question 1, for example, in a more direct form: "[...] What aspect of Levinas' thought do you consider inherent to the visualization of ...".

Here, we recall the previous answers focusing on some points, looking for connections and possibly sketching unitary discourses. This has the ideal purpose to broaden the dialogue we had with the participants also to the readers of these Proceedings.

About question 1, participants A and B use terms that are particularly meaningful in the Levinassian perspective, that is: "Infinity" and "other"; the latter sketches a reasoning about images as connections with the Other. Participants C and D prefer to highlight the use of images in mathematics, specifically in geometry, in order to facilitate students' understanding.

Question 2. After B, students could track the other through the unusual symbols and expressions, and also through particular ways to write the text they see in the historical document. From the previous historical document, two examples of particular ways to write a text can be the absence of a typographic distinction between problem and solution in Pacioli's excerpts (Fig. 4.1 and Fig. 4.2) and Fermat's use of equalities in the "*adequating*" method. After A, the student can learn that also in mathematics there are different viewpoints. C highlights the pedagogical opportunity to refer to the context in which mathematical concepts were used, so that they appear as the expressions of a specific historical period.

Question 3. The answers underline the possibility that students be discouraged while working with historical documents, but individual differences can be found. By means of the keywords "little" and "clear", the answer of B gives a hint to discuss the ways by which students are used to approach mathematical tasks. We think that this can suggest that students are not used to tackle complexity. In addition, C (speaking of understanding "languages") introduces the theme of the difficulties that students meet and have to overcome. Both these answers pose an educational problem that is how the teacher can help student facing non-trivial mathematical problems (we consider, among them, also the interpretation of historical documents). Students' curiosity or specific tools such as visualizations can be resources to be exploited. E discusses how students live alterity: his general utterance seems to derive from the specific cases of documents interpretation. Alterity is identified with "learning something new" and the necessity for students to always overcome new "barricades". This suggests reflecting on an attitude for school success, i.e. the willingness to tackle difficulties. After Levinas (2010, p. 213, English edition) "[t]he relation with the Other, discourse, is not only the putting in question of my freedom, the appeal coming from the other to call me to responsibility, is not only the speech by which I divest myself of the possession that encircles me by setting forth an objective and common world, but is also sermon, exhortation, the prophetic word". We would like to read this passage in these terms: "the relation with the Other", present in the historical

document, who offers him/herself to me through a discourse inherent in mathematics, “is not only the putting in question of my freedom”, i.e. staying in my closure and ignorance, without attention for Other’s, conditioning, proposal, “the appeal coming from the other to call me to responsibility”, i.e. to being open to the content of the document and being ready to tackle difficulties and surmount obstacles that I can meet in understanding, “is not only the speech by which I divest myself of the possession that encircles me”, by which I put in discussion my previous knowledge about historically contextualized mathematics, “by setting forth an objective and common world”, by accepting to discuss the problems the document presents to me, “but is also sermon, exhortation, the prophetic word”: the Other, I recognise inside the document, is the mirror of the *Third Party* that is the community of mathematicians. The Other offers himself to me without violence, shows me a goal, leads me through his/her proposal and so gives me motivation. So, a problem with the class rises: are we able to help students to see in mathematics (in history and documents) the Other who can “exhort” them in their effort with mathematical problems?

Question 4. D proposes a general reflection on Levinas’ ethics referred to the dialogue with the Other in learning. He relates the behaviour of Katia and Mitia with Levinas’ ethics. B seems to give the reason for considering their behaviour as ethical: because they are working with reference to conceptual aspects and not (only) to procedures.

Question 5. The term “Otherness” is present in two answers (B and D). The last contribution (E) mainly refers to violence (to deepen this, see Guillemette, 2017).

Question 6. The answers confirm that this question could be considered a synthesis of the previous five. Again, they propose the theme of alterity (“relationship”, “other”, “otherness”) again. We note references to history and mathematics education in the use of the following key words recalling the answers to question 2: problems and alternatives, different period texts.

Question 7. The answers report three levels of agreement. The last one focuses on educational problems and suggests a few more examples presumably regarding mathematics as a socio-cultural process. We expected that the document from Levinas’s *Difficult Freedom* could lead participants’ reflection toward explicit remarks regarding the social use of science.

Participant F opens and closes his “thoughts” by referring to the concept of violence. The first reference recalls what Levinas says in his Preface to *Totality and Infinity*: violence toward persons is “making them carry out actions that will destroy every possibility of action”. Let us consider students who are requested to perform methods or procedures they do not understand: will they be able to act by using them in a new situation, for example in solving a problem? We believe the answer is no. Nevertheless, we, as teachers, too often force our students to follow rules that, in their eyes, have neither justification nor usefulness. Insisting on our reflection on the fact that students “want a method” to follow, we can say that they seem to have internalized the customs of the person (teacher) who commits violence against them. They seem to require behaving like that person even if, in that way, they suffer violence.

We propose here a categorization of written and oral answers, considering the different kinds of elaborations that the participants offered. In brackets, for the written

answers, the question numbers are written close to the participants' capital letters; for the oral commentaries, O is followed by the participants' letters. Underlined are those answers that touch one or more aspects - out of a, b, c, d, e, f – in case they do not match the corresponding question. We consider that question 1 is inherent in the aspects a, f; question 2 in a, d, f; question 3 in a, d, f; question 4 in a, d, f; question 5 in a; question 6 in a, f; question 7 in a, e. Please, note repetitions highlighting the fact that some answers touch different aspects; remind that F wrote some thoughts without reference to any specific questions. Some answers that express agreement without any specifications do not appear in the following categorization.

Categorization of written and oral answers:

- a. References to the Levinas' thought (1A, 1B, 4D, 5A, 5B, 5D, 5E, 6A, 6B, 6D, OG, OH, OI, OL, OM, ON, OO)
- b. Connections with other authors (ON,OO)
- c. References to mathematical contents (1B)
- d. References to students or class (1B,1C,1D, 3B, 3C, 3D, 3E, 4B,6D,7C, F, OG, OP)
- e. References to mathematics education (3C,3E,4D,5E, F, OG, OH, OI, OL, OM, OO)
- f. References to the history in mathematics education (2A, 2B, 2C, 2D, 3C, 3E, 6A, OG, OP)

The above categorization suggests some reflections about the outcomes of the dialogue that took place during the workshop. The added categories b and c touch aspects that we recognize absolutely relevant to the topic of the workshop. The small number of participants who referred to them does not diminish their value, being suggestions that only indirectly derive from the questions we posed. *A posteriori*, we recognize that the presence of b in some questions could have led the participants to a critical position with our presentation of Levinas' thought. Concretely, the reference to other authors could have been limited to the quotations of their points of view, excluding a further comparison between their theories. The presence of c in some questions could have given more concreteness to the reflection, but it is not to forget that the aim of the workshop was to focus on methodological aspects.

The fact that some participants did not choose to write their answers, preferring instead oral interventions, suggests the limits of writing as recording tool with respect to others, i.e. audio or videotaping.

It is noteworthy that one of the participants helped us, by email, to 'interpret' his own written answers a few weeks after the end of the workshop.

The answers are often short and seem only to give strength to what the corresponding questions suggested. We can note that, in the cases in which the answers are wider, the participants either chose to answer only some of the questions or left out the reference to specific questions. That fact could suggest the organizers to further diminish the number of the questions and to make them less analytic. For example, it would have been preferable to unify the first two questions rather than refer to the historical documents given during the presentation of the workshop, and also should have been more important to insist on alterity in historical documents. Question 5 should have occupied the first place and, moreover, question 6 should have contained the final line of question 4. A different possibility might have been the

one of substituting all the questions with keywords, with an introductory statement requiring the participants to write down some reflections or suggestions inherent in them, or their free observations about the contents and structure of the workshop. Question 7 could have been deleted because the reflection on mathematics education it proposes seems too wide and even too vague.

7 Conclusions

When we got the announcement for ESU8, at the beginning we thought to submit a proposal of an oral presentation, but to be structured in a different way that is in dialogical form. We thought that brevity would help us to reduce complexity in our experiment. Dialogue would give the possibility to introduce ‘Levinas in action’, as interactions between the same and the others. This would subvert the traditional structure of oral presentations in ESU Conferences and would contrast with the participants’ expectation. We concluded that a workshop could be the best location for a dialogue on Levinas and for approaching his thought, in case for the first time. Moreover, organisers suggested a two hours’ workshop, instead of one and a half-hour one. We were aware of our responsibility in organizing more complex activities. We were also aware that participants would tackle a difficult task, even having a longer time at their disposal. We were afraid for the fact that none of them might have ever approached Levinas before: what could be the best way to introduce him? Would the traditional presentation be enough? What kind of activities to involve participants? And in case of a not immediate positive response?

According to the seminal character of the workshop, a more realistic aim was arousing interest in the theme and stimulating involvement in the activities. We consider that this aim was reached. Moreover, in the last part of the workshop we were asked by a participant for some suggestions about how to deepen Levinas’ thought. We concluded that articles and books regarding mathematics teaching or history in mathematics education could help to focus Levinas’ role in those perspectives (*e.g.* Boylan, 2016; Maheux, 2013; Radford, 2012; Roth, 2011; Roth & Radford, 2011). Moreover, other secondary sources about Levinas’s works could help to see him inside the history of philosophy, comparing him, for example, with Descartes, Husserl or Heidegger, in order to focus on the aspects of innovation in his thought and on his prophetic message.

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