Views on usefulness and applications during the sixties

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Abstract

At the Royaumont Seminar (1959) the New Math reform was officially launched. In the decade between Royaumont and the first ICME congress in Lyon (1969), many mathematics educators were involved in actions to facilitate the implementation of the New Math reform. The New Math advocates were convinced that a deep knowledge and understanding of the structures of modern mathematics was a prerequisite to arrive at substantial applications, but in actual classroom practices the applied side of mathematics was often completely neglected. But already in Royaumont there were alternative voices who pleaded for taking the role of applications seriously. We investigate the arguments for integrating applications in mathematics education, as well as the kind of (new) applications that were envisaged, at the Royaumont Seminar and in the decade thereafter.

Keywords: applications and modelling; Hans Freudenthal; ICME-1; Modern mathematics; Royaumont Seminar

Introduction

The OEEC Seminar, held from November 23 to December 5, 1959 at the Cercle Culturel de Royaumont in Asnières-sur-Oise (France) is considered as a turning point in the history of mathematics education in Europe and in the United States (De Bock & Vanpaemel, 2015a). As Bjarnadóttir (2008, p. 145) stated: "The Royaumont Seminar can be seen as the beginning of a common reform movement to modernize school mathematics in the world". Or in the words of Skovsmose (2009, p. 332): "After the Royaumont seminar, modern mathematics education spread worldwide, and dominated a variety of curriculum reforms". The famous slogan "Euclid must go!", launched at Royaumont by the Bourbakist Jean Dieudonné, became a symbol of the radical modernization of school mathematics. Most of the Royaumont proposals were strongly influenced by Bourbaki, the French structuralist school whose members or adherents, such as Gustave Choquet, Jean Dieudonné, Lucienne Félix and Willy Servais, were well represented at the Seminar. According to these scholars, the basic model for modernizing school mathematics should be the academic discipline of mathematics, as re-constructed and formalized from the late 1930s on by Bourbaki.

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Less well known is that alternative reform proposals, emphasizing the role of applications, were also voiced at Royaumont. These too, were inspired by new developments in the field of applications during World War 2. The application-oriented proposals were however less decisive for developments during the 1960s than the dominant structuralist ideas. These ideas, especially on what should be taught at school about the (axiomatic) basis of mathematics, determined the debate. In this chapter, we first discuss the kind of (new) applications that were envisaged by some Royaumont lecturers, as well as their pleas for integrating the applied side of mathematics in secondary school curricula. This discussion will be based on *New Thinking in School Mathematics* (OEEC, 1961a), the official report of the Royaumont Seminar. Second, we examine the views of the more radical New Math reformers on applications or more generally, on the usefulness of mathematics. Third, we follow the debate on applications and modelling in the mathematics education community between Royaumont and the first ICME congress in Lyon (August 2431, 1969). For several reasons, this decade is less well documented in the history of our field. At that time, *L'Enseignement Mathématique*, the only international journal on mathematics education until the late 1960s, had become a purely mathematical journal (see Furinghetti, 2009) and the international conference series which are now strongly established in our field (ICME, PME, ICTMA, HPM, …) had not yet started. An exception might be the annual meetings of the International Commission for the Study and Improvement of Mathematics Teaching (CIEAEM), founded in the early 1950s, but CIEAEM only started publishing Proceedings of their meetings in 1974 (Bernet & Jaquet, 1998).

Although there were no strong international communication channels in the mathematics education community during the early- and mid-1960s, it was a very rich period of noteworthy international activity, including several seminars and symposia organised under the auspices of OEEC/OECD, UNESCO or ICMI (see, e.g., Furinghetti et al., 2008). These meetings mainly focused on issues related to the forthcoming introduction of New Math (program development, renewal of geometry teaching, teacher (re-)education and new didactical methods), but occasionally, concerns and proposals about the integration of applications in the curricula were expressed too. On the basis of the Proceedings and other edited documents from these meetings, we more generally review the visions on the role of applications in the international mathematics education community of that time.

By the end of the 1960s the debate on applications and modelling gained momentum. Hans Freudenthal, at that time president-elect of ICMI, organized the international colloquium 'How to Teach Mathematics so as to Be Useful' (Utrecht, August 21-25, 1967). The contributions to that colloquium were published in the first issue of *Educational Studies in Mathematics* (May, 1968). In his introductory address, Freudenthal took the opportunity to explain his views on the colloquium's theme. He argued that students could not be expected to (be able to) apply the mathematics they had been taught in a purely theoretical way. Instead, to enable students to apply the mathematics they have learned, mathematics education should start from concrete contexts and patiently return to these contexts as often as needed (Freudenthal, 1968). It is the beginning of a new era in which applications and modelling gradually became an essential part of mathematics education. In the Netherlands, the theory and practice of 'Realistic Mathematics Education' (RME) were developed and inspired the teaching of mathematics in a large number of countries worldwide (Van den Heuvel-Panhuizen, 2018). We conclude this chapter with a more detailed discussion of Freudenthal's ideas on applications and modelling in the years preceding ICME1.

The focus of this chapter is on what happened in Europe and North America between 1959 and 1969. As for other aspects of mathematics education, the role of applications during this decade is not well documented, with exception of a paper in French by Hélène Gispert (2003). On one side, Gispert's contribution has a broader scope: it provides more information about the decade preceding Royaumont, for instance, about the discussion on applications of mathematics during the International Mathematical Union (IMU) congress in Amsterdam (1954). On the other side her contribution goes into more detail about what happened in France, more specifically with respect to Bourbaki, who was mostly interested in the general structural aspect of mathematics and not in mathematics education at the secondary level. With respect to the decade between Royaumont and Lyon she focuses on the OECD conference in Athens (1963), where not much progress was made with respect to the teaching of applications of mathematics. According to Gispert, the reason was that mostly professors in pure mathematics participated and discussed the direction of the New Math reform.

Applied mathematics at the Royaumont Seminar

The Royaumont Seminar was organised by the Office for Scientific and Technical Personnel (OSTP) of the Organisation for European Economic Cooperation (OEEC; later joined by nations outside Europe to form the Organisation for Economic Co-operation and Development, OECD). The Office was created for the purpose of promoting international action to increase the supply and improve the quality of scientists and engineers in OEEC countries (OEEC, 1961a). The main motive for OEEC/OSTP to organise a Seminar aimed at upgrading mathematics education was clearly economic: industry and other branches of economic activity were confronted with new applications of mathematics leading to a demand for more mathematicians with new kinds of skills. Therefore, a re-appraisal of the content and methods of school mathematics was needed. In his opening address, Marshall H. Stone, at that time president-elect of ICMI, formulated the functional argument as follows:

> […] the usefulness of mathematics in practical matters has been an added factor in its vitality as a component of the school curriculum. In this period of history, it is the rise of modern science and the ensuing creation of a technological society which compels us to give increasing weight to the utilitarian arguments for the more intensive teaching of mathematics (OEEC, 1961a, p. 17).

Stone also emphasized the need for a better coordination between mathematics and science teaching: "It is not going to be sufficient to improve the mathematical curriculum as an isolated part (…). It is of the first importance that instruction in mathematics and in the various sciences should be adequately co-ordinated" (OEEC 1961a, p. 21).

In view of the above, the Royaumont Seminar should thus have been a breakthrough of an applied and interdisciplinary perspective in mathematics education, but it turned out differently. Due to the dominance at the Seminar of professional mathematicians, most of them members or adherents of the French structuralist school, pure academic mathematics was de facto adopted as a model for school mathematics and most participants only paid lip service to the active application of mathematics. For instance, Dieudonné admittedly referred to applications to theoretical physics as a main argument for the inclusion of new topics in university courses of analysis, but left open the question whether any kind of 'applied mathematics' should already be integrated in the secondary school programs. Nevertheless, he believed that a favourable consideration of his reform proposals, having a clear Bourbaki orientation, would already provide the theoretical foundations for teaching questions of applied mathematics (OEEC, 1961a).

An alternative voice at Royaumont was that of Albert W. Tucker, a Canadian mathematician at that time working at Princeton. Tucker discussed the aspect of new uses of mathematics and their implication for mathematics education. Rather than study problems which involve two variables – or at most three or four – as most problems in classical physics, new branches of mathematics were developed to deal with complex realities involving several variables, which often occur in the social sciences, for instance in economics and psychology. Within these realities, Tucker distinguished problems of 'disorganised complexity' and problems of 'organised complexity'. The first category referred to problems with numerous variables and asked for techniques of probability theory and statistical interference, being effective for describing 'average behaviour'. Problems of organised complexity involved a sizable number of factors which were inter-related into an organic whole and required, among other things, a knowledge and use of matrix algebra. Tucker exemplified this last category with a problem of linear programming, utilising inequalities, intersections, graphic methods, and unique algebraic procedures for solving equations. According to Tucker, integration in all secondary-school programs of these newer types of mathematics, in a suitable form, was feasible and desirable. He however acknowledged that an effort was needed to enhance teachers' knowledge about modern mathematics and its applications to teach the subject well (OEEC, 1961a).

Tucker's plea for the integration of probability theory and statistics in secondary-school curricula was supported by Luke N. H. Bunt from Utrecht University (the Netherlands). Bunt presented at Royaumont the outline of a syllabus on this subject matter taught in a Dutch experiment for the alpha streams of secondary schools (for more details on this experiment, see, e.g., Bunt, 1959):

- Some elements of descriptive statistics, such as frequency distributions, histograms, mean, median, and standard deviation.
- 'Classical' probability theory, with proofs of some of the elementary theorems.
- Intuitive treatment of binomial probability distributions; application to physics.
- Testing of a hypothesis (Bernoulli type of distribution); null hypothesis; level of significance; sample space; critical region; confidence limits; sign test; rank correlation. Only Type I errors (accepting a false hypothesis) are considered.

(OEEC, 1961a, p. 91)

For Bunt the problem of estimating some characteristics of a population on the basis of the values of these characteristics in a sample should be the dominant objective of a course in statistics. Bunt's proposal went against the general trend of the Royaumont Seminar because (a) he did not primarily focus on those mathematically gifted students that would become mathematicians or engineers, but on future students in economics, psychology and other social sciences, and (b) his didactical approach was pragmatic rather than mathematically rigorous (see also De Bock & Vanpaemel, 2015b).

New Math reformers' view on applications and modelling

Although at Royaumont and in the decade thereafter, there were several calls for mathematical instruction to take applications of mathematics seriously (Niss et al., 2007), New Math, strongly focussing on theoretical academic mathematics, was – at least in continental Europe – the dominant reform paradigm. Originally, the ambitions of the New Math reformers and practitioners' call for a focus on useful mathematics were not in contradiction, or as stated by Niss (2008):

> It is worth noticing that despite the strong theoretical orientation of the New Math movement, its founders insisted that one of the points of the reform was to provide an ideal platform for dealing with the application of mathematics to matters extra-mathematical (p. 72).

Claims about the omnipresence and increased usability of (modern) mathematics can be found in many contemporary sources. In the *Charte de Chambéry*, a main French reform document prepared by the 'Commission Lichnerowicz' and adopted by the Association des Professeurs de Mathématiques de l'Enseignement Public (APMEP), the broad usability of modern mathematics is emphasized (and used as a main argument for the reform of mathematical teaching at all educational levels).[1]

> Contemporary mathematics is useful in many fields: theoretical physics of course, but also computer science, operational research, stock management of companies, organization charts of big administrations, planning for major projects, sociology, linguistics, medicine (diagnosing), pharmacy ... (Charte de Chambéry, 1968).

Georges Papy, the architect of the new mathematical curriculum in Belgium and president of the CIEAEM during the mid-1960s, wrote in the Preface of *Mathématique Moderne 1,* the first volume of his pioneering textbook series:

> The scope of the material studied in the first 13 chapters [sets and relations] goes far beyond the boundary of mathematics. The student is initiated into types of reasoning constantly used in all spheres of thought, science and technology (Papy, 1963, p. vii).

In 1968, Papy further clarified his position. In his view, the 'mathematization of situations' was the way education should prepare students to applications of mathematics:

> Thus, students are immediately accustomed to an approach which is essential for applications: the mathematization of situations. Obviously, it is difficult to predict the kind of mathematics that will be used by the students later. In the modern world, mutations are common. Many people, during their lifetime, have to change of profession several times and, in any case, of technical skills in their own profession. Mathematics does not escape from this phenomenon. […] We do not know how to predict which situations will be mathematized later, nor which mathematics will be used for that purpose, but we know that the mathematization of situations will remain fundamental. It is therefore essential to accustom our students, from the beginning, to this important strategy of the mind. By the active mathematization of situations, one substitutes 'learning' for 'teaching'. The ultimate goal of teachers is not to teach, but to enhance understanding and to learn learning (Papy, 1968, pp. 7–8).

A closer look at Papy's approach, as elaborated in his textbooks, revealed that Papy indeed occasionally leaves the pure mathematical path to pay attention to the 'mathematization of situations': he regularly presented 'daily-life' situations to prefigure new mathematical concepts and structures. However, these situations did not

1 The next four quotations were translated from French by the second author.

incorporate realistic or authentic problem situations to be solved with mathematical tools. Their only purpose was to facilitate comprehension of an abstract formalized definition of the mathematical concept or structure that was targeted. Moreover, the newly learned mathematics was never (re)invested to analyse and to solve new challenging problems outside mathematics.

To better characterize the role of extra-mathematical situations in New Math courses of the 1960s, Hilton's (1973) distinction between 'illustration' and 'application' might be helpful. The point Hilton made is essentially the following. A situation, within or outside mathematics, is an illustration of a mathematical theory if and only if that situation clarifies the theory. A situation is an application of a mathematical theory if and only if that situation is clarified by the theory. For the high-order mathematical structures of the New Math, such as groups, fields or vector spaces, no applications were available for the early-aged students to whom these structures were taught and thus, these structures only could be illustrated with concrete instantiations (e.g. concrete materials or games especially constructed for that purpose). Although New Math advocates often referred to the universal applicability of the powerful structures of the modern mathematics in today's science and technology, they were unable to demonstrate this applicability to their students, for them they were just words, no tools for real problem solving, application analysis or modelling.

> [Mathematical] structures are great and admirable machines, but in early mathematics education, they can only produce too little things and too little effects. These little things are the naive examples of structures that embellish textbooks in modern mathematics and are designed specifically for students (Rouche, 1984, p. 138).

Applications and modelling in the post-Royaumont era

Based on a survey of 21 national reports, Kemeny (1964) observed that the main interests of the international mathematics education community in the late-1950s/early-1960s were one-sidedly directed towards pure mathematics. The debate was focussed on the type of new mathematical subjects that could find a place in secondary school programs, on how the teaching of traditional topics could be improved by the adoption of modern ideas, on the 'right' way of teaching geometry, … With the exception of a widely supported plea for teaching some notions of statistics at the secondary level, relatively little attention was paid to the applied side of mathematics. Also at the international meetings on mathematics education of the 1960s, organised by OEEC/OECD, UNESCO or ICMI, only occasionally ideas for integrating applications of mathematics in secondary school curricula were voiced. In the next paragraphs we briefly discuss three main sources of applications that, aside from statistics, were mentioned at these forums.

First, reference was still made to applications of mathematics to classical physics. The Group of Experts, that met in Dubrovnik (1960) for the purpose of preparing a detailed synopsis for modern secondary school mathematics, as stipulated in one of the Royaumont resolutions (OEEC, 1961a), insisted once more on the need for a better coordination between the teaching of mathematics and the teaching of science (particularly of physics), but provided little or no concrete suggestions to put that coordination into practice. An exception might be the early introduction of vectors and the systematic development of their algebraic and geometrical properties in a modern curriculum for school geometry, which they considered, at least potentially, of the greatest use to the students and teachers of physics (OEEC, 1961b). From physical scientists, an increasing pressure was felt to teach a more or less intuitive introduction to calculus in secondary schools – which was not the case in many countries – but mathematics education reformers of that time did not have clear ideas how such introduction could be properly integrated in a modern mathematical curriculum (Kemeny, 1964).

Second, there is the mathematics related to the upcoming computing machines which began to fundamentally impact secondary school mathematics. Examples of new computer-related applications and their curricular impact were thoroughly discussed at the OECD conference in Athens (1963). That conference provided a special section on 'applications in the modernisation of mathematics' (OECD, 1964) in which Henry O. Pollak (USA), at that time Director of Mathematics and Statistics Research at Bell Laboratories and one of the pioneers in the field of applications and modelling in mathematics education, examined, among other things, new areas of mathematics motivated by computer sciences. He stated that the basic notions of programming, including the use of flow diagrams in the construction of algorithms, should be essential parts of secondary school curricula. In an interview with Alexander Karp, Pollak also mentioned the importance of the relationship between mathematics, computers and computing (Karp, 2007). He further told about his pioneering activities with respect to modelling and applications at the undergraduate level and about his vision on these issues:

> The point of all of what we were trying to do in mathematics itself is understanding, understanding when and how and why this stuff works. […] The point of applications of mathematics is also understanding. The difference is that w're trying to understand something outside of mathematics rather than inside (Karp, 2007, p. 77).

Pollak and his collaborators collected a series of engineering problems that led to nice mathematical formulations. The problems served as a basis for an early book on applications of mathematics (Noble, 1967). Freudenthal invited Pollak to speak at the ICMI colloquium in Utrecht (1967) and at the first ICME congress in Lyon (see the next section).

Hermann Athen, a German contributor to that Athens conference argued, computers may have a much broader impact on human thinking:

> A factor not to be neglected is the technical and economic revolution which is taking place as a consequence of the big automatic computers. This revolution in psychic and intellectual functions of human thinking and computing is continuously leading to new investigations in the fields of logic and the analysis of thinking. There is practically no field of mathematical investigation which is not dependent upon the use of computers, e.g. many problems of the social, behavioural, managerial and economic sciences (OECD, 1964, p. 245).

Other topics, in some way or another related to computers or their use, were suggested at that time, for instance binary representations of numbers, coding, numerical analysis, discrete mathematics, electrical circuits, logic and Boolean algebra.

Third, as a genuine application to economics and other social sciences, linear programming was repeatedly mentioned. The topic fitted well within a modern course of linear algebra, but also could strengthen students' numerical skills related to solving equations and inequalities. Moreover, it opened a window to operational research, a recent field of applied mathematics that deals with the application of advanced analytical methods to help make better decisions given certain constraints. Probably more than other fields of application, optimization involves mathematizing and modelling, i.e. interpreting a real-world situation in terms of a precisely formulated mathematical model (OECD, 1964). Modelling and models were not yet widespread notions during the 1960s, but they gained ground. In his Introduction to the Proceedings of the UNESCO colloquium in Bucharest (1968), Nicolae Teodorescu observed that the notion of 'model' had acquired universal presence and circulation, and already acknowledged the cyclic nature of modelling processes.

> Modelling the complex, heterogeneous reality is the deliberate aim of any modern research method in sciences of nature, in social sciences and in humanities. The victorious penetration of mathematics in other scientific domains is accounted for by modelling which, repeated successively, leads to mathematical models (International UNESCO Colloquium, 1968, p. 27).

Freudenthal and the emerging RME movement

The end of the 1960s was characterized by an increased interest for the didactics of mathematics, particularly at the micro level. Not only the purely mathematical subjects, but the way a child learns, became a main guiding principle for developing mathematics education. This new trend was reflected in a growing number of (international) congresses and meetings in the field. In the context of this chapter, the ICMI colloquium initiated by Freudenthal around the theme 'How to Teach

Mathematics so as to Be Useful' (1967), deserves our special attention. It was the first meeting in which an international panel discussed the differences in opinion about the role of the use of mathematics (La Bastide-Van Gemert, 2015). In his opening address Freudenthal sketched, in a general way, his views on mathematics education. He explained that teaching mathematics 'so as to be useful' is not the same as teaching useful mathematics:

> Useful mathematics may prove useful as long as the context does not change, and not a bit longer, and this is just the contrary of what true mathematics should be. Indeed, it is the marvellous power of mathematics to eliminate the context. […] In an objective sense the most abstract mathematics is without doubt the most flexible. In an objective sense, but not subjectively […] (Freudenthal, 1968, p. 5).

He further argued that we should neither teach 'applied mathematics', nor 'pure mathematics' (and expect that the student will be able to apply it later). Mathematics is rather learned by doing, as a human activity, as a process of mathematizing reality and if possible, even of mathematizing mathematics.

> The problem is not what kind of mathematics, but how mathematics has to be taught. In its first principles mathematics means mathematizing reality, and for most of its users this is the final aspect of mathematics, too. For a few ones this activity extends to mathematizing mathematics itself (Freudenthal, 1968, p. 7).

Freudenthal's colloquium was sometimes regarded as the symbolic beginning of a new era in the history of applications and modelling in mathematics education, the so-called 'advocacy phase' (Niss et al., 2007) in which advocates of applications and modelling provided arguments in favour of the serious inclusion of such components in the teaching of mathematics. In several countries (such as, for example, the UK and the US), this phase was quickly followed by a second 'development phase', mainly characterized by the development of new educational materials to put such teaching into practice, sometimes by institutes especially created for that purpose. In the Netherlands, for example, the implementation was driven by the Institute for the Development of Mathematics Education (IOWO) – founded in 1971 by Freudenthal, nowadays the Freudenthal Institute[2] – which shaped the philosophy and practice of RME.

But these developments did not yet reflect a global trend in mathematics education during the late-1960s. When in 1969 the first ICME congress was held, only two of the twenty published papers were related to applications and modelling (Editorial Board of Educational Studies in Mathematics, 1969; Gispert, 2003). Arthur Engel

2 The Freudenthal Institute is nowadays more than the successor of the IOWO. It also includes research groups on Science Education and on History and Philosophy of Science.

(Germany) talked about the relevance of modern fields of applied mathematics, such as operations research, computer science, stochastic and game theory, for the teaching of mathematics. He argued that not the new techniques per se are important, but the new 'modes of thinking' they provide to cope with the real world (Engel, 1969). Henry O. Pollak (1969) classified and discussed different types of realistic and not so realistic (word) problems that are used to involve students in applications of mathematics at different educational levels.

Conclusion

The New Math movement, launched in 1959 at the Royaumont Seminar but with roots in the mid-1950s (Gispert, 2003), was dominated by academic mathematicians who had a genuine interest in education, but most of them were rather involved in pure than in applied mathematical research. Applications were not their first concern. Moreover, they were convinced that a thorough insight in the structures of the New Math was a solid and necessary basis for teaching questions of applied mathematics. In daily-school practice New Math adherents rather used real-world situations to illustrate than to really apply mathematical structures. During the 1960s calls for taking applications – and later also mathematical modelling – seriously grew louder and culminated in Freudenthal's colloquium 'How to Teach Mathematics so as to Be Useful'. This meeting marked the beginning of a new and more favourable period in the history of the teaching of mathematical modelling and applications. Although the first ICME congress in Lyon (1969) can be seen as a sequel of Freudenthal's colloquium, the presence of modelling and application was not impressive.

In sum, it can be stated that the teaching of applied mathematics was not a primary concern during the early- and mid-1960s. But although most leading reformers of that time focussed on pure mathematics and only paid lip service to the active application of mathematics, some mathematics educators highlighted the role of (new) applications – and later also that of modelling – and regarded it as an essential element in the modernization of mathematics teaching.

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