Norms and practices of secondary teachers' formation. The Portuguese case (1915-1930)[1]

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Abstract

This paper discusses norms and practices of secondary teacher education in mathematics in the beginning of the 20th century in Portugal. Following the educational proposals of the new republican order, Normal Higher Schools (Escolas Normais Superiores), created in 1911 and operational from 1915 until 1930, play a central role in the training of professionals for teaching secondary disciplines, particularly mathematics. For the first time at the secondary level, teacher education comprises a theoretical study of subjects akin to their future profession and a practical initiation at the school level. We focus on the analysis of norms and practices leading to the professional knowledge formation of secondary school mathematics teacher. Norms are assessed through the study of legislative documents, regulations and bureaucratic materials. Practices are determined following the analysis of several kinds of students' work required for the diverse disciplines (exercises, tests, conferences, written assignments, etc.) and the extensive reports produced at the end of their internship. We found these schools fostered the development of a pedagogical knowledge influenced by the New School movement which was based on the ideas of Pestalozzi, Rousseau, etc.; the innovative international mathematical tendencies supported by ICME: and the appreciation of the social utility of school mathematics

Keywords: history of mathematics education; teacher education; internationalization

Introduction

In Portugal, from the beginning of the twentieth century, access to teaching positions at secondary schools required specific training. In 1901 a Secondary Teaching Formation Course [Curso de Habilitação para o Magistério Secundário] was created with a duration of four years (Pintassilgo, Mogarro & Henriques, 2010). This training model required the future teachers to obtain the scientific training at the university level during the first three years and, in the fourth, a pedagogical training took place in Lisbon. Until then, access to the secondary teaching profession was done through examinations for which a university education was not necessary.

The model was further developed after the implantation of the Republic in 1910 with the creation in 1911 of two Higher Normal Schools [Escolas Normais Superiores, ENS] attached to the Faculties of Letters of Coimbra and Lisbon, which

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were completely functioning in 1915 (Gomes, 1989). Born of the republican desire to value education, these schools intended to give the dignity of higher education to professional teacher education and played a central role in the formation of professionals to teach mathematics in secondary schools (, 2014). Later, the dictatorial regime created by Oliveira Salazar (1926-1974) would take a set of measures that lead to their extinction from 1930.

The ENS were initially studied by Joaquim Ferreira Gomes (1989) who presented an exhaustive survey of documentation concerning the two schools, in particular Coimbra. A more analytical approach was developed by Joaquim Pintassilgo, Maria João Mogarro and Raquel Henriques (2010) who related the legislative intentions and other documentation with the tendencies of pedagogical thought of the time. Maria João Mogarro (2012) also studied the relations between the formation in the ENS and the New School trend. This movement gathers a cluster of ideas disseminated from the end of the eighteenth century that essentially put the student at the centre of the educational process. It intended to base teaching and learning in children's experiences and valued the importance of active methods rejecting authoritarian pedagogical approaches centred on the transmission of abstract knowledge. The proposals of Jean-Jacques Rousseau, Johann Heinrich Pestalozzi, among others, are associated with this movement. Our previous work on this topic focused on the development of teachers' professional knowledge (Matos, 2017) and on the ENS's role in the circulation of ideas (Santiago & Matos, 2018). ENS represent a significant point in the development of the autonomy of school knowledge (Julia, 1995) in Portuguese secondary education. For the first time, contexts for reflections and practices focusing on specific contents of school mathematics and the related teaching methods were available (Matos, 2015).

The purpose of this text is the analysis of norms and practices (Julia, 1995) leading to the formation of professional knowledge of secondary school mathematics teachers in Portugal from 1915 until 1930. Norms include knowledge to teach and behaviours to be inculcated and they will be assessed through the study of legislative documents, regulations and bureaucratic materials. Practices that allow the transmission of this knowledge and the incorporation of these behaviours will be assessed through the analysis of several kinds of students' work required for the disciplines (exercises, tests, conferences, written assignments, etc.) and the extensive reports produced at the end of their internship. This paper is based on documentation preserved at the archives of the University of Coimbra, at personal archives, and in Biblioteca Nacional in Lisbon that was not previously studied, as far as mathematics education is concerned. This research is somehow unbalanced, as the materials found come mainly from the ENS in Coimbra.

We will survey the path students took as they entered these institutions: entrance examinations, first year and its disciplines, teaching practice of the second year, final

examination and the required dissertations. For each of these steps we will analyse their norms and practices.

Entrance examinations

To be admitted to the ENS, prospective secondary teachers should have had a college degree and have passed an entrance examination. In the case of mathematics, candidates should have had at least a "Bacharelato" in mathematics (3 years). This degree had a roughly similar structure at the University of Coimbra and at the two polytechnic schools of Lisbon and Porto. It included the following disciplines (Quadro de equivalências, 1915):

- Higher Algebra, Analytic Geometry, Spherical Trigonometry
- Differential, Integral, and Variations Calculus, Probability,
- Rational Mechanics,
- Descriptive Geometry,
- Astronomy, Geodesy,
- Physics, Chemistry, Mineralogy, Geology, Zoology, Botanic,
- Drawing.

Entrance examinations were composed of two parts. An eliminatory general part, common to all prospective teachers, intended to verify the degree of general culture of the candidates (Decree n. 2.646, 26/9/1916). It was composed of a written essay about Portuguese history (3 hours) and a translation of a text from Portuguese to French (1 hour). These examinations also included a special part akin to the discipline the candidates intended to teach. In the case of mathematics, this part was composed of a practical test, which involved solving a problem of algebra or geometry and two oral examinations on analysis and geometry. Candidates could also present other elements (books, texts…) and some in fact wrote special 'mémoires' for the exam on such topics as set theory, conics, complex numbers, and complex functions.

The archives keep some of the written tests including students' answers. Examinations for the special part in mathematics that were found included problems of geometry, analytic geometry, analysis, algebra, and numerical computations with a difficulty level near to the one they had experienced at the Bacharelato. We present some of them below.

> Discuss the curve represented by the equation $y=x^3-2x^2+3x+1$ and show the form of the curve. (Entrance examination, Special part, 7/12/1917)

Given the equation of the ellipse y^2-3xy+5x^2+2y-3x-5=0, determine the diameter of the cords parallel to the line y=2x-1 and the curve's axes. (Entrance examination, Special part, 26/1/1918) [2]

Calculate with the support of logarithmic tables the expression

$$\frac{4.5832}{\sqrt[5]{28\sin 24^\circ 15' 8''}} : \frac{\sqrt[3]{\pi}}{e}$$

(Entrance examination, Special part, 20/3/1920)

A folder of documents was found (Box 1) that allows us to understand the procedures for the entrance examinations of the special part of mathematics for the school year 1925-26. Although the school year should have started in October, these exams only took place between 23/11/1925 and 12/12/1925 and the jury convened several times to prepare lists of problems and questions that were later distributed among the candidates. Classes for the first year must have taken place between January and June of 1926.

One handwritten document lists the problems of analysis and algebra that were given in the afternoon of 2/12/1925. These included properties of polynomials and proofs involving series and integrals which the candidates had two hours to complete. Below are two examples of these questions:

If P and Q are two relatively prime polynomials, having degrees m and n respectively, there are two other polynomials P_1 and Q_1, of degrees m_1<m and n_1<n, such that

$PQ_1 + P_1Q$=1.

The system P_1, Q_1 is unique.

(Entrance examination, analysis part, 2/12/1925)

Any fraction smaller than the unity can be univocally developed in a series of the form $\frac{1}{2^{\alpha_1}} + \frac{1}{2^{\alpha_2}} + \cdots + \frac{1}{2^{\alpha_n}} + \cdots$ with different integer numbers α_n.

Prove this proposition and deduce a rule to approximately evaluate the side of the regular pentagon, using only the subdivision of segments in 2 equal parts.

(Entrance examination, analysis part, 2/12/1925)

A typewritten document listing the distinct geometrical problems proposed to the candidates in the afternoon of 3/12/1925 was also found containing problems of

2 *Diameter of the cords* may refer to the family of cords parallel to the given line.

descriptive, projective, and analytic geometry. Candidates had two hours to solve one of these. Below are two examples.

> [Trace perpendiculars from one point to two straight lines.] Find the nature of the locus of the points such that the distance between the intersecting points of the perpendiculars to the straight lines is constant.
>
> (Entrance examination, geometry part, 3/12/1925)
>
> Show that, if an angle of fixed size is moving on a plane, such that the vertex describes a fixed straight line and a side passes a fixed point, the other side has a parabola as its envelope.
>
> (Entrance examination, geometry part, 3/12/1925)

We also found another typewritten document listing the questions posed at the oral examinations of 1925, which took place after the written parts. It includes the following topics: analytical study of conics with centre; analytical study of the circle and its relationship with conics; homogeneous coordinates in the plane and "in the star"[3], transformation formulas of these coordinates; polarity in the fundamental forms of the second kind, applications to the study of conics; classification and diametric properties of quadrics; tangent planes to the revolution surfaces by the methods of descriptive geometry. In the middle of December the process was concluded. Figure 1 shows the results of the oral and written tests of the four candidates and the final results.

All the candidates passed the test but their final grades (in the scale 0-20, in which a grade below 10 means a failure) were not high. We are aware of 47 prospective secondary mathematics teachers that passed these exams but we do not have information of how many candidates failed the tests.

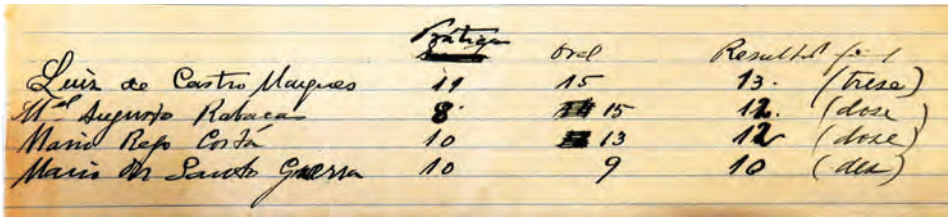

Fig. 1. Summary of admission examinations 1915.

Apparently, even if these tasks included standard problems studied at the universities, which we did not verify, the examiners intended to check mathematical knowledge of the candidates beyond an elementary level.

3 May refer to coordinates used in cosmography.

The disciplines of the first year

According to the founding decree of the ENS (Decree with force of law, *Diário do Governo*, *129*, 1911, 2081-3), after entrance, students attended a two-year course that included an initiation to pedagogical practice in the secondary schools (Liceus). Given the shortage of teachers in some areas, it is likely that some of the candidates already had teaching experience.

The curriculum of the first year was composed of the following disciplines: Pedagogy (with exercises in experimental pedagogy) (annually), History of Pedagogy (annually), Child Psychology (semesterly), Theory of Science (semesterly), General Methodology of Mathematical Sciences and Natural Sciences (annually), Organization and Comparative Legislation of Secondary Education (quarterly), General Hygiene and especially School Hygiene (semesterly), Moral and Higher Civic Instruction (semesterly). [4]

This curricular plan valued the general and specific pedagogical training on the basis of the assumption that education and psychology are experimental sciences. Hygiene and moral education, the topics dear to the republican spirit of the time, were also included; the latter was considered an important element of instruction of republicans and secular citizens (Pintassilgo, Mogarro, & Henriques, 2010). The General Methodologies were new because they focused on the specific professional knowledge of each school discipline.

Legislation included recommended teaching methods for the ENS. The instruction was supposed to include traditional magisterial lessons, conferences followed by discussions or sets of practical assignments, especially written exercises to be done in class, exercises on experimental pedagogy, and studies of children´s pedagogy.

The archives provide us with information regarding the practices of these disciplines in Coimbra. As we are looking for the professional knowledge developed in these schools, we will focus firstly on Pedagogy and History of Pedagogy considering especially the ways in which these were shaping pedagogy as a new area of knowledge and secondly on the content of Methodology of the Mathematical Sciences.

Pedagogy

The disciplines of Pedagogy and Psychology in the ENS of Coimbra were taught essentially by Augusto Joaquim Alves dos Santos (1866-1924) until his death and later by José Joaquim de Oliveira Guimarães (1877-1960). The former was a pioneer

4 The curricular plan underwent minor changes three years after schools started, abolishing the Theory of Science, dividing the discipline of Legislation in two semesters and creating a discipline of General Methodology of the Sciences of the Spirit (Decree nº 4,649, *Diário do Governo*, 157, 1918, pp. 1311-4).

of Experimental Psychology in Portugal. A colleague of Édouard Claparède at the Jean-Jacques Rousseau Institute in Geneva, he created the first laboratory in this subject area in the country and was well acquainted with the leading European psychologists of his day.

The discipline of Pedagogy contained what was believed to be the core of knowledge related to education: its foundations, curricular organization, and teaching methods (Pintassilgo, Mogarro, & Henriques, 2010). In 1917 and 1930, topics of students' written essays and conferences encompassed themes such as measurement of memory and auditory attention, ludic activities, psychological conditions of attention, pedagogical optics. Most notably, some of these essays report exercises of experimental pedagogy. For example, the essay *Measurement of auditory attention* written in 1917 by Francisco Ferreira Neves (Box 1) describes how he measured reaction times of blindfolded "subjects" hearing a series of strokes. He explains how the experiment was designed and reaction times to six series of strokes varying in rhythm (constant/varying) and warning (given or not given prior to the beginning of the experiment) were measured. Distractions were added. Several tables and graphics were included and a descriptive statistical analysis supported the conclusions. After discussing the limitations of the study, he concluded that warnings seemed to shorten reaction times.

History of Pedagogy

Luciano Pereira da Silva (1864-1926), a mathematician, and Joaquim de Carvalho (1892-1958), a philosopher, taught History of Pedagogy in the ENS of Coimbra. The essays kept at the archives address topics as History of instruction in Portugal, Religious congregations in the 17th century, The New School, Russian Pedagogy after 1918. The ideas of pedagogues were also discussed, namely: Pestalozzi, Rousseau, Rabelais, Montaigne, João de Barros. In a given school year, the same topic was assigned to all students in the class.

There are differences between these two disciplines. On the one hand, an enthusiastic teacher, who sought to introduce students to the ways in which a scientific (experimental) methodology could be used in an innovative field, gave the discipline of Pedagogy a very advanced status. On the other hand, History of Pedagogy was a topic not even close to the scientific interests of the teachers in charge, and appears to be much more conservative approach.

Methodology of Mathematical Sciences

The discipline Methodology of Mathematical Sciences is of special interest. It was taught in the ENS of Lisbon by Eduardo Ismael dos Santos Andrêa (1879-1937), a teacher in secondary schools who taught calculus in the University of Lisbon

and authored successful textbooks. In the ENS of Coimbra Methodology of Mathematical Sciences was taught by Luciano Pereira da Silva (1864-1926), also a teacher of calculus at the University of Coimbra, author of books about history of mathematics, astronomy and navigation, and Director of ENS, and by João Pereira Silva Dias (1894-1960), a teacher of Calculus, Geometry and Physics at the University of Coimbra who was appointed in the 1940s to official posts related to education and culture.

We investigated the essays produced in Coimbra in detail. Prospective science secondary teachers (Physics and Chemistry) and future teachers of mathematics in normal schools also attended these classes, but the content was essentially focused on mathematical topics. Most of them discussed methods of the mathematical sciences such us inductive, deductive, analytic, graphical, and other methods and a special attention was given to the laboratory method. For example, in 1917, by the end of May, a written exercise on "The laboratory method" was proposed to the class, probably as a take-home exam. The three handwritten essays in the archives (ranging from 4 to 16 pages) focused on the contemporaneous trends in school education in England and the United States, on how a logical approach to mathematics was being changed to a method grounded in Psychology and taking into account children's interest (today we would say 'motivation'). All supported the importance of planning mathematical instruction as a move from the concrete to the abstract and stressed the importance of linking together distinct mathematical topics and of integration of mathematics with other subjects. In 1921 another series of essays and conferences was devoted to "The value of graphics in the laboratory method".

Mathematical methods *per se* were also discussed. For example, in 1923, essays written in class focused on the distinction between the inductive and the deductive method. In the same year, the laboratory approach was also a topic for the take-home essays. In 1924, several methods were discussed.

The proposals of the New School movement underlay these essays. In Portugal, this perspective was widely spread from the end of the nineteenth century as Education (or Pedagogy) and Psychology started seeking acceptance as scientific fields. The republican movement used the New School banner ("intuitive teaching") as a perspective that would improve schools (Mogarro, 2012).

The second year, pedagogical practice

Under the legislation, the second year was occupied by an initiation to the pedagogical practice developed at the Liceus accompanied by a secondary school teacher. Until December candidates had to assist to the classes of their advisers and teach occasionally. The advisers had to teach them the special methodology of the discipline. During the rest of the year candidates would have full teaching responsibilities

supervised by their advisers. They also had to participate in other school activities. University professors also had to participate in these educational activities.

In Coimbra, pedagogical practice was conducted at Liceu José Falcão. Alberto Álvaro Dias Pereira (1891-1984) was the adviser for most of the time. He was a former teacher of mathematics in several schools, a congressman, and also the Rector of the Liceu. He was dismissed as a teacher for political reasons in 1936. Occasionally other teachers took this role: António Tomé e Aníbal do Amaral Cabral, José Custódio de Morais.

In Lisbon the pedagogical practice was conducted in three Liceus and the advisers were: Adolfo Bernardino de Sena Marques e Cunha (1872-1927) at Liceu Pedro Nunes (he had a background in medicine and was a prominent member of a religious group); Domitila Hormizinda de Carvalho (1871-1966) at the female Liceu Maria Pia who graduated in mathematics and philosophy, and had a doctorate in medicine (she was the first woman to officially graduate in the University of Coimbra and one of the first women deputy at the national assembly); and José Ferreira de Carvalho e Santos at Liceu Passos Manuel.

We do not have much information about the actual procedures of the pedagogical practice (class attendance, interactions between advisor and students, etc.), nor do we have any evidence that university teachers actually participated in the activities of the second year. We only know that prospective teachers took the classes of the advisors and their attendance was monthly reported to the ENS, and, at least from 1927, students were required to write their teaching plans for specific topics.

The dissertations and the final exam

After finishing the pedagogical practice of the second year of the course, State Examination [Exame de Estado] took place and the jury appreciated the overall merit of the candidate's work. The exam included: two discussions of half an hour each, a lesson given to a class in the Liceu followed by its pedagogical discussion, and the presentation of a dissertation on a topic of didactics of secondary teaching at the candidate's choice (Decree 2646, 26/9/1916). The dissertation was one of the significant elements together with the performance during the teacher practice. From 1927 the dissertation was replaced by a report on the pedagogical practice (Decree 13296, 17/3/1927) of which we only have an example (Santos, 1929).

Joaquim Ferreira Gomes found the titles of numerous dissertations, essentially produced by ENS students from Coimbra (1989). This listing was enriched with other titles collected through the consultation of legislative acts and of the existing documentation in Biblioteca Nacional.

Themes of the dissertations

We are currently familiar with the titles of 42 dissertations related to the teaching of mathematics at ENS in Coimbra — which correspond to almost all of those presented in State Examinations — and two from ENS in Lisbon (table 1).

Table 1. Themes of the titles of the dissertations.

Theme	Number
School mathematics topics	
Arithmetic	5
Algebra	2
Geometry	5
Analysis	8
Analytic Geometry	4
Cosmography	3
Other	6
General educational topics	10
Teaching methods[5]	1 (+4)
Total	44

As might be expected, two-thirds of the papers have titles that indicate the development of topics in school mathematics (*The Teaching of Derivatives in Liceus* or *On Extending the Idea of Number*, for example) and were aggregated according to mathematical themes of the time. Some remaining titles were placed under the theme "Other topics". These are subjects such as trigonometry, logarithms, complex numbers, indeterminate analysis or numerical approximations. Most of the rest are titles of a general scope (*Mathematics in secondary education*, etc.). Table 1 also highlights titles that denote a concern with teaching methods (e.g., *The heuristic method in the study of fractions* or *The slide rule*).

We found 15 of these dissertations. Physically, they are texts between 20 and 110 pages of various sizes, some printed in book printing shops, others typewritten. Students assumed their choice of the theme, sometimes manifesting a strong conviction about its importance. In none of the texts do we find a reference to either a professor of the ENS or their adviser at the Liceu. Apparently the students chose the topic with a reasonable degree of freedom. These dissertations allow us to go beyond the titles and actually study their content.

5 We include here four dissertations on mathematics topics that were counted above because they also explicitly propose a teaching method.

Development of mathematical topics

About one third of the 15 dissertations that we found were focused on mathematical topics, usually at the level of an undergraduate university course. Usually a justification of the author's choice of topic was provided. For example: authors argue that analytic geometry can better be taught if it integrates functional, algebraic, and geometrical dimensions; graphics allow the use of geometrical intuition; transformation geometry provides a better system of axioms.

Some authors are aware of the international centrality of the study of analysis as they develop mathematical texts on functions or on differential and integral calculus. They usually include references and quotations from contemporary texts by Henri Poincaré, Jules Tannery, Emanuel Beke, Félix Klein, Bertrand Russell, and Federico Henriques. In fact, this was a subject being discussed in the Commission Internationale de l'Enseignement Mathématique (CIEM) (Zuccheri & Zudini, 2014) and partially integrated in the Portuguese high school programs since 1905 (Aires & Santiago, 2014). These dissertations draw on the work of Félix Klein, a known advocate for introducing the topic in the secondary curriculum, and Jules Tannery, one of the proponents of the introduction of analysis in French curricula. Henri Poincaré is also referred to, to justify the need for an approach that, while using a logical treatment, does not let students forget intuition.

Rego (1918) in particular develops the theme supported by numerous quotations from the report presented by Emanuel Beke to the 'Conference internationale de l'enseignement mathématique[6]' organized by the CIEM in Paris four years earlier, which discussed precisely the progress of the reforms of the teaching of the analysis underway in several countries. His work is important to us, as it is the only dissertation for the ENS of Lisboa found so far. We know that his teacher of Methodology of Mathematical Sciences, Eduardo Andrêa, was the author of one of the rare articles on mathematical education (1905), precisely supporting the teaching of analysis in secondary schools. Apparently, the topics discussed in Lisbon were not far from the ones discussed in Coimbra.

The geometry of transformations is another innovative topic that can be found in three dissertations. Henriques Júnior (1921) and Tavares (1927) explore alternative axiomatic systems for geometry based on geometric transformations and, as in the previous case, this theme has been debated internationally (Barbin & Menghini, 2014). Much influenced by the proposals of Bertrand Russell and Federico Henriques, Tavares (1927) in his dissertation, written as a college level mathematics textbook, is aware of its philosophical implications and identifies his study as metageometry. Henriques Júnior (1921), in a text closer to secondary school mathematics, but still

6 According to Zuccheri and Zudini (2014), this report was published in *L'Enseignement Mathématique*, *16*, 285-89.

containing the definitions, axioms and theorems relevant to support a perspective of geometry based on transformations, presents the 'gonio', a mathematical instrument (also known as the compass of parallels) which he developed at the Physics Laboratory of the University of Coimbra during his training at ENS, and his dissertation is part of a book dedicated to the subject. Also the work of Silva (1920) briefly discusses, among other subjects, the controversy about the foundations of geometry and explores some elementary transformations.

Mathematics and its methods

One third of the dissertations found do not focus upon a mathematical topic. Instead we could say they discuss philosophically the science of mathematics and its methods. The centrality of mathematics among the sciences is highlighted, but the dominant theme of several dissertations is the presentation of arguments to counter the idea of the primacy of logical deduction in the production of mathematical knowledge, an idea they associate with current teaching methods at Liceus. As Gersão (1917) says, to recommend the mathematicians to follow the deductive method would be to suggest that they search for a needle in a haystack. An inductive path is much more productive. Neves (1919) deepens this idea:

> Mathematical sciences are commonly characterized by their exclusively deductive method. Thus, as deduction is a reasoning that goes from the general to the particular, it would result that mathematics would teach little (...). But on the other hand, we constantly hear that the tendency of mathematics is generalization. In effect it is: for example, from the fundamental notion of integer, we pass to that of a fractional number; then that of an irrational number. (...) This is how primitive simple science, *arithmetic*, undergoes a generalization: *algebra*; which then undertakes a new extension: *analysis*; and who knows what will be the last word in mathematics? (Neves, 1919, pp. 31-2, italics in the original).

Neves continues and, based on Poincaré, argues that along with deduction, mathematical induction and reasoning by recurrence are equally fundamental mathematical methods. As for the nature of the axioms, he distinguishes the positions of Kant (axioms are evident truths or a priori), Stuart Mill (experimental acts), and Poincaré (conventions).

Silva (1920) and Santos (1929) produce similar arguments. Santos adds that the methods of mathematical investigation are: the analytical, the synthetic, the reduction to the absurd, the deductive, the inductive, that of the indeterminate coefficients, and that of the infinitely small, thus not limiting mathematics to a by-product of logic. Many dissertations state similar classification of mathematical methods, argue that mathematics is not exclusively deductive, and stress intuition, observation, and experimentation. Immanuel Kant, John Stuart Mill, and Henri Poincaré are

often referred. This concurs with the contents of students' essays for the discipline titled Methodology of Mathematical Sciences.

Teaching methodologies

One third of the found dissertations explicitly address teaching methodologies. Almost all argue that no desirable learning comes from an accumulation of notions, and on the contrary, they are generally concerned with students' tendency to memorize concepts they forget on the first occasion. Such is the case of Silva (1920) who believes that basing teaching on definitions would be counterproductive. Alternatively, a justification part should precede and prepare the general notions that would only be presented after the consideration of several particular cases. Similar opinion has Beirão (1921) for whom

> the abstract notions of mathematics must rest on sensible notions which aid their understanding; and thus, by geometric processes we will interpret the different notions ... and in this concrete way we can be able to arouse the interest of students, who feel a natural repugnance for abstract notions, for them always vague and indefinite. (Beirão, 1921, p. 8)

Five dissertations, with a remarkable uniformity of views, distinguish three teaching methods. The first, the didactical method, is described as "the teacher exposes the questions, directs them to show clearly what he has in view. Students listen, follow the teacher's reasoning and arrive at the same time as him to the conclusions" (Gersão, 1917, p. 6). In the heuristic method, the teacher "expose truths inductively and lead students through a series of well-directed questions to discover the truth" (Gersão, 1917, p. 7).

> The student walks by himself, feeling the ground, standing here and there, but supported by the teacher, who guides him, overcoming the main obstacles, when he alone is unable to do so. By means of insinuations, suggestions, remembrances of principles forgotten by the teacher, the pupil will follow the path of truth in front of the teacher, and, once the purpose of the work was achieved, the student is left with the salutary and stimulating impression that he discovered it; (...) the key is the method or the know-how that the student grasped. (Neves, 1919, p. 40)

Some students believe this method came from Immanuel Kant, Jean-Jacques Rousseau, and Johann Heinrich Pestalozzi. Finally, comes the laboratory method, which:

> seeks to establish an intimate approximation between Mathematics and other sciences, using natural phenomena, which gave rise to the emergence of certain theories, now purely abstract, to achieve them. (Abreu, 1922, pp. 45-6)

With regard to mathematics teaching, the graphic process is presented as an example of using the laboratory method. The millimetre paper, "which students should always have at hand" (Abreu, 1922, p. 47) allows pupils to know "by sight, relations of greatness existing between certain objects, relations that would have no representation in their spirit, if were given by means of numbers" (Abreu, 1922, p. 47). The graphs are pointed out as an essential element for this understanding, which allows abstract notions, such as function, to have a concrete meaning (Neves, 1919). One of the students (Abreu, 1922) explicitly associates the laboratory method with the term "lessons of things" [lições das coisas], referring to one of the key terms popular among republican educators simpathetic of the New School movement (Mogarro, 2012). Some of these dissertations ascertain the origin of the laboratory method in the works of Eliakim Hastings Moore, John Perry, and Jacob William Albert Young.

Neves (1919), Abreu (1922), and Santos (1929) agree that in the first three classes of Liceus the laboratory method should be preferred, in the fourth and fifth the heuristic method and, in the sixth and seventh, the didactic method, which should also be used in the University. Abreu justifies this sequence with what might be called a "theory of the mental development of the child":

> Teaching methods must be in accordance with the stages that the child goes through during her development, from childhood to puberty. During the time from infancy to puberty, they must give education a purely concrete feature; during puberty, the concrete must operate with the aid of the abstract, but it must predominate; from puberty teaching must be characterized by the mastery of the abstract. (Abreu, 1922, p. 43)

The social utility of mathematics

One of the main advocates of the laboratory approach is John Perry. He envisioned mathematics laboratories where problem-based approaches could incorporate the technological developments associated with graphical analysis and the use of slide rules. He proposed the regular use of graph paper, in which the integrating element would be the concept of function. This movement has expanded in several countries and has naturally been reflected in the work of ICME (Furinghetti, Matos and Menghini, 2013).

At the basis of this position was the idea that the teaching of mathematics should primarily concern itself with utility. Mathematics should therefore be taught experimentally in laboratories and not through abstraction (Howson, 1984). We find a reflection on this theme in three dissertations (Guardiola, 1921, Neves, 1919, Rego, 1918). Neves explains his position upfront:

> In writing this work, one idea constantly guided me: this idea is that man must be educated in order to be **socially efficient**. (...) Man is an element of society, to whom is distributed a function that he will perform without friction and with a maximum income to be distributed by himself and by society. (Neves, 1919, p. v, bold in the original)

> It is therefore incumbent upon the educator to ensure that the share of education made at the expense of those sciences is maximal, so that together with the results obtained by other means, the individual is enabled to produce a maximum revenue in his work, compatible. of course, with the general conditions of existence. (Neves, 1919, pp. vii)

A similar position is adopted by Guardiola (1921). Rego (1918) completes this idea, arguing that the secondary school should provide elements of a general culture, without losing sight of the usefulness of the knowledge imparted.

Conclusion

ENS schools were intended to replace the system under which access to the position of secondary teachers depended solely on approval on a state exam. These new schools included demanding entrance requirements both in general knowledge and mathematical proficiency. We also found that the disciplines of the first year paved the way for the development of a pedagogical knowledge, shown especially in the dissertations, based on:

- 1) the ideas of the New School movement that looked for the educational enterprise from children's perspective, valued the need for a concrete basis for the abstract knowledge (and thus the rejection of logic as a starting point for mathematics teaching), supported the centrality of "intuitive teaching", the importance of relating teaching to the real world ("lições das coisas"), and channelled these ideas into a laboratory approach to mathematics;
- 2) the innovative international mathematical tendencies supported by ICME, especially related to the importance of the inclusion of analysis in the mathematics curricula and on the axioms for geometry;
- 3) and for some of the prospective teachers, the appreciation of taking into account the social utility of school mathematics

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Box 1: Depósito IV, Secção 2ª.E, Estante 9, Tabela 5, nº 19. Escola Normal Superior de Coimbra. Arquives of the University of Coimbra.

Box 2: Depósito IV, Secção 2ª.E, Estante 9, Tabela 5, nº 21. Escola Normal Superior de Coimbra. Arquives of the University of Coimbra.

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