Building new mathematical discourse among practitioners in restoration and enlightenment England 1650 – 1750

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Abstract

By the mid seventeenth century England was becoming a more diverse society where new political developments fostered a more secure and forward-looking society, and the idea of universal education was developing. From the late 16th century a community of mathematical practitioners flourished, and from these, specialist instrument makers had arisen so that by the early 17th century this community had grown considerably. The education system was unable to provide the 'useful' knowledge required for the new demands of an emerging practical society, so independent teachers began to provide the mathematical skills required. From the mid-17th century 'Dissenting Academies' were established to provide Nonconformist students, excluded by their religious beliefs from Oxford and Cambridge, with higher education. These became places where people could learn more 'useful' subjects; mathematics, foreign languages, geography and natural science, thus usurping and expanding the role of the universities. Practical instruction on a range of mathematical and astronomical instruments and new concepts and techniques provided the public needs by authors, showing their readers how to deal with Compound Interest and Annuities, and how to keep accounts "after the Italian manner" with exercises to improve the organisation of trade and business. New publications were intended to persuade the middle classes that the work of the artisan was to be valued. Joseph Moxon's Mechanick Exercises *(1703) encouraged interest in the work of the artisans like smiths, builders, and others. Moxon believed in the inclusiveness of types of knowledge where the skills of the Blacksmith could be seen involved in a range of related occupations including those required in working with fine metals in watch-making, and supported his claim by reference to Francis Bacon's mechanical philosophy. This paper presents a discussion of the varieties of access and involvement in mathematics with related artisanal and industrial activities, as seen by the participants themselves, involved in the emergence of a technical curriculum, where mathematics is applied to, and becomes a result of, many different activities.*

Keywords: industrial, discourse, practical, gentlemen, culture

The legacy of Francis Bacon (1561-1626)

The intellectual atmosphere of the sixteenth century aided by new kinds of scientific endeavour and founded on the new developments in astronomy and mathematics, had its most popular and powerful expression at the beginning of the next century in Francis Bacon's *Great Instauration* of 1620, containing the *Novum Organon*. In this reorganisation of the search for knowledge Bacon developed a new system of Natural Philosophy that was coherent with the social ethic of Protestantism. He paid attention to the sociology of knowledge by focusing on ideological obstacles,

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barriers that were preventing the transformation of intellectual values from one area to another. Bacon produced a model for a reform of natural philosophy and other sciences where technological case-histories were used to demonstrate that the intelligent application of advances in one area, could lead to economic progress and intellectual advancement in another. In this way, Bacon issued a 'manifesto' for the experimental method. By attacking preconceived ideas scientific work should not seek 'final causes' but use logic and rhetoric in organising what we already know, so the advancement of the 'mechanical sciences' was attributable to the accumulation of small improvements; essentially empirical, collaborative and democratic, in contrast to the authoritarian beliefs passed from master to pupil. (Rogers, 2016). The inductive method was consistent with the democratic methods of the mechanic because the only sure means of a secure understanding was an experience with the object or situation itself. By the rigorous application of 'experimental method'[1] it was hoped to ascertain conditions necessary and sufficient for the production of natural phenomena, whereby it would be possible to imitate nature by producing a grand catalogue of phenomena enabling new theories to explain the confusing data of our senses. This encouraged people's active participation in empiricism by bringing the natural philosopher and the craftsman together.

Mathematics, instrument makers, and the restoration mathematical community[2]

The mathematical community of the late 17th and early 18th century England was far broader than hitherto portrayed in general histories of mathematics; it involved a wide range of people with very different backgrounds and very different approaches and understandings of mathematics. An increasingly sophisticated level of practical mathematical activities grew in accountancy, commerce, navigation, and instrument making. New mathematical learning was discussed and employed in workshops, warehouses, dockyards, and coffee houses, and was disseminated in printed books, leaflets, journals and letters, so the proliferation of these works enabled cross-fertilisation of ideas, particularly ideas about the establishment of scientific beliefs and technical practices.

By 'mathematical community' (Wenger, 1999), I indicate that group of people who were connected with mathematical practices or study in any way. It includes mathematical practitioners (Rogers, 2012) such as surveyors, navigators and astronomers;

1 Bacon's 'experimental philosophy' included craft skills as legitimate areas of study from which 'natural philosophy' could learn and improve its own methods. He is often regarded as the first English empiricist philosopher.

2 The Restoration was the period from 1660 to about 1720 when the Scottish and Irish monarchies were restored under Charles II, until the accession of the Hanoverian dynasty.

teachers of practical mathematics, from schools to Dissenting Academics[3]; makers of mathematical instruments and booksellers publishing mathematical texts, and a large number of amateurs among the gentry and nobility, who patronised the mathematical sciences. This included the local conditions, the espoused and enacted beliefs of individuals, and the social, political, and economic influences on the establishment of particular kinds of knowledge. It is important to realise that this also included well-known mathematicians of the time who often took part in discussions and collaborative activities.

In the study of the practice of mathematics in 17th and 18th century England, the emerging epistemological varieties appear in the views of the nature and processes of formation of the mathematics and its uses that were espoused by the different agents whose views depended on their ideologies and were inevitably involved in the production of their texts. The purposes of mathematics that were perceived, and the uses to which it was put, to some extent determined the kind of mathematics that an individual was motivated to learn and to apply, and these needs were felt in different ways in particular sections of society. A significant aspect of this was a deep reflexive relation between the Master craftsman and the Instrument. Customers asking for instruction on and improvements to instruments prompted new developments, and craftsmen talking to each other and their clients were driving modifications that challenged the user to explore connections between the practical use of instruments, and the supporting mathematics. In that 'Baconian' discourse, geometry, trigonometry, logarithms and algebra were among the more theoretical aspects of mathematics; while astronomy, navigation, surveying (both civil and military), geography, gunnery, dialling, accounting and book-keeping, gauging, and other forms of mensuration, all fell within the field of practical mathematics.

Familiar books from the 16th century were reissued with additions and modifications: Billingsley's (1570) edition of Euclid, with Dee's Mathematicall Praeface was still being read (Ramping, 2011), both Richard Norwood's *The Sea-Mans Practice* and Edmund Wingate's *Arithmetique made easie, including the use of logarithms*, appeared in many different editions and were still being printed well into the eighteenth century. At another level appeared instructions on the use of particular instruments, books containing mathematical tables, or any kind of simple 'ready-reckoners' for measurement. For example, Mr Hoppus' *Measurer Practical measuring made Easy to the Meanest Capacity* (1736) contained instructions for reducing mensuration to the simplest measurements, rules and tables thus avoiding difficult calculation. Extensive proliferation of 'pseudo-applications' of mathematics and science appeared in Almanacs, in

3 The 'Dissenting Academies' provided university-level education for those protestants of various persuasions, who were not members of the established church, barred from entering the universities of Oxford and Cambridge (Parker, 1914).

calendars with astronomical calculations, planetary motions and horoscopes, mixed with medical and religious information.

Expanding subject areas

Arithmetic found its most frequent new application in accounting, book-keeping and calculating interest; older texts were re-edited to include additional sections. Edward Hatton edited the last issue of Recorde's *The Ground of Arts* in 1699, and then went on to produce *An intire system of arithmetic, or Arithmetic in all its parts: containing Vulgar, decimal, duodecimal, sexagesimal, political, logarithmical, lineal, instrumental and algebraical arithmetic* in 1721. This included negative numbers, 'approximation' (using converging series) "the whole intermix'd with rules new, curious and useful ……", thus advertising the contents. Cocker's *Arithmetic* edited by Hawkins (1703) has tables of interest and rebate, logarithms, and 'algebraical' arithmetic, but new areas of interest and applications of mathematics began to appear with John Graunt's *Natural and Political Observations Made upon the Bills of Mortality* (1662/3), and William Petty's inventions in *Political Arithmetic* (1690).

Geometry, and its applications were being used in surveying and architecture; trigonometry appeared in navigation and astronomy using constructions with sine, cosine and tangent lines on sectors and similar instruments. Geometry traditionally dominated the discourse, particularly when trigonometry was incorporated, and was still the most important part of mathematical theory related to applications.

Algebra rarely found any real applications in mathematics at this time. There was a belief that algebra was important, and there were publications that 'explained' algebra and began to include new useful subjects in the same volume. John Ward produced in 1698 a *Compendium of Algebra*, *Exemplifying Plain and Easie Rules for the Speedy Attaining to that art* …. including Equations, Squaring the Circle, making Sines, Tangents and Logarithms with an Appendix on Compound Interest and Annuities. However, William Webster dismissed algebra entirely:

> Algebra, indeed is not properly a Science distinct from Arithmetic, but is only a different Method of Computation, performed by substituting letters in the Place of Figures, and expressing the several Parts of the Operation by Symbolical Characters ……. But however useful it may be to those who understand it, it is certainly an unintelligible jargon to the mere Numerist, and can give him very little satisfaction when laid before him as a Demonstration (Webster 1751 Preface A3).

Others like Edward Wells (1714) in *The Young Gentleman's Course of Mathematicks Containing the more Useful and Easy Elements* …. neglected algebra almost entirely, dealing with the more popular or 'mechanical' subjects like optics and dialling. In most contexts, virtually all of the practical problems that might have been susceptible to algebraic equations were still being solved by the application of proportional analysis.[4]

Joseph Moxon (1627-1691) was a prolific author of the later 17th century, taking up Bacon's theme that craft knowledge was equally as valuable as 'theoretical' knowledge. He produced the first English mathematics dictionary and the first detailed instructional manual for printers, together with other books on a wide range of subjects, from mathematics, astronomy, and architecture, to works about artisans and craftsmen. He regarded the 'trades' as legitimate occupations and worked to break down social and political barriers, providing a unifying account of people working together for the common good, following Baconian principles. Typical of this position was his *Doctrine of 'Handy-Works'* where he described smithing as an important craft skill which encompassed "not only the Black-Smiths Trade but takes in all Trades which use either forge or file, from the Anchor-Smith to the Watch-Maker." thus including all types of metals in use, and justifying his claims:

> The Lord Bacon, in his Natural History, reckons that Philosophy would be improv'd by having the secrets of all Trades lye open; not only because much Experimental Philosophy is coucht amongst them; but also that the Trades themselves might, by a Philosopher, be improv'd. Besides, I find, that one Trade may borrow many Eminent Helps in Work of another Trade (Moxon 1703 Preface).

In 1678, he became the first tradesman to be elected as a Fellow of the Royal Society.

Tools for practical mathematics

Edmund Gunter (1581-1636) is remembered for developing many practical instruments which include the 'Gunter's Chain", 'Gunter's Quadrant', and 'Gunter's scale'. In 1619 Sir Henry Saville, wishing to improve the state of mathematical studies in England, funded the first two chairs of astronomy and geometry at Oxford University. Gunter applied to become professor of geometry, bringing some of his instruments to the interview, but Saville, requiring the applicants to be acquainted with traditional sciences, dismissed him, accusing Gunter of showing 'tricks', not true geometry.[5] This attitude, also expressed by William Oughtred (1535-1660) showed an innate suspicion of the technically skilled:

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4 For example; The Rule of Three, or double and triple proportion.

5 There is a well-known story (see MacTutor History of Mathematics archive) about how Gunter took his instruments to demonstrate but Saville, a rather opinionated character, rejected him since he had no obvious knowledge of traditional geometric reasoning. Successful use of an instrument does not necessarily mean the operator knows the theory behind it.

> That the true way of Art is not by Instruments, but by Demonstration: and that it is a preposterous course of vulgar Teachers, to begin with Instruments, and not with the Sciences, and so, in-stead of Artists, to make their Scholers only doers of tricks, and as it were Iuglers: to the despite of Art, losse of precious time, and betraying of willing and industrious wits, vnto ignorance, and idleness. That the use of Instruments is indeed excellent, if a man be an Artist: but contemptible, being set and opposed to the Art (Oughtred 1632 sig.A3 verso)

In the same year, supported by Henry Briggs, Gunter was appointed professor of Astronomy at Gresham College[6]. In 1620 he published his *Canon triangulorum.* a method of calculating logarithmic tangents and in 1624 a collection of his mathematical works entitled *The description and use of sector, the cross-staffe, and other instruments for such as are studious of mathematical practise*. It was a manual for sailors and surveyors in the real world and published in English. Gunter wrote*:*

> Not that I think it worthy either of my labour or the publique view, but to satisfy their importunity who not understand the Latin yet were at the charge to buy the instrument.

He invented the terms cosine and cotangent and some of his most notable practical inventions were the Gunter chain, the Gunter Quadrant and the Gunter scale. The 'Gunter chain' was meant for surveying and triangulation with intermediate measurements indicated, giving a new unit easily converted to area[7]. Gunter's quadrant contained a stereographic projection of the sphere on the plane of the equinoctial. With the eye placed in one of the poles, the tropic, ecliptic, and horizon form arcs of circles. This is used to find the hour of the day, the sun's azimuth, etc. and to find the altitude of an object in degrees. Gunter's scale: on one side are placed the lines of chords, sines, tangents, and rhumbs, etc. on the other side the corresponding artificial or logarithmic lines. Using this instrument questions in navigation, trigonometry etc., are solved with the aid of a pair of proportional compasses. This is a predecessor of the slide rule, commonly used from the 17th to the mid-20th century. His instruments were commonly used and they reflected the practical nature of his teaching that was linked to the more scholarly work of his time.

Advertising your wares

Social gatherings spread information as new instruments became available; advertisements were carried in books or by pamphlets produced by the instrument makers themselves. Whether the purveyor was expert enough in different areas to explain

6 Gresham College was founded in 1597 to give free lectures to people of the city of London. It was the first 'university' outside Oxford and Cambridge.

7 Traditional English units: 66 feet or 20 m long. An area of land of 10 square chains gives 1 acre.

the technical details to a customer is an open question. Seth Partridge (1603-1686) described himself as a surveyor, offering land-measuring, but he seems to have been involved in various branches of mathematics and its applications. In the end-page of his *Rabdologia: or The Art of Numbring by Rods* (1648), his self-praise is typical: declaring himself capable of offering instruction in arithmetic (whole numbers and fractions , decimals, roots, astronomical fractions, algebra, arithmetical rods [i.e. Napier's bones]; geometry (principles, gauging, surveying, use of the plane table, circumferentor, theodolite, circular scale, quadrant, semicircle, peractor, sector, circles of proportion and Wingate's lines of proportion); trigonometry (use of logarithmic trigonometric tables, measuring of heights etc., doctrine of triangles; navigation (including the use of instruments, maps and charts); cosmography (use of globes) the armillary sphere, the astrolabe, and dialling.

John Blagrave (1558-1612) was a gentleman[8] and self-taught professional mathematician. His work included land-surveying and the design and erection of sundials. He became involved in inventing and modifying new mathematical and navigational instruments. His books accompanying his *Familiar Staffe* (1584) (proportional compasses) and the *Mathematical Jewel* (1585) (a modified armillary sphere) contain instructions for operating the instruments, with basic geometrical and astronomical information on the setting up and use of the instrument in a variety of situations. Typically, these instruments, although often capable of producing more accurate results, never became established being complicated and difficult to set up. His *Mathematical Jewel* appears in Seth Partridge's advertisement in the *Rabdologia* in 1648.

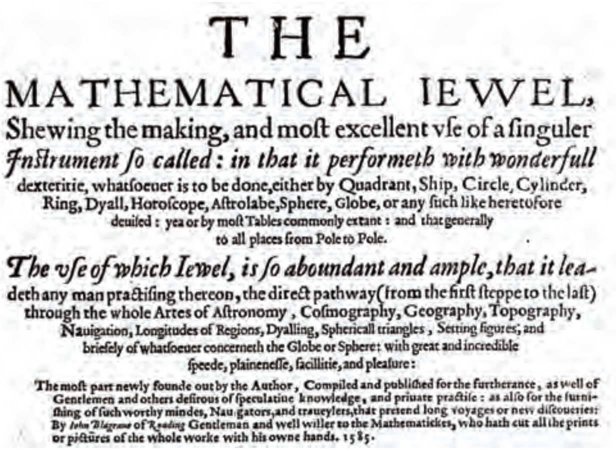
THE
MATHEMATICAL IEWEL,
Shewing the making, and most excellent vse of a singuler
Instrument so called: in that it performeth with wonderfull
dexteritie, whatsoeuer is to be done, either by Quadrant, Ship, Circle, Cylinder,
Ring, Dyall, Horoscope, Astrolabe, Sphere, Globe, or any such like heretofore
deuised: yea or by most Tables commonly extant: and that generally
to all places from Pole to Pole.
The vse of which Iewel, is so aboundant and ample, that it lea-
deth any man practising thereon, the direct pathway (from the first steppe to the last)
through the whole Artes of Astronomy, Cosmography, Geography, Topography,
Nauigation, Longitudes of Regions, Dyalling, Sphericall triangles, Setting figures; and
briefely of whatsoeuer concerneth the Globe or Sphere: with great and incredible
speede, plainenesse, facillitie, and pleasure:
The most part newly founde out by the Author, Compiled and published for the furtherance, as well of Gentlemen and others desirous of speculatiue knowledge, and priuate practise: as also for the furnishing of such worthy mindes, Nauigators, and traueylers, that pretend long voyages or new discoueries: By *John Blagraue* of *Reading* Gentleman and well willer to the Mathematickes, who hath cut all the prints or pictures of the whole worke with his owne hands. 1585.

Fig. 1. John Blagrave's Advertisement from his Mathematical Jewel. Upper half of title page from Blagrave, 1585, recto.

8 The term Gentleman (sometimes 'the gentry') originally referred to anyone who had a private income, owned land, or was an ordained member of the Church of England. Later, merchants owning property became included.

Edmund Wingate (1596-1656) went to Paris, where he became teacher of English to Princess Henrietta Mari[9]. He had learned the 'rule of proportion' (the logarithmic scale) recently invented by Edmund Gunter which he communicated to mathematicians in Paris. He quickly published a description, *L'usage de la règle de proportion en arithmétique*, Paris, 1624, and an English version as *The Use of the Rule of Proportion*, London, 1626, demonstrating the principle of the slide rule. Later, he produced *Logarithmotechnia, oringate. The construction, and use of the logarithmeticall tables by the help of which, multiplication is performed by addition, division by subtraction, the extraction of the square root by bipartition, and of the cube root by tripartition* …. 1635, to secure priority of publication. His most significant work was *Of Natural and Artificiall Arithmetique*, London, 1630, 2 parts. Part I. had been designed "onely as a key to open the secrets of the other, which treats of artificial arithmetique performed by logarithms"[10], it was re-issued and published in 1652 as *Arithmetique made easie*. This book reappeared in many versions until 1753.

Henry Sutton (1624-1665) was a typical example of an English instrument maker working in London from ca. 1650 to 1661. His work was advertised mostly by word of mouth, and it was known that he could make all of the usual mathematical instruments. Known for his high-quality engravings of scales and quadrants, he sold books, and understood the mathematics enough to collaborate with mathematicians in the design of new instruments. He was reputed to be the most talented and original mathematical instrument maker in London in the middle of the seventeenth century and was mentioned in Samuel Pepys' Diary. His place of work and range of stock is shown from an advertisement, written on a quadrant dated 1658, "This Instrument or any of the Mathematiques are made in Brass or Wood by Henry Sutton Instrument maker behind the royall exchange".[11] One of Sutton's most important projects was engraving the plates for John Collins's *The Sector on a Quadrant* (1658) with the design prints of the 'Sutton quadrant'.

This quadrant, made by Henry Sutton (1657), is a conveniently sized and high-performance instrument. The front is designed as a 'Gunter' quadrant and the obverse as a trigonometric quadrant. The side with the astrolabe has hour lines, a calendar, zodiacs, star positions, astrolabe projections, and a vertical dial. The side with the geometric quadrants (shown here) features several trigonometric functions, rules, a shadow quadrant, and the chord line. This 'Sutton-type' quadrant, for astronomical calculations was similar to existing quadrants, but extensively promoted,

9 Princess Henrietta Maria of France, a Catholic, became the wife of Charles 1 in 1625. She bore him two children, Charles, (to be Charles II) and Princess Henrietta.

10 Interesting to observe here that that the use of logarithms for calculation made arithmetic 'artificial'.

11 The Royal Exchange is located in the City of London near St Paul's and the Bank of England. This area was frequented by many who were making or purchasing mathematical instruments in the 17th and 18th centuries

so by the end of the 17th century it was one of the most common astronomical calculating devices (Jardine, 2006). Sutton died of the plague in 1665.

Fig. 2. Astronomical Quadrant made by Henry Sutton 1657. MHS. Oxford

Promotion and dissemination

Edmund Stone (1700–1768) and Nicolas Bion[12] (1652–1733).

Nicolas Bion's *Traite de la Construction et des Principaux Usages des Instruments de Mathemtique* (1709, 1752) was also published in German. Edmund Stone published his first translation of Bion's work in 1723: *The Construction and Principal Uses of Mathematical Instruments Translated from the French of M. Bion, Chief Instrument-Maker to the French King. To which are added, The Construction and Uses of such* INSTRUMENTS *as are Omitted by M. BION*, particularly those invented or improved by the *ENGLISH*. By EDMUND STONE. The whole book is illustrated with thirty Folio Copper Plates containing the Figures of the several INSTRUMENTS. (2nd Edition 1758).

12 Very little is known about Edmund Stone or Nicolas Bion. Stone was the son of a gardener and taught himself to read and write and learnt Latin and French at an early age. His Patron was the Duke of Argyll, who supported his talents. There are records of his works in the Royal Society, but outside his publications, nothing seems to be known. Bion is equally unknown. As an instrument maker with a workshop in Paris he advertised himself as an instrument maker to the King, but as far as my French colleagues can determine, there are no supporting records.

Thanks to this translation, and the influence of his patron, the Duke of Argyll, Stone was made a Fellow of the Royal Society in 1725. In 1758 he published a further edition, including *A Supplement concerning a further account of the Most Useful Mathematical instruments as now Improved*. This is a remarkable book, with 326 pages of descriptions and 30 engravings of instruments including many geometrical diagrams, giving the method of construction and mode of operation of every measuring and calculating instrument in use during the 17th and 18th centuries. Nowhere else in this period can be found such a wide-ranging, and detailed collection showing the geometrical principles necessary for their construction. Stone regarded mathematics as a useful and necessary part of a proper education, defining mathematics both as a science, with regard to the theory, and as an art, with regard to the practice. Mathematical instruments connected these important and inseparable sides of mathematics:

> Mathematical Instruments are the Means by which those Sciences are rendered useful in the Affairs of Life. By their assistance, it is that subtile and abstract Speculations reduced into Act. They connect, as it were, the Theory to the Practice, and turn what was bare Contemplation, to the most substantial Uses. The Knowledge of these is the Knowledge of Practical Mathematics: so that the Descriptions and Uses of Mathematical Instruments, make, perhaps, one of the most serviceable Branches of Learning in the World (Stone 1758 Preface. [v])

John Collins (1625-1683) was an important figure in the mid-17th century[13]. He is most known for his extensive correspondence with many leading scientists and mathematicians in England and on the continent, providing details of the many discoveries and developments made in his time. He was exchanging notes on mathematical books, problems or inventions with many people who had an interest in both theory and applications of mathematics. Apprenticed to a bookseller, and later employed as a clerk, he obtained some basic instruction in mathematics. He went to sea, where he devoted his leisure to the study of mathematics and merchants' accounts. On leaving his ship in 1649 he taught in London. In 1653 he published *An Introduction to Merchants' Accounts*, republished in 1664 and 1674. He next wrote *The Sector on a Quadrant, or a Treatise containing the Description and Use of three several Quadrants*[14] (1658). In 1659 his *Geometricall Dyalling, or Dyalling performed by a Line of Chords only,* appeared, and *The Mariner's Plain Scale new Plained*, a treatise on navigation, these were written for the East India Company.

After the Restoration, Collins was appointed accountant to the excise office, and accountant to the Royal Fishery Company. Collins helped support many

13 For a comprehensive account of Collins' life and work, se Beeley 2017.

14 A note includes the inscription: "To be sold by George Hurlock book-seller at Magnus Corner, by William Fisher at the Postern near Tower-Hill, and by Henry Sutton mathematical instrument maker, at his house in Threadneedle Street behind the Royal Exchange."

important publications and was particularly active in seeing Jeremiah Horrocks'[15] work *Astronomical Remains* through the press. He contributed to *An Account concerning the Resolution of Equations in Numbers*, a survey of recent algebra improvements made in England, completed by John Kersey in 1685. His *Arithmetic in whole Numbers and Fractions, both Vulgar and Decimal, with Tables for the Forbearance and Rebate of Money*, &c., was published in 1688. For his zeal in collecting and diffusing scientific information, Collins was styled the 'English Mersenne.' He was constantly stimulating others to become involved in useful inquiries and pointing out defects in different branches of science. For this work, Collins was elected a fellow of the Royal Society in October 1667.

Qualitative language, new investigations and new applications

John Graunt (1629–1674) developed early statistical and census methods that later provided a framework for modern demography. Graunt's book *Natural and Political Observations Made upon the Bills of Mortality* (1662/3) used the records of deaths in London in an attempt to create a warning system for the onset and spread of plague in the city. As a result, Graunt produced the first Life Insurance Table recording the likelihood of human survival at each age. His work also resulted in the first statistically based estimation of the population of London. Graunt's life tables presented mortality in terms of survivorship, where he predicted the percentage of persons that will live to each successive age and their life expectancy year by year. He managed to produce a realistic estimate for the population of 17^{th} century London as about 600,000 persons. For this work he was elected to the Royal Society in 1662.

William Petty (1623-1687) was an important figure in the history of social science, a proponent of the new empirical and mechanical philosophy and one of the founders of the Royal Society. Petty's invention in *Political Arithmetic* (posthumously published in 1690) was an early form of economic theory that applied the methods and concepts of seventeenth-century natural philosophy to the challenges of Great Britain as a monarchy governing a colonial empire, consisting of English rule in Ireland, colonies in the Americas, trade in the East Indes, and the politics of religions in England, Scotland and Ireland. Petty developed a sophisticated monetary theory; he reduced political, religious, and ethnic differences to demographic measures. After Petty's death his 'instrument of government' through demographic engineering was rearticulated as a mode of statistical analysis, an early form of social science.

These new areas with applications of mathematics were growing out of the newly perceived needs of a more sophisticated population. People were involved

15 Jeremiah Horrocks (1618-1641) was a British astronomer who predicted, and with his friend William Crabtree, were the first to view the Transit of Venus in 1639.

in developing new ways of talking about problems and using mathematics to solve them. It is interesting to note that a parallel movement was happening in early education theory. George Snell produced *The Right Teaching of Useful Knowledge to fit Scholars for some honest Profession*. (1649) Snell and others argued that instead of taking so much time over teaching of the classics, 'useful' knowledge should be taught and there was a deliberate movement to establish what knowledge is useful for a contemporary society in order to develop a new, relevant curriculum.

Technology, culture and the Royal Society

From the beginning of the Civil War (1642) many Oxford academics lost their posts for their political and religious views, and replacements were sent to match the current government's agenda[16]. The New Commonwealth government was ineffective, purges were badly organised, so Oxford became a place where different opinions, ideas and personalities were brought together in a potentially dangerous situation. John Wilkins (1614-1672) became Warden of Wadham College in 1648 and managed to encourage political and religious tolerance by drawing people into a group that became known as the Oxford Philosophical Club meeting regularly to discuss natural philosophy and experimental science. This group included Robert Hooke, Christopher Wren, Seth Ward, and John Wallis. After the Restoration (1660) there were similar regular meetings at Gresham College[17] in London. Wilkins brought together those studying different disciplines and encouraged them to work collaboratively, forging friendships, intellectual connections, and a spirit of enquiry that would foster the formation of the Royal Society in 1660 as a 'College for the Promoting of Physico-Mathematical Experimental Learning', an important group for discussion and promotion of scientific ideas.

Robert Hooke (1635-1703) had experimented with the new microscope constructed by Christopher White of London[18]. Hooke used this microscope for the observations that formed the basis of his *Micrographia*, published in January 1665, the first major publication of the new Royal Society.

16 A complicated area of English history covering the period of the Royalist government, the rise of the Parliamentarians and the Commonwealth of England after the execution of Charles 1.

17 The Will of Sir Thomas (1519-1579) an English merchant and financier, provided for the setting up of Gresham College in Bishopsgate as a place for free public lectures on scientific subjects and frequent discussion of new ideas. The college played an important role in the formation of the Royal Society.

18 Compound microscopes, with an objective lens and an eyepiece appeared in Europe in the early 17th century. A Dutch inventor Cornelis Drebbel is said to have had one in London in 1619. In that context, Christopher White could easily have built and improved the instrument.

This remarkable book containing Hooke's drawings of flies, fleas and lice, revealed an unknown world, inspiring a wide public interest in the new science of the microscope. The popularity of the book helped further the society's image and mission as England's leading scientific organisation, and the illustrations of the miniature world captured the public's imagination in a radically new way. Hooke's Preface to the *Micrographia* explains his rationale and offers some fascinating insights into the scientific method of the day, where he talks about an "inlargement of the Dominion of the senses":

> The next care to be taken, in respect of the Senses, is a supplying of their infirmities with Instruments, and, as it were, the adding of artificial Organs to the natural; this in one of them has been of late years accomplished with prodigious benefit to all sorts of useful knowledge, by the invention of Optical Glasses. By the means of Telescopes, there is nothing so far distant but may be represented to our view; and by the help of Microscopes, there is nothing so small, as to escape our inquiry; hence there is a new visible World discovered to the understanding. …….

And he declares the new kinds of knowledge gained from the experimental methods:

> From whence there may arise many admirable advantages, towards the increase of the **Operative, and the Mechanic Knowledge**, to which this Age seems so much inclined, because we may perhaps be enabled to discern all the secret workings of Nature, ….. (Hooke Micrographia (1665) Preface) (emphasis by author)

The scientific method encouraged technical advances and supported empiricism. Optical instruments were improved, and new applications developed. Newton invented the reflecting telescope and investigated the use of a prism to produce his *Opticks* (1704), a record of experiments and deductions made from them that were originally reported to the Royal Society in 1672. The significance of this work was that, by experimental method and scientific induction the dogma originating from Classical Philosophy was overturned. Light was a series of colours emanating from the physical property of the phenomenon itself, depending on the angle refracted through the prism, so that colour is a sensation of the mind, not an inherent property of the material objects.

However, once the basic ideas of empiricism had been set out by Francis Bacon, philosophers began to explore and advance the study of the human mind and its behaviour. The experiences of the instrument makers, reported and recorded in publications, and the discussion of the means and consequences of actions and observations provided the data for more philosophical discussion. In 1690 John Locke published his *Essay Concerning Human Understanding*, a seminal source that concerned the foundation of human knowledge. He described the mind at birth as a blank slate to be filled later through experience. However, Bishop Berkeley's *Theory of Vision*

(1709) and *Principles of Human Knowledge* (1710) refuted Locke's theory of human perception and sought to prove that the outside world (the world which causes the ideas one has within one's mind) is also composed solely of ideas. David Hume in his *Treatise of Human Nature* (1739) adopted Locke's theory of ideas but disputed them on the grounds that there was no perceptual experience that conveys the idea of self. 'Common sense' certainty of one's existence as found in Descartes and Locke was disputed and Hume declared that experience of causality arises only through the 'observed sequencing of events'. But the 'observed sequencing of events' was the foundation of the experimental evidence. At this time, apart from Berkeley's *The Analyst*, there was little direct effect of these ideas on the practice of mathematics or its applications.

Discussion and conclusion

In this period, re-examination of classical concepts emphasized the fact that reason requires knowledge, and that reasoning skills are necessary for success in any society by building necessary knowledge and testing emergent hypotheses as entrepreneurial activity develops. A major aspect of this paper is the way in which we see craftsmen become innovators, as many 'traditional' activities like ship-building (Rogers, 2012) and monument-building (Moxon, 1673) become a design activity and develop new ideas by forward planning using mathematics and scientific acumen. Clearly, this applies to the generations of Instrument-makers whose innovative and cooperative work was driving forward the development of science and many different new technologies. The basis for a large part of these advances are the improvements in technologies themselves (Rogers, 2016) Metal-working of all kinds, from new furnace design, smelting, and fashioning to engraving metals is not specifically mentioned here, but as crafts emerge from cottage industries we see the slow, steady early progress towards industrialisation that demands attention from the population.

It became clear that the few educational institutions were quite unable to deal with the demand for appropriate education, to adapt to the changing economy and expanding culture. There was a particular demand for mathematics and its developing specialisations; new methods of calculating and applying mathematics in novel situations and ways of measuring all kinds of objects and events. The political and religious circumstances drove the foundation of the Dissenting Academies to satisfy the needs for the expanding economy, offering occupational and new useful professional training, based on rational philosophy and egalitarian principles,[19] adopting English as the new language of science.

19 In effect, these Academies, growing outside the universities, became the Mechanics Institutes and Polytechnics of the late 18th and 19th centuries.

While it seems that 'pure' mathematics has been ignored in this account, it cannot be neglected, where people like John Collins, Edmund Gunter, and Henry Sutton were living in a community that included people like Harriot, Oughtred, Wallis, Hooke, Wren, and others who were clearly present and involved in exchanges of information and useful techniques, to a greater or lesser degree. Exposing this mathematical community focuses on the changing discourse as we move from individual craft-apprentice through master technician and business organiser to instrument user, recognising the development of professional practice, and the acquisition of new forms of knowledge and their consequent epistemological basis together with the development of the language involved. Between each occupation there can be seen a symbiotic relation where the experience of one informs and improves the other (Meli Bertoloni, 2006). Using the instrument develops knowledge, and in turn, the instrument inspires new ideas. Augmenting the senses develops operative knowledge (Hooke) and people are beginning to recognise that theory and practice lie on a continuum so that each individual can choose where to reside. Overall, we see the gradual evolution of a new curriculum of useful knowledge that laid the foundations of the technical and school curriculum from the early 19th century.

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20 EEBO stands for Early English Books Online. This is found through the Bodleian Library Oxford, (solo.bodleian.ox.ac.uk), https://eebo.chadwyck.com/

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