HERMENEUTICS, AND THE QUESTION OF "HOW IS SCIENCE POSSIBLE?"

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ABSTRACT

The present paper discusses the educational potential of reading historical sources in the mathematics classroom under the viewpoint of hermeneutics. In a first step, key concepts of Gadamerian hermeneutics will be discussed, 'application', 'hermeneutic circle' and 'prejudice'. Taken in their entirety they characterize what Gadamer calls the 'historicity of understanding' which reflects the intellectual situatedness of any reader and provides a foundation to the idea that any text allows a multitude of possible interpretations. In a second step, three readers of ancient Greek texts on astronomy are studied in order to exemplify the functioning of Gadamer's hermeneutic conception. The readers are students (grade 9), the pedagogue Martin Wagenschein (1896 – 1988) and the philosopher Karl Raimund Popper (1902 - 1994). We show their different intellectual backgrounds, different questions and problems they were interested in and the quite different ways of reading the sources. Nevertheless, there exists an important point of convergence. They all understood the epistemological relevance of their ancient sources culminating in the question of "How is science possible?". Indeed, this question is and should be a central theme of education.

1 Introduction

The present paper discusses the educational potential of reading historical sources in the mathematics classroom under the viewpoint of hermeneutics. In recent times, reading historical sources has become a fruitful avenue of theoretical and practical educational activity on all levels of the teaching of mathematics. Some references might highlight the scope and variety of work (Arcavi & Isoda 2007, Kjeldsen & Blomhøj 2012, Barnett et al 2013, Jahnke 2014, Chorlay 2016). A nearly overview on these activities is given by Jahnke et al (1990), a more recent survey can be found in Clark et al(2016)

Beyond that, there is in many countries a trend towards a stronger emphasis on activities of modelling. Frequently, this requires tasks with extensive texts for detailing circumstances and conditions of the modelling problem in question. Therefore, interpreting and writing texts enters the mathematics classroom from this side, too. In sum, language, reading and writing will take up a more extended space in today's teaching of mathematics.

We approach our problem by looking in more detail on the process of reading and interpreting a text and ask what hermeneutics, that is the theory and methodology of interpreting texts, has to contribute to this problem. We shall see that hermeneutics will allow a twofold perspective. There is a *dynamical* perspective insofar reading and interpreting changes the text and the reader. After the very act of reading, the text is not the same as before, and also the reader has changed. Thus, hermeneutics makes understandable the necessary *variety of possible interpretations* which is not an outcome of vague thinking but of a process involving rigour and imagination.

The argument of this paper will be unfolded in four steps. First, some key concepts of Gadamerian hermeneutics will be discussed. Afterwards, we study three (types of) readers illuminating Gadamer's concept of the 'historicity of understanding' and leading to the

theme of the variety of possible interpretations. We study students (grade 9) reading an ancient source on astronomy, a pedagogue and a prominent philosopher. This part of the paper has some relations to Fried (2018) though its conceptual frame is different. In the end we shall return to the educational potential of reading sources in the mathematics classroom.

2 Gadamer's hermeneutics

Modern hermeneutics originated from methods used for interpreting theological and legal texts. It was only in the 17th and 18th centuries that some authors formed the idea of a general theory and method of interpretation independent of the subject to which it was applied. In Germany, probably for the first time, the Latin term 'hermeneutica' in this general sense can be found with Johann Conrad Dannhauer (1603–66) whereas Johann Martin Chladenius (1710–59) und Georg Friedrich Meier (1718–77) wrote influential books on this topic. (Joisten 2010, ch. 6). It was Chladenius who introduced the concept of 'Sehe-Punkt' as translation of the Latin 'scopus' (engl. 'point of view') which aimed at the idea of different interpretations depending on the 'Sehe-Punkt' of the reader. (Joisten 2010, 88ff, Gadamer 1990, 186). Modern hermeneutics then as an established field of study emerged in the beginning of the 19th century with the romantic movement and historicism in the work of theologian F. Schleiermacher (1768-1834) (cf. Schleiermacher 1977) and others.

Gadamer's (1900 – 2002) 'philosophical hermeneutics' can be considered as an attempt to mediate between the 'hermeneutical philosophy' of young Heidegger (1889 – 1976) and traditional hermeneutics of the 19th century. The relation between these two points of reference still seems to be a philosophically unsolved problem (Scholz 2011, 444). Thus, it is not without difficulties to refer to Gadamer's *Truth and Method* (Gadamer 1990, originally published in 1960, English translation Gadamer 2004)¹ as a theoretical frame for analysing processes of understanding texts. The present paper tries to circumvent systematic philosophical problems by pragmatically limiting and concentrating the discussion on three key concepts of Gadamer's hermeneutics, namely 'application', 'hermeneutic circle', and 'prejudice'. These concepts are introduced and elaborated in the middle part of his book in chapter II 'Elements of a Theory of Hermeneutic Experience' (TaM, 203ff) ('Grundzüge einer Theorie der hermeneutischen Erfahrung' (WuM, 270ff)).

Application. With religious and legal texts the problem of arriving at an unambiguous and 'correct' interpretation receives a special urgency. Laws should be binding rules of behaviour within a community or a state which are valid for everybody. The same is true for religious texts. Obviously, a community can only function when laws are applied to any concrete case in the same way. But what could 'the same way' mean? Already in the 14th century, Italian jurist Bartolus distinguished between three possible types of interpretation, a 'declarative' one explaining only the meaning of the words, a 'restrictive' one limiting the set of cases to which the law was to be applied, and an 'extensive' interpretation which widens the number of cases. (Schröder 2011, 206). In principle, when the meaning of the words did not suffice to apply a law jurists had to take into account the 'sense' of the law. In such a situation, jurists would appeal to 'reason' or to general

¹We refer to Gadamer (1990) as: WuM, to Gadamer (2004) as: TaM. We shall quote only the English translation TaM to which we add the page numbers of the German original WuM.

principles of jurisdiction and frequently arrived at rather extensive interpretations. But from the times of renaissance up to the 19th century there happened a shift of argument and jurists would point more and more to the will of the legislator and the special circumstances under which the law was issued. This shift contributed considerably to a growing awareness that understanding a text must take into account its author and his times and, thus, has a necessary historical dimension. This development substantially furthered the emergence of general hermeneutics (cf. Schröder 2011).

Gadamer is well aware of this development, but in his considerations on 'application' he stresses another point of view. He considers 'application' "just as integral a part of the hermeneutical process as are understanding and interpretation" (TaM, 318; WuM, 313). As he explains, the decisive point with 'application' is

"that understanding always involves something like applying the text to be understood to the interpreter's present situation. Thus we are forced to go one step beyond romantic hermeneutics, as it were, by regarding not only understanding and interpretation, but also application as comprising one unified process" (loc.cit.).

The insight that understanding a text necessarily involves considering the historical situation and the author as a historical person was already reached in 19th century romantic hermeneutics and historicism. To Gadamer the new element was the application of a text "to the interpreter's present situation". This point of view he exemplifies with several cases. An interpreter in a negotiation should not simply repeat what one of the partners says in the discussion he is translating, but he should say it in a way that seems most appropriate to him, since he alone knows both languages (loc.cit.).

In a similar vein a reproduction of a historical piece of music is more than a mere reproduction. Rather, the player has to take into account "that the stylistic values of one's own day... set limits to the demand for a stylistically correct reproduction" (TaM, 320; WuM, 315).

Returning to the relation of theological and legal hermeneutics to general hermeneutics he says:

"The fact that philological, legal, and theological hermeneutics originally belonged closely together depended on recognizing application as an integral element of all understanding. In both legal and theological hermeneutics there is an essential tension between the fixed text—the law or the gospel—on the one hand and, on the other, the sense arrived at by applying it at the concrete moment of interpretation, either in judgment or in preaching. A law does not exist in order to be understood historically, but to be concretized in its legal validity by being interpreted. Similarly, the gospel does not exist in order to be understood as a merely historical document, but to be taken in such a way that it exercises its saving effect. This implies that the text, whether law or gospel, if it is to be understood properly—i.e., according to the claim it makes—must be understood at every moment, in every concrete situation, in a new and different way. Understanding here is always application" (loc.cit., 318/9; WuM, 314)

Thus, the core of application is the fact that a text must be understood "at every moment, in every concrete situation, in a new and different way." Whether we read a love poem or a historical document on an administrative act we inevitably relate it to our situation, our ideas, concepts, emotions, phantasies, former experiences, former studies etc. That is, we apply the text. And by applying the text we add to it connotations and dimensions of meaning the author necessarily could not have thought of.

Hermeneutic Circle. In Jahnke (2014, 84pp) and Fried et al (2016, 216ff) the reader will find a short account of the hermeneutic circle applied to reading sources in the mathematics classroom. Hermeneutics distinguishes systematically between the author and the reader of a text and their different perspectives. This causes an experience of dépaysement (Barbin 1994). Gadamer speaks of the 'temporal distance' ('Zeitenabstand', TaM, 303; WuM, 296) and of different 'horizons'. Understanding amounts in his terminology to a 'fusion of horizons' (TaM, 310ff) ('Horizontverschmelzung', WuM, 305ff). In hermeneutics the process by which the fusion of horizons occurs is described by a spiral, the so-called 'hermeneutic circle' which points to the necessity of already possessing an interpretation of a text in order to gain a new interpretation. A reader starts with a certain expectation what the text might be about. Then s*he reads the text and realizes that some aspects of the expectation, read again, modify and so on until s*heis satisfied with the result. Here is one of Gadamer's descriptions of the spiral:

"We know this from learning ancient languages. We learn that we must 'construe' a sentence before we attempt to understand the linguistic meaning of the individual parts of the sentence. But the process of construal is itself already governed by an expectation of meaning that follows from the context of what has gone before. It is of course necessary for this expectation to be adjusted if the text calls for it. This means, then, that the expectation changes and that the text unifies its meaning around another expectation. Thus the movement of understanding is constantly from the whole to the part and back to the whole. Our task is to expand the unity of the understood meaning centrifugally. The harmony of all the details with the whole is the criterion of correct understanding. The failure to achieve this harmony means that understanding has failed" (TaM, 303/4; WuM, 296)

We single out from this quotation two aspects.

(1) The hermeneutic circle is a process of adaption. Successful interpretation means that the harmony between the expectations of the reader and the text is step by step enhanced. This might be visualized by a diagram.



Figure 2.1: Hermeneutic Circle (cf. Glaubitz 2011, 61)

(2) Gadamer describes the process of adaptation as a dialectical oscillation between whole and part. This might refer to the interplay between the meaning of a single word and the meaning of a phrase in which a word occurs. In further steps the reader has to take into account the meaning of, say, a paragraph in its interplay with the whole text in front of her/him. In this way ever larger units of text have to be taken into account, say, a collection of texts up to reconstructing, for example, the philosophical thinking of Greek antiquity. The dialectic of part and whole is a principle problem of understanding, experienced when reading a piece of literature as well as of mathematics.

Seen from the dialectic of part and whole it is teachable to consider a subtle difficulty in the English translation above. The English speaks of a 'change' of expectation whereas the German original uses the word 'umstimmen' instead of 'change'. The literal translation of 'umstimmen' is 'retune', a musical metaphor which expresses much better the intended holistic meaning.

In regard to the hermeneutic circle I would like to point at the fact that it is quite analogous to the spiral of modelling and can be considered as a process in which a hypothesis is put up, tested against the (empirical) data, modified, tested again and so on until the reader arrives at a satisfactory result. With modelling, too, it is an important point of view that it aims not only at a better and better representation of the problem in question, but that it is also dependent of the situation and the needs of the creator of the model.

Considered in this perspective hermeneutics can be related to the "hypotheticodeductive method". This is basically the argument by Foellesdal (1979) who elaborates it by interpreting a piece of literature (Ibsen's Peer Gynt). Gadamer, too, in his late years agreed that there is a certain reconciliation of his philosophical hermeneutics and certain directions of thinking in modern analytic philosophy (Gadamer 1976, 1070).

The conception of the hermeneutic circle was already well known in the 19th. But people at that time thought that in the hermeneutic spiral the reader approaches better and better the 'real meaning' of a text by putting oneself better and better into the historical situation of its author. Though 'real meaning' might only be a regulative ideal which can never be reached, it was the aim of interpretation in the eyes of Schleiermacher and Dilthey. Gadamer's conception is radically different. He thinks that interpretation and understanding necessarily involve also an adaptation of a text to the thinking of its reader such that a new meaning emerges which the original author never would have thought of. This is a motive we have already seen in the concept of 'application'. It will become fully clear with the concept of 'prejudice'.

Prejudice. With the concept of prejudice Gadamer takes up an essential element of the philosophy of his teacher Heidegger. There is a whole word field of interrelated concepts stemming from Heidegger and being used by Gadamer. They are: Vor-Urteil = pre-judgement, Vor-Meinung = fore-meaning, Vor-Entwurf = fore-projection, Vor-Sicht = fore-sight, Vor-Griff = fore-conception. The basic message enshrined in these concepts is "to elevate the historicity of understanding" to the status of a hermeneutic principle (TaM, 284; WuM, 270). Historicity here does not mean to consider a text as an historical document, but the fact that the very act of understanding itself happens at a certain moment in time and is an act in history. Any act of understanding starts with a certain "fore-conception" or "pre-judgement" (the P1 in fig. 1).

"A person who is trying to understand a text is always projecting. He projects a meaning for the text as a whole as soon as some initial meaning emerges in the text.

Again, the initial meaning emerges only because he is reading the text with particular expectations in regard to a certain meaning. Working out this fore-projection, which is constantly revised in terms of what emerges as he penetrates into the meaning, is understanding what is there." (TaM, 285/6; WuM, 271)

Thus, only "the recognition that all understanding inevitably involves some prejudice gives the hermeneutical problem its real thrust" (TaM, 286; WuM, 274). Any reader of a text has his personal intellectual history which itself is embedded in the culture of his time. This determines the perspective under which s*he approaches a text. Prejudices are not an obstacle, but a condition of understanding. However, prejudices become a problem when we remain unconscious of them. "It is the tyranny of hidden prejudices that makes us deaf to what speaks to us in tradition" (TaM, 286; WuM, 274).

In order to become conscious of our prejudices we have to uncover them as much as possible.

"Indeed, what characterizes the arbitrariness of inappropriate fore-meanings if not that they come to nothing in being worked out? But understanding realizes its full potential only when the fore-meanings that it begins with are not arbitrary. Thus it is quite right for the interpreter not to approach the text directly, relying solely on the fore-meaning already available to him, but rather explicitly to examine the legitimacy—i.e., the origin and validity—of the fore-meanings dwelling within him" (TaM, 284; WuM, 272).

Hermeneutic understanding requires from the interpreter "to be open to the meaning of the other person or text" (TaM, 285; WuM, 273) and to be permanently aware of the alterity and temporal distance of the text (or the other person).

In a separate chapter Gadamer discusses "The discrediting of prejudice by the Enlightenment" (TaM, 288ff; WuM, 276ff) and by historicism and maintains that being conscious of our prejudices neither "involves ... 'neutrality' ... nor the extinction of one's self, but the foregrounding and appropriation of one's own fore-meanings and prejudices" (TaM, 286; WuM, 274). The term "self-extinction" had been used by historian Ranke to designate the neutrality of a researcher in doing history (TaM, 229; WuM, 215). Gadamer uses a whole chapter to uncover the hidden prejudices in the views of historians Ranke and Droysen in order to show the illusionary character of the idea of a 'neutral' observer (TaM, 215-233; WuM, 177-221).

On the basis of the crucial role of fore-meaning and prejudice Gadamer proceeds to a rehabilitation of authority and tradition (TaM, 292ff; WuM, 305ff) and the principle of 'history of effect' ('Wirkungsgeschichte') (TaM, 292ff; WuM, 305ff). In this way, he thought to have established a new foundation for the human sciences different from Dilthey's. We are embedded in tradition, and tradition suggests concepts and questions we pose in regard to the texts we study.

To sum up, according to Gadamer 'application' and 'prejudice' are inherent components of every act of interpretation and responsible for the 'historicity of understanding' and explain the necessity and legitimacy of different understandings of a text. Interpreting a text is not adequately described as a reconstruction of the original 'true' meaning, but a construction of a new meaning the original author of the text would not have thought of.

Gadamer is vague in delimiting 'application' from 'prejudice'. When a reader thinks of an example for a statement in a text, does s*he apply the text or embed the text in her/his intellectual background and, thus, refer to her/his prejudices? A first approximation might be provided by looking at the human sciences as a whole. On the one hand there is the internal functioning of science, and a scientist studying a text would first ask for the state of research and embed her/his research problem into this context. On the other hand, there is the 'application' of human sciences which consists above all in contributing to societal debates on, say, ethics, aesthetics, politics and culture in general.

Résumé. (a) In a principle way Gadamer's conception of hermeneutics leads to an unlimited *variety of interpretations* of a text. This is a necessary consequence of his concepts of **application** and **prejudice** and the resulting **historicity of understanding** which reflect the intellectual situatedness of the reader. Since in many cases it is difficult to distinguish between application and prejudice we shall henceforth only talk of the historicity of understanding.

(b) The variety of interpretations does not mean that interpretations are arbitrary. On the contrary, implicit in Gadamer's approach is a high demand on the **internal quality of argument**. We mention three points. (1) The spiral process of the 'hermeneutic circle' is in principle infinite and requires ever subtler and precise arguments for reaching harmony between part and whole, and may require the consideration of ever larger collections of texts. Gadamer does not give "rules" of interpretation, and it is a matter of judgement whether harmony is reached and an interpretation is successful. (2) Any interpreter is subject to the requirement that he has to get conscious of his prejudices and to make them explicit. (3) Any interpretation has to respect the temporal distance and the 'alterity' of the text. In a principle way, the distance between the text and an interpretation cannot be reconciled. Out of these insights we conclude that in a very high amount interpretation requires what a mathematician should have learnt, namely rigour.

In the following we study three different types of readers in order to explore the relevance of Gadamer's concept of 'historicity of understanding', and begin with schoolchildren.

3 Students (grade 9) as Readers

The following analysis refers to a teaching experiment described in Glaubitz & Jahnke (2003a). The experiment was conducted in a classroom of 26 students aged 15 to 16 years



Figure 3.1: The source

and comprised 6 lessons. The students were to read two fragments from an ancient Greek booklet on astronomy, 'The Heavens' by Cleomedes (English transl.: Bowen & Todd 2004; German transl.: Kleomedes 1927). Since nothing is known about the author the dates of his life are quite controversial, estimates running from 100 BC to 400 AD (the latter guess by O. Neugebauer). Today, there exists consensus that he should have lived in the second century AD. Textual analysis shows him as an adherent of stoic philosophy (Bowan & Todd 2004, 5ff). Cleomedes' booklet gives a survey on astronomy for the educated public of his time, themes ranging from shape, position and size of the earth over movements of moon, sun, planets, fixed stars to special questions like f.e. the difference between solar and sidereal day. The students read fragments about (1) the shape of the earth and (2) the method of Posidonius to determine the size of the earth. On concrete experiences with reading sources on ancient astronomy and about the importance of this topic cf. Hosson 2015 and Tzanakis 2016.

(1) In regard to the shape of the earth Cleomedes discusses the alternatives of being a plane, a bowl, a cube, a pyramid, or a sphere (Kleomedes. 1927, 26ff; Bowen & Todd, 2004, 65ff). We used an abbreviated version of the respective passages in Cleomedes' book. On account of observations on the visibility of sun, moon and stars the first four alternatives are excluded, thus the earth is a sphere. If, for example, the earth were a plane the sun and the stars would rise and set at all places at the same time. Since, however, it is well known that the sun rises in Persia four hours earlier than in Spain this cannot be the case.



Figure 3.2: Student drawings. The earth as plane, bowl, sphere

(2) Posidonius' (135 - 51 BC (?)) method for determining the size of the earth uses the bright fixed star Canopus (Kleomedes, 1927, 33ff; Bowen & Todd, 2004, 78ff). At the time of its declination it is elevated 7.5° above horizon in Alexandria, whereas in Rhodos



Figure 3.1: Parallel rays



Figure 3.2: Nonparallel rays, drawing by a student

it is just visible at horizon. Since Alexandria and Rhodos have approximately the same longitude it follows that these cities have a difference of latitude of 7.5°. According to Posidonius/Cleomedes the distance between Alexandria and Rhodos amounts to 5 000 stades which implies a circumference of the earth of 340 000 stades. Figure 3.3 shows the situation with the simplification of parallel rays from Canopus to Alexandria and to Rhodos, whereas figure 3.4 is a student drawing with non-parallel rays from a pointshaped Canopus.

The students had substantial difficulties to deal with the simplification that the rays from Canopus to Alexandria and to Rhodos can be considered as 'practically parallel' (figure 3.3) because of the enormous distance of the fixed stars from the earth (cf. similar experiences in Hosson 2015). They were not ready to accept it as legitimate since it seemed to contradict the absolute precision characteristic of mathematics. On the other hand, it seemed much more acceptable to them to consider Canopus as a mathematical point (figure 3.4) in spite of its enormous size. Of

course, this is understandable because fixed stars appear to us visually like points at the sky (Glaubitz & Jahnke 2003a, 83pp).



Figure 3.3: sun and cube-shaped earth

Even more difficult was the discussion on Cleomedes' argument against a cube-shaped earth. In this case, according to Cleomedes, the sun would be above horizon at every place only for six hours a day (figure 3.5). But this is only true when the sun at every moment illuminates only one face of the cubic earth, and this again is valid only when the sun moves on a special circle which touches the edge of the cube at the moments

of rising or setting. But in general the sun would illuminate three faces at the same time. Thus, Cleomedes' argument is highly problematic. Even more problematic are his reasons against a bowl-shaped earth (Glaubitz & Jahnke 2003a, 81). Thus, the students found themselves in a situation which required from them a critical evaluation of a text which at first they must have considered as a scientific authority simply because their teachers had given it to them.

The source required from the students in a considerable amount to revise their expectations or in Gadamerian terms their prejudices. The students expected that the ancient Greeks considered the earth as a plane. The source taught them that they considered it as a sphere. The students expected that there was only the alternative plane vs. sphere. The source discussed five alternatives, plane, bowl, cube, pyramid, sphere. Against their expectations of an earth travelling around the sun, according to the source the earth doesn't move. Angles are not measured in degree, but in ratios of the zodiac. The students thought that Cleomedes was an authority whose arguments are always correct. Instead of this, they had to accept that he used superficial, even incorrect arguments. Students were convinced that mathematics is always exact. They learnt that (applied) mathematics makes use of simplifications.

Résumé. The students had to cover a considerable distance from their original expectations to an understanding of the source. There was a permanent tension between their modern astronomical knowledge, their prejudices, and the statements and arguments of the source. In a certain amount they were aware of the historicity of the source. Their guiding question however seemed more that they wanted to learn about the astronomical problems discussed than to learn about history. Obviously, they applied the text. Nevertheless, the historical distance was present to them. This became clear from a small questionnaire they were asked to fill in. There one can find several statements of the type: "I found fascinating how somebody could arrive at the idea to prove that the earth is not flat by such simple things." (Glaubitz & Jahnke 2003a, 88pp) This shows, some students realized that reading such texts is also teachable under an epistemological point of view.

4 A Pedagogue as Reader

The pedagogue considered is Martin Wagenschein (1896 – 1988). He took a PhD in Experimental Physics at the university of Gießen in 1921. From 1924 to 1933 he worked at the 'Odenwaldschule', a boarding school and prestigious project of 'Reformpädagogik'. In the 1920s the school had a high reputation with many visitors from abroad. Children of quite a number of prominent people attended the school. The time there became the pedagogically formative period of Wagenschein's life. In his autobiography he spoke of the "magic of this school" (Wagenschein1989,34). "At the Odenwaldschule 'exchange',

not 'instruction' became for me the unshakable base of teaching." (loc.cit., 38) In 1927, also mathematician Otto Toeplitz visited the school for two days, and attended among others a course by Wagenschein (loc.cit., 34).

In 1933, the founder and director of the Odenwaldschule, Paul Geheeb, emigrated from Germany, together with some teachers and students, in order to found a new school in Switzerland, and Wagenschein changed to a position at a state driven school. In his autobiography he described the Nazi-time in a chapter headed "Wartezeit" ('waiting period'), and the time immediately after the war by the verb "aufatmen" ('drawing a deep breath'). He continued teaching at school until 1957, and gave courses for teachers of physics and mathematics at the Technical University of Darmstadt, the University of Tübingen and other institutions of teacher training quite until his old age. It was only after 1950 that he became a visible and prolific writer of articles and books on didactics of physics and mathematics.²

Obviously, Wagenschein had a strong and coherent vision of good teaching, and he described it in a language full of metaphors. He taught at a regular school, but criticised the established school system in a rather fundamentalist manner speaking f.e. of the "tragedy of the teaching of mathematics" (UVeD I [1961], 417-428;) which consisted in his eyes in an one-sided emphasis on 'passive knowledge' (he used the German word 'Stoff'). According to him, students frequently cannot connect their knowledge to real phenomena, and he complained about too early a formalization.



Figure 3.4: Martin Wagenschein in 1983. Source: Wikipedia

It is not easy to describe Wagenschein's approach to teaching for the simple reason that he never developed a 'system of pedagogy'. This might have been a consequence of the fact that he never held a professorship at a university, but it is more probable that creating a 'system' did not fit to his way of thinking and his personality. Thus, it seems appropriate to begin with some observations in order to get an image of his visions.

Figure 3.6 shows Wagenschein in a teaching situation at his age of 87. One sees children dealing with some artefacts

of a physical experiment (the theme was Pascal's barometer) and, presumably, describing what they observe. Wagenschein himself is attentively listening, and this is the message of the picture. A teacher should, first of all, be able to remain silent and to listen to his students. This is so because for Wagenschein the main problem of teaching was to connect concepts and theoretical insights with the phenomena they are to explain. In order to become really aware of the phenomena there should be broad opportunities for students to describe them in their own language, and independent of the 'official' language of science.

Out of this motive Wagenschein experimented with language. A wonderful example worth reading is a small piece of two pages with the title "Das große Spüreisen" ("The big

² Many of Wagenschein's papers are reprinted in Wagenschein 1970a (=UVeD I) and Wagenschein 1970b (=UVeD II). For reasons of space we do not mention the title of every paper, but always add in square brackets the year of its first publication. All translations into English by the author.

feeling iron") (UVeD I [1951], 175pp) which at the time of its publication caused a real scandal among physicists and teachers of physics (Wagenschein 1989,79). The German word 'Spüreisen' is a creation by Wagenschein whose English translation is only a first approximation. The word is intended to arouse a connotation of magic like f.e. the English word 'divining rod'.

The 'feeling iron' is an oversized magnetic needle of 1 meter length whose slow oscillation around the north-south direction Wagenschein described in a completely animistic language attributing something like a free will to it. The description ends with the sentence "It lasted nearly a quarter of an hour until our feeling iron came to rest. It had to work hard to find its peace." (UVeD I [1951], 176).

Wagenschein has written many of his texts in a pronounced emotional and existentialist language, take as an example the title of another paper from 1951 "Mind and heart in the acquisition of exact scientific knowledge" (UVeD I [1951], 181ff). Therefore, it is no wonder that he frequently made reference to Simone Weil's book (1949) on "Enracinement", since the 'enracinement' ('taking roots'; 'Einwurzelung') of phenomena in the minds of students was his great theme. (f.e. UVeD II [1966], 60-62)

In the course of time Wagenschein standardized his approach by using the triad of concepts "genetic – Socratic – exemplary" (see Wagenschein 1968). The three concepts have a long tradition in pedagogy, as a triad they might be adequate to characterize his thinking. Wagenschein explains: "Pedagogy has to deal with genesis: the growing human being and ... the genesis of knowledge inside him. The Socratic method is involved since genesis, the awakening of his intellectual forces, happens most effectively in a conversation or a dialogue. The exemplary principle is involved because a genetic-Socratic procedure must and also can confine itself to limited themes ('Themenkreise')" (UVeD II [1966], 68). In regard to a Danish reception of Wagenschein's ideas on exemplary teaching cf. Blomhøj & Kjeldsen (2009).

The language motive for reading sources. In the course of his life Wagenschein seems to have read quite a number of original texts by famous scientists, f. e. Aristarch, Foucault, Galilei, Leonardo da Vinci, Kepler, Lichtenberg, Newton. The list is by far not complete. In his autobiography, in the chapter on his university studies he deals extensivelywith reading such sources under the revealing heading 'Lichtenberg and other masters of language' (Wagenschein 1989, 25-30). Language and the distance to textbook knowledge seemed to interest him most. As a consequence, historical sources became part of his intellectual life, and reading them was subordinated to his search for authentic, peculiar, even idiosyncratic linguistic representations of observations and phenomena far off the standardized language of modern science.

Wagenschein stated at several places "Genesis is not history" quoting explicitly Otto Toeplitz (f.e. UVeD II [1965], 78). "The history of his science ... is for the teacher not a mere subject but helps him to take the questions of his students as serious as they are meant." (loc.cit.) At another place one finds a sentence like this one. "We recognize this less from textbooks than out of the history. ... The 'old' researchers are in reality the young ones, the early ones." (UVeD II [1967], 26;).

There is a piece of eight pages presumably written in 1962 and published as appendix C to Wagenschein (1995) under the title "Genetic teaching (history of human ideas). The didactical significance of studying sources, shown by the example of the law of inertia". To my knowledge this is the only paper where he explicitly discussed a possible

significance of sources for teaching. The general message of this paper is twofold. (1) He addresses the teacher, not the student as a possible reader of a source. (2) For the teacher sources should be a key to the questions and the thinking of his students. As far as I can see, he never used sources in teaching at school, but there is at least one reference to a seminar he gave to future teachers on "Didactical suggestions from the writings of the scientific pioneers of the 17th century" (Wagenschein 1995, 288).

The epistemological motive for reading sources. Beyond his interest in the not standardized language of historical sources there is also another fundamental reason why he was interested in the history of physics and mathematics. This is an epistemological one and closely connected to the great importance he attributed to ancient astronomy as a subject of teaching mathematics and physics. Time and again he returned to Aristarchos' method of determining the relative distances of sun and moon from the earth (cf. Jahnke 1998). As is well known, this method consists in observing sun and moon at the moment of half-moon in a case when moon and sun are both visible. From the angle moon (M) – earth (E) – sun (S), one can determine the ratio of the distances of moon and sun from the earth (Figure 3.7).



Wagenschein is not so much interested in the numerical side of Aristarchos' method, but in the underlying geometric and qualitative understanding of the phases of the moon. Many students and many adults after a long time of attending school have a lot of astronomical textbook knowledge. They know about the sun as the centre of the solar system, the earth travelling around the sun, the moon travelling

Figure 3.5: Aristarchos' method

around the earth, the size of the sun being many times larger than the size of the moon and the earth, etc. However, when asked to explain the phases of the moon a majority of students and adults would refer to the shadow of the earth as a cause for the crescent figure. Since on the other hand it is very easy by observing sun and moon when they are simultaneously visible at the sky to realize that there is no shadow of the earth involved and to arrive at the 'right' idea this is paradigmatic for the basic failure in the teaching of mathematics and physics. In a sense textbook knowledge prevents students from observing and thinking (UVeD II [1966], 59). At some places Wagenschein uses the concise wording "verdunkelndes Wissen" ('obscuring knowledge') to designate this phenomenon (UVeD II [1966], 58). Thus, we have to regain "the primary phenomenological reality" (Wagenschein 1995, 291) and Wagenschein calls this process the "genetic metamorphosis of science" (UVeD II [1965], 87).

To regain the primary phenomenological reality meant to "let students understand how human beings can come to know such things"or to answer the question **"How is science possible?"** (Wagenschein 1995, 292). This Kantian-type question was a running theme in his pedagogical thinking, and he explained it in a pathetic, but illuminating way in an early publication:

"The relation of human beings to nature is mysterious since it touches deeply the enigma of their own existence: they belong to it [to nature] and, yet, are able to pose themselves opposite to it. Therefore, the question of how we gain and pass on scientific knowledge is a humanist question; it concerns the whole human being ... science is the trace of a one-sided ... expression of human nature... It concerns all

the more the whole human being since s*he must know this one-sidedness when s*he wants to remain whole..." (UVeD I [1951], 182)

Thus, answering the question of how human beings can come to know is a requirement of humanism, and this in turn implies the fundamental importance of ancient astronomy for teaching science and mathematics. In this area one can observe with the naked eye or with very simple instruments and derive from these observations by means of simple geometry far-reaching consequences about the structure of our universe.

This means in regard to the example of the phases of the moon that the teacher should not start with the above figure 7, or alternatively, with a lamp (representing the sun) being moved around a tennis ball (the moon) and try to let students see the crescent figure. According to his experience, this will not work. Instead: when we suppose that moon and sun are really physical bodies (which was not obvious to the Greeks) and that the sun is a shining body whereas the moon receives light from the sun, then we can conclude by pure imagination a fundamental fact: **always** is one hemisphere of the moon illuminated by the sun, and one hemisphere is dark. Only on the rare occasions of an eclipse of the moon this is not the case.

With this fundamental idea in mind students are asked and guided to observe the moon every day during a certain period, say 2 weeks. In fact, they are not asked to observe only the moon, but to observe the pair moon – sun. When both are visible simultaneously, that is during day-time, this is no problem. But what about when it is night? In this case we have to add the sun in our mind's eyes to the visible configuration by looking in which direction the illuminated hemisphere of the moon is pointing. When we prolong this direction beyond horizon we will intuitively get an estimate of the position of the sun and after a time intuitively realize that the sun must be distant from the earth many times farther than the moon, and, since its apparent size is equal to that of the moon, must be many times larger than the moon. Thus, in a combination of imagination, thinking and observation anybody can get, after some training of his eyes and his imagination, a correct qualitative intuition of the configuration earth – moon – sun without any measurement, any technical instrument and without any numerical calculation. Obviously, measuring and calculating are necessary further steps leading to new insights and to new surprises (UVeD I [1951], 184ff).

The phases of the moon are a fundamental paradigm for Wagenschein's views on history of science and of 'regaining the primary phenomenological reality'. No wonder then, that he was interested in authentic descriptions of this phenomenon he found in historical sources. In an Italian – German edition of Leonardo da Vinci's "philosophical diaries" he hit upon a short remark of da Vinci's (Ms. Arundel 94r) on the phases of the moon he liked so much that when quoting it he rearranged it in a way that it looked like a poem (UVeD II [1966], 67). Here it is.

"Der Mond hat kein Licht von	"La luna non ha lume da sè,	"The moon has no light out of
sich aus,	se non quanto ne vede il	herself,
und soviel die Sonne von ihm	sole,	and as much as the sun sees of
sieht,	tanto l'allumina;	her,
soviel beleuchtet sie;	della qua lluminosità,	as much he illuminates;
und von dieser Beleuchtung	tanto ne vediamo	and of this illumination
sehen wir soviel,	quanto è quella cheve de	we see as much
wieviel davon uns sieht."	noi."	as much of it sees us."

We come back to the motive of 'poem' in the next section of this paper.

Résumé. Wagenschein had a strong vision of good teaching formed by his personal experiences at the 'Odenwaldschule' and he was immersed in the pedagogical tradition of 'Reformpädagogik'. He quoted Toeplitz' statements implying that genesis is different from history. He saw historical sources as a valuable component of the intellectual life of a teacher of mathematics and physics, but, presumably, did not think about reading sources with children and students at school. His central problem was the regaining of the primary phenomenological reality and in this context attributed a high relevance to the history of science and, especially, to ancient astronomy. He considered answering the question of "how is science possible" as a requirement of humanism. He was also very much interested in the linguistic dimension of historical sources, and, by this, also the literary and artistic quality of sources became important.

5 A Philosopher as Reader

The philosopher considered is Karl Raimund Popper (1902 - 1994). Since 1918 he took courses on mathematics, physics, psychology, philosophy at the University of Vienna, and in the early 1920s he engaged in a social-democratic student organisation, worked in street construction and made an apprenticeship as cabinet maker. In 1924 he passed an examination as a teacher at elementary schools, andin 1928 he earned a PhD in psychology under Karl Bühler. From 1929 to 1937 he worked as a teacher of mathematics and physics at a secondary school. In 1937 he obtained a position as lecturer of philosophy at the university of Christchurch/New Zealand. In 1945 he changed to the London School of Economics, and in 1949 was appointed professor of logic and scientific method at the University of London (cf. Popper 1976b and the article on Popper in the English Wikipedia).

Popper was one of the most influential philosophers of the 20th century who is wellknown to a broader public far beyond specialized philosophy. His book *The Open Society and Its Enemies*', published for the first time in 1945, in which he criticized the in his view totalitarian components of the philosophies of Plato, Hegel and Marx, fitted to the situation between west and east in the 1950s and, surely, contributed a lot to this public visibility.

In 1934 appeared his major work *Logik der Forschung* ('The logic of scientific discovery') which established his philosophy of 'critical rationalism' and, biographically, helped him to get the position in New Zealand and to emigrate shortly before the Nazis entered Austria.

Base and starting point of Popper's 'critical rationalism'is his fierce rejection of traditional inductivism and sensualism. Instead, he held the view that theories can never be (finally) confirmed by empirical evidence – a philosophical position which came to be called 'fallibilism'. Popper considered scientific progress as guided by a permanent interplay between the creation of theories as bold conjectures and the search of scientists torefute them, be it by critical argument or by falsifying evidence (cf. the title of Popper1976a).

Relation to ancient astronomy. According to his own testimony Popper was interested in and deeply impressed by the Presocratic philosophers since the time when he was 16 years

old, and he cultivated this interest all of his life (Popper 2006. 88). But it was only in 1956 that he published a paper entitled 'Back to the Presocratics' in his *Conjectures and Refutations* (1976a, 1st edition in 1956). Since the 1970s, on advise of the later editor A. Petersen, Popper began to think seriously about writing a book on Parmenides and the Presocratics (Popper 2006. 9ff). He wrote and rewrote a number of papers including a preface, but the book itself appeared only after his death (Popper 1998).

Popper read fluently ancient Greek, all translations of Presocratic Sources into English used in the book are his own. He knew much of the philological and philosophical literature on Presocratic philosophy and did not hesitate to enter even philological arguments where he found this necessary.

Parmenides (520? – 450? BC) was a student of Xenophanes, the teacher of Zeno and lived in the newly founded Greek colony of Elea in Southern Italy (Popper 1998, 139).We know of Parmenides' philosophical thinking by a poem in hexameters in the style of Homer and Hesiod of which only 180 lines out of estimated 800 lines have been passed on to us. According to Popper, Parmenides' work "is beset with problems that perhaps will never be solved" (loc.cit.). Nevertheless, Popper tried to develop an interpretation whose tentative character becomes clear from the fact that his book on Parmenides contains several different attempts.

An overarching motive in Poppers interpretation was the thesis that Parmenides was "essentially a cosmologist" (Popper 1998, 143), in contrast to many modern philosophers who consider him above all as an 'ontologist'.

Parmenides' poem contained a revelation from the goddess Dike in two distinct parts. In the first part, the goddess reveals the truth about what really exists. This is called the **way of truth**. In the second part, the goddess speaks about the world of appearances, the illusory world of movement, change, development (loc.cit., 138). This way is called the **way of opinion**. The following is a short account of Popper's interpretation.

The way of truth proceeds **purely rational** and by **logical proof**. It is the way of the Gods. The Way of Opinion is the way of the mortals who believe in **sensory perceptions**. Popper interprets Parmenides as a rationalist who rejects explicitly sensualism (loc.cit., 103).

The **real world** which the way of truth uncovers by rational deduction is very simple. It is a universe without change or movement. "This universe consists of one well-rounded spherical block that is completely homogeneous and structureless. It has no parts: it is one. It has no origin and thus no cosmogony, and it always was and is and always will be at rest, changeless and colourless." (loc.cit., 140)

By contrast, the **appearing world** as it is seen by the mortals is a universe in which there is change, movement and development. The Goddess describes also this world in detail, and this part of the poem contains important and original ideas, such as the doctrine of the spherical shape of the earth and a theory of the moon. (loc.cit., 139). Thus, any interpreter of Parmenides' poem is confronted with the paradoxical situation that the best achievements of Greek astronomy of the time which in part might have been reached by Parmenides himself are attributed to the illusory world of appearances. To say it in Popper's words: "Why does the goddess expound part 2 at all, stressing that it is mistaken?" (loc.cit., 124)

According to Popper "Parmenides was the first who consciously placed reality and appearance in opposition and consciously postulated one true unchanging reality behind the changing appearance" (loc.cit., 140).

How did Parmenides arrive at this distinction and to understand its importance? Popper's attempt to answer this question points to the *phases of the moon*. This is an essential issue in his book. There are three chapters around this idea with slightly modified titles and representing different attempts at describing the idea and its context. The titles are "How the Moon might shed some of her light upon the Two Ways of Parmenides" (1992), "How the Moon might throw some of her light upon the Two Ways of Parmenides" (1989) and "Can the Moon throw light on Parmenides' Ways?" (1988).

Popper's explanation is rather straightforward. "Parmenides discovered that the observation ... that the Moon — Selene — waxes and wanes during the course of time is false. ... She does not change in any way. Her apparent changes are an illusion." (loc.cit., 108). "The moon does not change. It is a material sphere of which one half is always illuminated, the other half is always dark." (loc.cit.) Therefore, in eternity the moon does not change. This eternal moon is 'being'. On the other hand, the changing shape of the moon from new moon to crescent to half-moon to full moon is mere appearance. It does not really exist; it is 'not being'.

Of course, to a modern reader and, may be, also to an ancient reader, this case might at best be an example making understandable the distinction between reality and appearance. It is a stupendous step from there to Parmenides' radical conception of a rigid universe without movement and change. Taking into account the fragmentary state of Parmenides' poem it is clear that much subtler arguments as are given here are required to bridge this gap, andany attempt at interpreting Parmenides is necessarily highly hypothetical as Popper himself stressed.

There is another important point. To realize that a hemisphere of the moon is always illuminated seems to be only a small step. But we have seen in the last section how excited Wagenschein was about it, so much so, that he rearranged a short remark of da Vinci expressing this in the form a poem. The same excitement we find with Popper/Parmenides. When reporting how, as a boy of 16 years, he hit upon the Presocratics Popper said: "The verses that I liked best were Parmenides' story of Selene's love for radiant Helios (DK 28 Bl4—15). ... before reading Parmenides' story it had not occurred to me to watch how Selene always looks at Helios' rays ...

Bright in the night with the gift of his light, Round the Earth she is erring, Evermore letting her gaze Turn towards Helios' ray"

(loc.cit., 88/89, Popper's translation of Parmenides). And he added: "Since the day when I first read these lines (in Nestle's translation), 74 or 75 years ago, I have never looked at Selene without working out how her gaze does indeed turn towards Helios' rays (though he is often below the horizon)." (loc.cit., 89). At another place he added: ",,I personally am indebted to him [Parmenides] for the infinite pleasure of knowing of Selene's longing for Helios..." (loc.cit., 130)

To consider Parmenides' phrases as a love poem about Selene (the moon) and Helios (the sun) is only weakly suggested by the Greek wording. Only the half sentence that

Selene is always looking for Helios' rays might be in favour of this interpretation. Therefore, it is a (de)construction by Popper, similar to Wagenschein's poetic deconstruction of da Vinci's short remark. Both, Wagenschein and Popper, show a sensitivity to the artistic quality of their sources and, thus, create a personal and individual relation to them.

Résumé. Popper's book on Parmenides and the Presocratics is a contribution to the history of human ideas. The author sees himself as part of the historiographic and philosophical tradition and meets philological standards. His guiding problem is the search for ideas which are essential in the context of his own philosophical outlook, critical rationalism. A key element of his argument is the thesis that the Presocratics, and, especially, Parmenides can best be understood from the point of view of ancient astronomy. Beyond the main line of argument, as a certain surprise, the artistic quality of Parmenides' poem plays an important part.

6 Conclusions

Gadamer's concept of the 'historicity of understanding' which he concretized in the two concepts of 'prejudice' and 'application' implies that there is necessarily an unlimited variety of legitimate interpretations of a text. Any interpretation depends on the intellectual background and the situatedness of the reader. Any new interpretation by a new reader deepens our understanding of a text, understanding is the 'sum' of all existing interpretations and, thus, open to future. At the same time, reading requires rigour and judgement.

Our three readers: the students, the pedagogue and the philosopher have, of course, different intellectual backgrounds, different questions and problems they were interested in and quite different ways of reading their sources. Nevertheless, there was an important point of convergence. They all understood the epistemological relevance of their sources. Indeed, to answer the question of "How is science possible?" is and should be a central objective of education. Ancient astronomy (not necessarily Greek as Hosson 2015 shows) is particularly suitable for this aim.

Wagenschein has made us sensitive to the special role of language. This is a particular quality of reading historical sources which cannot be replaced by any other educational activity. The language of a historical source provides opportunities of reflection which cannot be arrived at by the standardized language of textbooks. Additionally, language, particular words or phrases, might cause the reader to feel touched and addressed in a special way. Then s*he will build up a relation to a text which may last over a long time as the 'poems' by the pedagogue and the philosopher show.

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