Arithmetic in Joan Benejam's La Enseñanza Racional (1888)

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Abstract

Joan Benejam was born on the island of Menorca in 1846. He became a teacher in 1866 and in 1874 he became elementary school teacher back in his home town, where he spent the rest of his life until his retirement in 1912. His life was completely devoted to teaching and education. He wrote papers, he gave lectures, he wrote children's plays, etc. He also founded and directed several journals addressed to elementary teachers. Here, we will focus on one of such journals published in 1888: La Enseñanza Racional *[The Rational Teaching]. This journal contained sections devoted to geography, grammar, history, arithmetics, etc. In this paper, we will describe and analyse the arithmetic section of this journal. This will give us an idea of the pedagogical ideas of its author as well as of the situation of arithmetic teaching at the time. Even if this is a very particular case, it constitutes an interesting example of how the ideas of the so-called* Regenerationism *(which tried to reform and modernize the country) were received and, in some sense, put into practice in the field of education.*

Keywords: arithmetic, journal, 19th century, Spain, Joan Benejam

Introduction

The 19th century in Spain was a period of great instability. The country was involved in several wars and many different governments followed on one another. By the end of the century, most of the overseas colonies had been lost and the formerly great Spanish Empire faced a clear decline (Esdaile, 2000).

This situation gave rise to an intellectual movement, the Regenerationism, which tried to understand the underlying reasons of the Spanish decadence in order to overcome them (Carr, 2001). The name of this movement came from the word 'regeneration' as opposed to 'decay' and one of its main characters was Joaquín Costa. Some of his most famous slogans were "school and pantry" or "close the tomb of El Cid with two locks", pointing out that it was time to stop looking back at a lost glorious past and to start looking at a future where the main goals should be to reduce the poverty and increase the education of the people.

Regenerationism was a very active movement which gave rise to many journals devoted to disseminate its ideas in different areas. As for education, it deeply influenced the so-called Institución Libre de Enseñanza [Free Educational Institution]. This institution, which existed between 1876 and 1936 tried to impose progressive ideas on the educational practices of that time. For example it proposed the

Bjarnadóttir, K., Furinghetti, F., Krüger, J., Prytz, J., Schubring, G. & Smid, H. J.(Eds.) (2019). "*Dig where you stand" 5. Proceedings of the fifth International Conference on the History of Mathematics Education*. Utrecht: Freudenthal Institute.

coeducation, the low importance of textbooks, the involvement of the family in the teaching process, the interest of non-formal education by means of excursions, poetry and theater sessions, etc. (Molero Pintado, 2000).

In this work, we are going to focus on a rather unknown and not very influential character, Joan Benejam Vives. He was an elementary school teacher in a very small town in the island of Menorca. He did not write important books and his ideas were not particularly groundbreaking, but his life and his work illustrate the importance of these somewhat anonymous people in the process of innovating and improving education. In fact, the role played by teachers like Benejam in order to improve education by disseminating and sharing innovative ideas was fundamental because they acted as 'loudspeakers' bringing these ideas from the main cultural centers to more isolated regions. The case of Benejam is an interesting example which is worth studying, in addition, due to his intense activity (as we will see in the next section) writing books and editing journals.

More particularly we are going to analyze the arithmetic section of one of the many journals that Benejam edited during his life. This section presents some interesting features that can be related with more general ideas to exemplify how the teaching of mathematics can also be used as a mean to educate in the broad sense of the term.

Joan Benejam Vives. Life and works

Joan Benejam Vives (Figure 1) was born in a small town of Menorca, called Ciudadela, on May 27, 1846. At the age of 16, he entered Barcelona's Normal School in order to become an elementary teacher. In 1866, he graduated and started his teaching career in Blanes, near Barcelona. Soon, in 1869 and back in his native town, he founded a private school called Colegio Ciudadelano [School from Ciudadela]. In 1874, he became a public servant gaining a position as elementary teacher at Ciudadela's public school. He kept this position until his retirement in 1912. For a short period of time, between 1912 and 1914, he lived in La Habana, where he was involved in adult education activities. Finally, from 1917 until his death in 1922, he was devoted to local politics at Ciudadela's town hall. For a more detailed account of Benejam's life we refer the reader to the works by Adrover, Vallespir and Villalonga (1986) or, more recently, by Vilafranca i Manguan (2002).

Fig. 1. Portrait of Joan Benejam.

As we have just seen, Benejam's life took place in a quite narrow geographical area. The years he spent as a student in Barcelona and as a teacher in Blanes (about 60 km to the north of Barcelona on the Mediterranean coastline of Spain) from 1862 to 1869 were the longest period he lived far from Ciudadela. After that, he only left the island for his two years stay in La Habana and for some short occasional journeys[1].

On the other hand, Benejam was a very prolific author. He published around 70 articles and more than 40 books. Most of them were devoted to pedagogy and teaching, but he also wrote on local culture (a Menorquin-Spanish dictionary, for instance) and he was the author of some poetry and plays.

If we focus on Benejam's strictly pedagogical books, he wrote about many different topics. We can mention, for example, books about curricular contents such as *Vulgarizaciones Científicas* [Scientific Vulgarizations] (Benejam, 1898) devoted to natural sciences or *La Aurora de la Lectura* [The Dawn of Reading] (Benejam, 1890) devoted to the teaching of reading. He also wrote books like *La Alegría de la Escuela* [The Joy of School] (Benejam, 1899), addressed to inexperienced teachers and even a sketch of a renewed educational system (Benejam, 1908).

Benejam founded and edited 5 journals of pedagogical content addressed to teachers and families. Some of the names of these journals clearly show their scope: *La Enseñanza Práctica* [The Practical Teaching] (1894-1895), *La Escuela y el Hogar* [School and home] (1906-1910), *Alma de Maestro* [Soul of Teacher] (1915-1916), etc. As we can see, most of these periodicals did not have a very long life, possibly because they were mainly personal projects that had to be abandoned due to the lack of the time or money required to continue them.

Finally, it is interesting to point out that Benejam even invented a device designed to teach geography, the Didascosmos (Benejam, 1893). Roughly speaking, it

1 In a local journal from 1899, for instance, we find a short note informing about his departure to Palma and Barcelona due to the summer season (*El Demócrata* [The Democrat], year L. Ciudadela, July 29, 1899. No. 13, p. 2).

was a model that included several geographical features such as mountains, islands, rivers, railways, tunnels, bridges, etc. The idea of the author is better expressed in his own words (Benejam, 1893):

> Maps, drawings […] are not enough to show the students the world we live in. It is necessary to get closer to reality: see and touch those things the books talk about […]. Excursions put the children in the presence of nature but only a few landscapes […] can be reached. But excursions hardly ever take place and only with great difficulties. Let us bring a small world to the schools… (p. iii)[2]

However, to date, we have not been able to find any surviving Didascosmos.

La Enseñanza Racional. An overview

The journal *La Enseñanza Racional* [The Rational Teaching] (Figure 2) was the first periodical edited by Benejam (Benejam, 1888). Only 24 issues, each of them containing 12 pages, were published during 1888 (and maybe 1889). The page numbering was continuous from 1 to 288. Its periodicity is unknown, but it is likely that it was irregular. In fact, the last issue of the journal ended with the expression "end of the first part". Its publication was discontinued and the second part never came out.

By that time, Benejam had already been teaching for more than twenty years, so he was an experienced teacher who had probably had the time to reflect about curricular contents as well as about his own teaching practice. In addition, by 1888, Benejam had already published books about the teaching of different topics like arithmetic, grammar or sciences. Thus, this journal might be worth analyzing in order to have a closer look at Benejam's pedagogical ideas.


LA ENSEÑANZA RACIONAL

REVISTA PEDAGÓGICA

Año I. Redactada por Juan Benejam Núm. 1.

Fig. 2. Top of the cover page of the first issue of *La Enseñanza Racional*.

The title of the journal literally means 'the rational teaching'. This name resembles that of an institution initiated by the anarchist pedagogue from Barcelona Francisco

2 Mapas, grabados, esferas, no alcanzan a llevar al ánimo y al convencimiento del niño la idea del mundo que habitamos. Es necesario acercarnos más a la realidad: ver, palpar las imágenes de aquellas cosas que nos hablan los libros […]. Las excursiones escolares colocan al niño en presencia de la naturaleza; pero solo es dable ver por su medio algunos paisajes […]. Pero las excursiones escolares no pueden practicarse sin grandes dificultades, por cuya razón apenas se practican. Llevemos un pequeño mundo corpóreo a los centros de enseñanza… (Translation by the author).

Ferrer i Guardia (Avilés, 2006), the Escuela Racional [Rational School]. This movement led to the so-called rationalist or modern school that spread through Europe and the United States (Avrich, 1980) and which promoted freethought and collaborative learning among other progressive ideas. Nevertheless, as Vilafranca i Manguan (2002, p. 320) points out, it is not possible to consider this journal as part of the rationalist movement since Ferrer's school was not created until 1901. In any case, many of the ideas of Ferrer's school were similar to those pursued by the Institución Libre de Enseñanza [Free Educational Institution] that we have presented in the introduction.

In the first issue of the journal, we can find a very short introduction where the author provided some information about his motivations, objectives or scope. The journal is described as "one of a kind" and it is addressed not only to teachers, but also to parents and to "friends of childhood". Benejam's main objective is to "introduce easy lessons and useful exercises in the school and at home". Finally, the author is aware of the novelty and difficulty of his proposals: "This teacher who is writing, understands how difficult it is to implement the proposed exercises and lessons due to the current organization of our schools[3]".

The journal contained several different sections. Now, we give their names and a brief description of them.

- A joyful lesson. This section presented a hypothetical dialogue between a teacher and one or more students trying to show an ideal lesson in the classroom. The topics were diverse but always related to geography, from 'The mountains' to 'A journey through Germany and Holland' passing by 'The movements of the Earth'.
- Grammar through examples. This section introduced the most important grammatical concepts using them in many examples.
- Writing exercises. This section usually presented some topics the students had to write about. It also provided some examples of compositions about the given topic and, in some issues, dictation exercises.
- Nature in the presence of the children. This section included the description of a lesson about natural sciences together with didactical suggestions for the teachers to carry them out.
- Religious, social and aesthetical education. In this section, with a structure quite similar to others, religious and ethical aspects of life were treated with the students.

3 In 1886 the freedom of teaching was abolished in Spain. This was, of course, an obstacle to innovation.

- Commented poetry. In this section, we can find short poems intended to be read by the students and some explanations about the meaning of the poem, the figures of speech that contains, etc.
- Arithmetic. This section dealt with elementary arithmetic, presenting both didactical suggestions for the teachers and problems to be proposed to the students. We will get back to this section later.

The order in which the sections appeared in the journal was the same in which we have presented them above. The first five sections were included in all the issues of the journal. The arithmetic section was not present in the last issues (21-24). The poetry section was not regular, appearing in the issues 4-6, 9-12, 14, 15, 18 and 20-23.

The arithmetic section

As we have already pointed out, this section appeared only in issues 1 to 20 and, if present, it was always the closing section of the journal. Benejam did not state the ages of the students to whom the contents of the journal were addressed. However, he was an elementary school teacher so, according to the Spanish system of that time, his actual pupils were between 6 and 9 years old. We may assume that the author had his own students in mind when writing the journal.

The contents of the arithmetic section were organized into three degrees, somehow according to their difficulty and depth. This organization into degrees, which we will further describe in a forthcoming section, was also used in the grammar section. In Table 1 we show the sequence of contents corresponding to each degree throughout the 20 issues.

Table 1. Sequence of contents of the arithmetic section.

	First degree	Second degree	**Third degree**
Issue 1	Decimal number system	Decimal number system	Decimal number system
Issue 2	Addition of natural numbers	Addition of natural numbers	Addition of natural numbers
Issue 3	Subtraction of natural numbers	Subtraction of natural numbers	Subtraction of natural numbers
Issue 4	Multiplication of natural numbers	Multiplication of natural numbers	Multiplication of natural numbers
Issue 5	Division of natural numbers	Division of natural numbers	Division of natural numbers
Issue 6	Fractions	Common fractions	Common fractions

Issue 7	The meter	Decimal fractions	Operations with fractions
Issue 8	The liter	Operations with decimal numbers	Operations with fractions
Issue 9	The gram	Decimal metric system	Decimal fractions
Issue 10	The peseta[4]	The square meter	The square meter
Issue 11	Syllabus	The cubic meter	Other units of surface
Issue 12		The liter	Volume
Issue 13		The gram	The liter
Issue 14		Monetary units	The gram
Issue 15		Review of operations	Miscellaneous arithmetic problems
Issue 16		The rule of three	
Issue 17		The rule of interest	
Issue 18		Interest and discount	
Issue 19		Discount	
Issue 20		Proportionality problems	

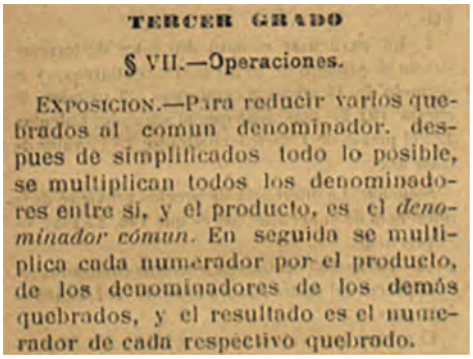
As we can see, the main topics covered in the section were natural numbers, positive rational numbers, monetary units and the decimal metric system and proportionality with its most common applications. Roughly speaking, these contents correspond to the arithmetic contents in the official curriculum of elementary primary education at that time.

In the case of the first degree, issues 11 to 20 were devoted to present a detailed syllabus. The contents corresponding to the second degree (third column of Table 1) show the usual sequencing: beginning with the number system, then natural numbers and their operations, fractions and decimal numbers, measurement and ending with the rule of three as the culmination of elementary arithmetic knowledge.

In addition to the degree structure, the arithmetic section was also divided into some kind of subsections or parts according to the type of material that they presented. Namely, we can find expositions, exercises and calculation exercises.

4 The peseta was the Spanish currency from 1868 (when it unified the 21 different currencies that were simultaneously used throughout the country at that time) until 2002 (when it was replaced by the euro).

The expositions were usually presented at the beginning of the section. They consisted in more or less detailed discourses proposed to the potential teacher to be used in the classroom. For example, Figure 3 shows the explanation of how to reduce a set of fractions to a common denominator.



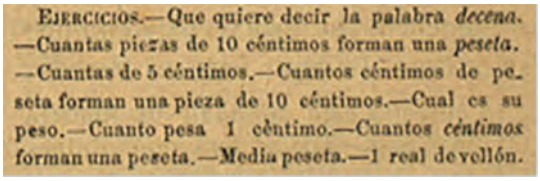
TERCER GRADO

§ VII.—Operaciones.

EXPOSICION.—Para reducir varios quebrados al comun denominador, despues de simplificados todo lo posible, se multiplican todos los denominadores entre sí, y el producto, es el *denominador cómun*. En seguida se multiplica cada numerador por el producto, de los denominadores de los demás quebrados, y el resultado es el numerador de cada respectivo quebrado.

Fig. 3. Reducing fractions to common denominator (Issue 7, p. 84)[5].

The exercises appeared just after the expositions and they were conceived as a practice to be proposed to the students as a revision or, sometimes, as an extension of the contents introduced during the exposition. The statements of these exercises were short and they consisted either of topics to be explained by the student or of short questions to be answered orally. In Figure 4, for instance, the students are asked, among other things, how many cents there are in a dime.



EJERCICIOS.—Que quiere decir la palabra *decena*. —Cuantas piezas de 10 céntimos forman una *peseta*. —Cuantas de 5 céntimos.—Cuantos céntimos de peseta forman una pieza de 10 céntimos.—Cual es su peso.—Cuanto pesa 1 céntimo.—Cuantos *céntimos* forman una peseta.—Media peseta.—1 real de vellón.

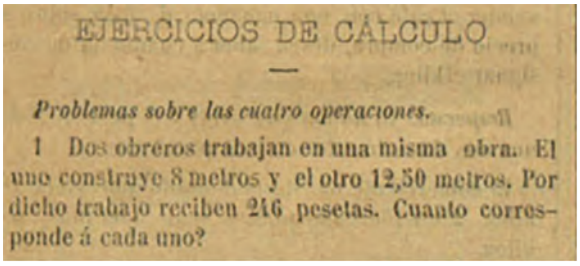
Fig. 4. Exercises about Spanish currency (Issue 10, p. 118)[6].

Calculation exercises were introduced, for each degree, at the end of the section after the exposition and the exercises. They were of a more applied nature and they mostly (but not always) consisted of arithmetic problems. More than 200 problems were proposed as calculation exercises and, in the case of verbal problems, they

5 To reduce several fractions to their common denominator, after simplifying them as much as possible, all the denominators are multiplied and the result is the common denominator. Then, every numerator must be multiplied by the denominators of the other fractions, the result being the numerator of each of the fractions. (Translation by the author).

6 What does the word 'ten' mean. – How many coins of ten cents are there in a peseta – How many of 5 cents. – How many cents are there in a dime. – How much does it weight. – How much does a cent weigh. – How many cents are there in a peseta. – Half a peseta. – 1 royal vellon. (Translation by the author).

were often presented in commercial, agricultural or industrial contexts. In Figure 5, we see an example where the payment for a job must be distributed among the two workers that did it. It is interesting to point out that nowhere the author states that the distribution must be proportional and, moreover, at that point proportionality had not been introduced yet. In fact, as we see in the figure, the problem appears in a list under the title "problems involving the four operation".



EJERCICIOS DE CÁLCULO

—

Problemas sobre las cuatro operaciones.

1 Dos obreros trabajan en una misma obra. El uno construye 8 metros y el otro 12,50 metros. Por dicho trabajo reciben 246 pesetas. Cuanto corresponde á cada uno?

Fig. 5. Problem about a proportional distribution (Issue 6, p. 71)[7].

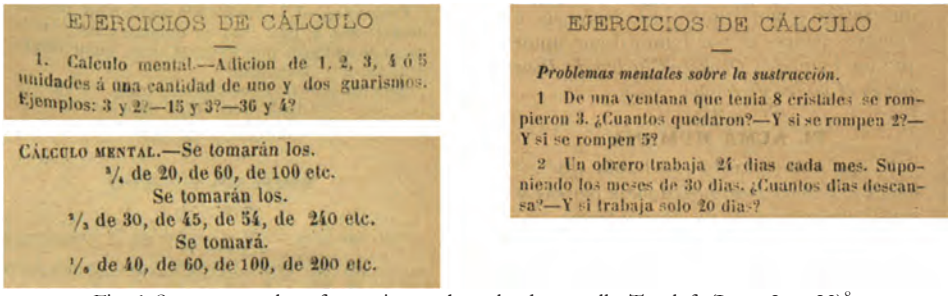
It is noteworthy that there are no figures or illustrations along with the problems. In fact, neither of the sections contained any kind of illustration. This was probably due to the printing costs, that Benejam was likely to pay himself.

A closer look at the calculation exercises

Karp (2015) points out the interest of studying problem sets. Thus, we are going to have a closer look at the calculation exercises that are part of the arithmetic section of Benejam's journal. The problems that can be found in it, have different degrees of difficulty and also seem to pursue several diverse goals.

Some of them were intended to be solved mentally. As we can see in Figure 6, when some problems had to be solved mentally, the author explicitly stated it. These problems could range from performing simple arithmetic operations (like the addition of 15 and 3 we see on the top of the left hand side of Figure 6, for instance) to solving simple verbal problems (on the right hand side of Figure 6 we read: "a window had 8 glasses and 3 broke up, how many of them were left?"). In some cases the exercises are proposed with the objective that the students develop strategies to carry out certain computations (like the ones involving fractions at the bottom of the left hand side of Figure 6, for instance).

7 Two workers are doing the same work. One of them build 8 meters and the other 12,50 meters. For this work they earn 246 pesetas. How much does it correspond to each of them? (Translation by the author).



EJERCICIOS DE CÁLCULO

1. Calculo mental.—Adicion de 1, 2, 3, 4 ó 5 unidades á una cantidad de uno y dos guarismos. Ejemplos: 3 y 2?—15 y 3?—36 y 4?

CÁLCULO MENTAL.—Se tomarán los.
¾ de 20, de 60, de 100 etc.
Se tomarán los.
⅔ de 30, de 45, de 54, de 240 etc.
Se tomará.
⅕ de 40, de 60, de 100, de 200 etc.

EJERCICIOS DE CÁLCULO

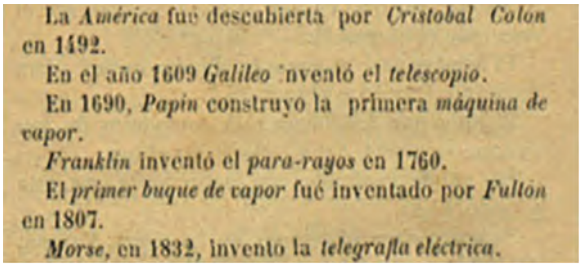
Problemas mentales sobre la sustracción.

1 De una ventana que tenia 8 cristales se rompieron 3. ¿Cuantos quedaron?—Y si se rompen 2?—Y si se rompen 5?

2 Un obrero trabaja 24 dias cada mes. Suponiendo los meses de 30 dias. ¿Cuantos dias descansa?—Y si trabaja solo 20 dias?

Fig. 6. Some examples of exercises to be solved mentally. Top left (Issue 2, p. 23)[8], bottom left (Issue 7, p. 82)[9] and right (Issue 14, p.167)[10].

Some easy exercises, particularly in those involving just the reading or writing of numbers in the first degree, the statements are used to introduce interesting or useful facts.



La *América* fué descubierta por *Cristobal Colon* en 1492.

En el año 1609 *Galileo* inventó el *telescopio*.

En 1690, *Papin* construyo la primera *máquina de vapor*.

Franklin inventó el *para-rayos* en 1760.

El *primer buque de vapor* fué inventado por *Fulton* en 1807.

Morse, en 1832, invento la *telegrafia eléctrica*.

Fig. 7. Learning from the statement of an exercise (Issue 1, p. 12)[11].

8 Mental calculations. – Addition of 1, 2, 3, 4 or 5 units to numbers of one and two digits. Examples: 3 plus 2? – 15 plus 3? – 36 plus 4? (Translation by the author).

9 Mental calculations. – Compute ¾ of 20, of 60, of 100 etc. Compute 2/3 of 30, of 45, of 54, of 240 etc. Compute 1/5 of 40, of 60 of 100, of 200 etc. (Translation by the author).

10 Mental problems about subtraction.

1 On a window containing 8 glasses 3 were broken. How many were left? – And if 2 were broken? – And if 5 were broken?

2 A worker works 24 days each month. Assuming months have 30 days, how many days does he rest? – And if he works 20 days? (Translation by the author).

11 America was discovered by Cristobal Colón in 1492.

In the year 1609, Galileo invented the telescope.

In 1690, Papin constructed the first steam engine.

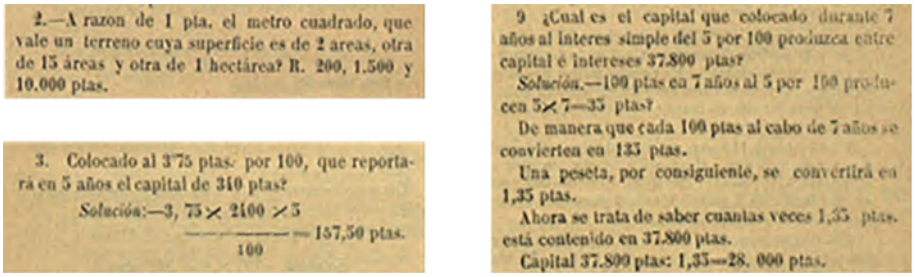
Franklin invented the lightning rod in 1760.

The first steam boat was invented by Fulton in 1807.

Morse, in 1832, invented electric telegraphy.

(Translation by the author).

In Figure 7, for example, we see some sentences that the students were supposed to read out loud. In doing so, they could learn that Galileo invented the telescope in 1609 or that the first steam boat was invented by Fulton in 1807. This practice was not uncommon in Spain at that time (Meavilla & Oller, 2014).


2.—A razon de 1 pta. el metro cuadrado, que vale un terreno cuya superficie es de 2 areas, otra de 15 áreas y otra de 1 hectárea? R. 200, 1.500 y 10.000 ptas.

3. Colocado al 3'75 ptas. por 100, que reportará en 5 años el capital de 340 ptas?

Solución:—3, $\frac{75 \times 2100 \times 5}{100}$ = 157,50 ptas.

9 ¿Cual es el capital que colocado durante 7 años al interes simple del 5 por 100 produzca entre capital é intereses 37.800 ptas?

Solución.—100 ptas en 7 años al 5 por 100 producen 5×7=35 ptas?

De manera que cada 100 ptas al cabo de 7 años se convierten en 135 ptas.

Una peseta, por consiguiente, se convertirá en 1,35 ptas.

Ahora se trata de saber cuantas veces 1,35 ptas. está contenido en 37.800 ptas.

Capital 37.800 ptas: 1,35=28. 000 ptas.

Fig. 8. Different degree of detail in the solutions. Top left (Issue 16, p. 191)[12], bottom left (Issue 17, p. 204)[13] and right (Issue 11, p.132)[14].

In the case of verbal problems, some of them were given without its solution, while in some other cases a solution was provided. When the solution was provided, the degree of detail was variable. In some cases (top left of Figure 8), only the numerical solution can be found. Sometimes, we can also find the arithmetical operations that led to the numerical solution (bottom left of Figure 8). Finally, in some cases, the author provided a completely worked-out solution that included explanations (right side of Figure 8).

Finally, a few problems included useless data; i.e., data not required to obtain the solution. This feature is still rather uncommon, even though it is generally accepted that presenting such kind of problems to our students can be very formative. In

12 At a price of 1 peseta per square meter, what is the value of a field with a surface of 2 areas, other of 15 areas and other of 1 hectare? R 200, 1500 and 10000 pesetas. (Translation by the author).

13 At an interest of 3,75 pesetas per 100, what will be the benefit after 5 years of an amount of 340 pesetas?

Solution: - $(3{,}75 \times 2400 \times 5)/100 = 157{,}50$ pesetas.

(Translation by the author). Note that the 2400 in the solution should be a 340. In addition, the final result is incorrect.

14 What is the capital that after 7 years at a simple interest of 5 per 100 produces 37800 pesetas between capital and interests?

Solution. – 100 pesetas in 7 years at an interest of 5 per 100 produce $5 \times 7 = 35$ pesetas.

So every 100 pesetas become, after 7 years, 135 pesetas.

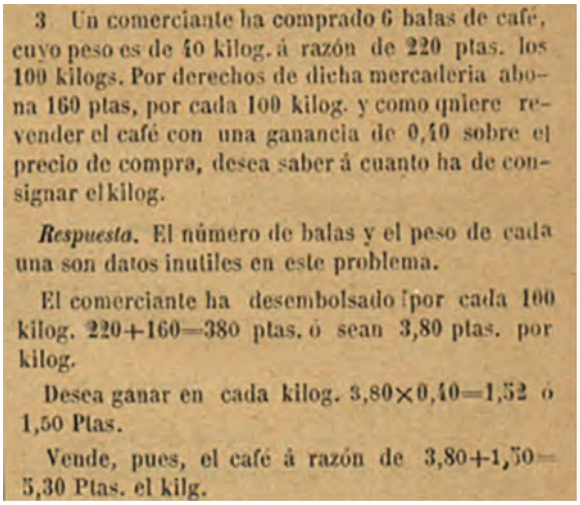
One peseta, consequently, will become 1,35 pesetas.

Now, we have to know how many times 1,35 pesetas is contained in 37800 pesetas.

Capital 37800 pesetas : 1,35 = 28000 pesetas.

(Translation by the author).

Figure 9 we see an example and how in the solution of the problem the author explicitly mentioned that "the number of packages and the weight of each of them are useless data in this problem".



3 Un comerciante ha comprado 6 balas de café, cuyo peso es de 40 kilog. á razón de 220 ptas. los 100 kilogs. Por derechos de dicha mercadería abona 160 ptas, por cada 100 kilog. y como quiere revender el café con una ganancia de 0,40 sobre el precio de compra, desea saber á cuanto ha de consignar el kilog.

Respuesta. El número de balas y el peso de cada una son datos inútiles en este problema.

El comerciante ha desembolsado por cada 100 kilog. 220+160=380 ptas. ó sean 3,80 ptas. por kilog.

Desea ganar en cada kilog. 3,80×0,40=1,52 ó 1,50 Ptas.

Vende, pues, el café á razón de 3,80+1,50=5,30 Ptas. el kilg.

Fig. 9. Problem including useless data (Issue 6, p. 72)[15].

Some words about the organization of the contents into degrees

As mentioned previously, the contents from the arithmetic section were organized according to three degrees. Only this section and the grammar section were organized in this way, even if other subjects like religious and moral education or natural sciences were very important at that time and received much attention in the journal. The reason for only organizing arithmetic and grammar in degrees may have been that it was easier to do so than in other subjects, due to the nature of their contents.

Benejam did not provide many explanations about his possible inspirations or motivations to follow such organization. In fact, neither the official curriculum nor the textbooks made that explicit distinction in degrees. The only information that we

15 A merchant has bought 6 packages of coffee, each weighting 40 kilograms, for 220 pesetas every 100 kilograms. He pays as taxes 160 pesetas every 100 kilograms and since he wishes to sell the coffee with a benefit of 0,40 with respect to the buying price, he wants to know at what price he must sell each kilogram.

Answer. The number of packages and their weight are useless data in this problem.

The merchant has paid for every 100 kilograms 220 + 160 = 380 pesetas, that is, 3,80 pesetas per kilogram.

He wishes to win in each kilogram 3,80 × 0,40 =1,52 or 1,50 pesetas.

So he sells the coffee at a price of 3,80 + 1,50 = 5,30 pesetas per kilogram.

(Translation by the author).

find appears in a short footnote on the first issue of the journal (p. 11), where the author described them as "three concentric circumferences". This description and the underlying idea make the degrees somewhat reminiscent of the modern concept of spiral curriculum (Bruner, 1960) since, in fact, it shares some of its features such as the revisiting of topics or the increasing level of difficulty.

Since the idea of progression is essential in designing a curriculum, it is relevant to analyze how Benejam constructed the different levels of difficulty. As an example, we will focus on the case of the addition of natural numbers as presented on the second issue of the journal (pp. 22-24). In Table 2 we see the main ideas regarding this topic, the sequence in which they are introduced and the degree each of them belongs to.

Table 2. Addition of natural numbers (Issue 2).

First degree	1. Example of how to perform the algorithm with only two summands
Second degree	2. Nature (abstract or particular) of the summands and the sum 3. Example of how to perform the algorithm with four summands
Third degree	4. Definition of problem, data and unknown 5. Consequence over the result of increasing one summand 6. Checking the correctness of a sum by rearranging the summands

The first degree dealt only with algorithmic issues, which were revisited in the second degree together with a first step into more conceptual aspects. Finally, the third degree mainly dealt with conceptual issues. This organization of contents let us infer that Benejam's idea of difficulty was mostly related to conceptual aspects rather than to reckoning abilities (Prytz, 2015). Contents evolve from performing simple procedures to the understanding of them and the third degree deals mainly with cognitive abilities. This stress on cognitive abilities can be seen as a particularity of Benejam's work, at least when compared with the Spanish official curriculum at that time and with the curricula from other countries. In the Swedish case, for example, Prytz (2015, p. 318) identifies only one term referring to cognitive abilities during the period 1850-1950.

As we mentioned before, there was no explicit indication in the journal about the age of the students. If we assume ages 6 to 9, it would be possible to identify the three degrees with ages 6-7, 7-8 and 8-9, respectively. This, of course, would coincide with the usual organization of students into years and the distinction into degrees would be less interesting. However, we do not think that this is what Benejam had in mind. In fact, we think that his idea was that the students could proceed through these degrees regardless their age. A gifted student of age 6 could rapidly advance to the third degree, while a student age 9 could still be struggling in the second degree.

This would be consistent with the author not mentioning ages and would oppose to the rigid curricular organization into courses according to age.

An example of innovation

As we have already mentioned, the arithmetic contents that we can find in the corresponding section Benejam' journal were essentially the same that could be found in the official curriculum. Nevertheless, the treatment of the contents was, in some cases, different from the usual.

Here, we are going to briefly present and analyze de case of the introduction of the number system. As seen in Table 1, this topic was covered in the first issue of the journal (pp. 10-12).

In order to see how this topic was usually introduced at that time, we can have a look at some textbooks. Here, we focus on two contemporary books published in 1899 (Olivares, 1899) and (Guerola, 1899). Both these books were written by elementary teachers (like Benejam himself was) and they were addressed to elementary primary school pupils.

In Table 3, we present the main concepts related to the topic of number system as well as the order in which they were presented by the three compared authors.

Table 3. Sequence of contents related to number system (Issue 1).

	Benejam	**Olivares**	**Guerola**
Digits	1	4	4
Names of compound units (from tens to millions)	2	3	3
Idea of decimal and positional system	3	5	6
Concept of unit	4	1	1
Numbers as sets of units. Abstract and particular numbers	5	2	2
Absolute and relative value of digits	6	6	5
Roman numerals	7	-	7

We can see that both textbooks give a very similar treatment to the topic. They begin presenting the concept of unit and define abstract numbers as set of units. Then, they give names to the compound units, introduce the idea of digit and finish with the notion of positional system. Roman numerals are introduced at the end of the chapter only by Guerola.

On the other hand, Benejam did not begin with the abstract concept of unit. Rather, he first introduced the digits as mere symbols as well as the naming and writing conventions for numbers on the first degree. After that, on the second degree, he introduced the concepts of unit and number, defining both abstract and particular numbers. Finally, on the third degree, Benejam focused on the relative value of digits and on the decomposition of a number into units, tens, etc. He concluded with a description of the roman numerals and the roman number system.

As for the exercises, in the first degree Benejam only proposed the writing and reading of small numbers, with at most two digits. In addition to some 'open' proposal like writing down the number of books that a student owns, we find some exercises whose learning goal is not only mathematical. Thus, Benejam proposed that the teacher exemplified the number 5 mentioning the five continents with their names, or the number 4 by the mention of the four cardinal points, etc. Exercises for the second and third degree proceeded in the same way but increasing the number of digits and also proposing exercises of a more theoretical flavor. For instance, on the third degree, Benejam proposed to discuss the values of 0 according to its position as an extension exercise.

Final comments

We think that Benejam's work in this journal provides a good example of the efforts of anonymous teachers to innovate and to disseminate their innovations in a period when education in Spain was trying, sometimes successfully, to modernize.

In 1921, Benejam published a short note of 2 pages in a journal called *El Menorquín* [The Menorquin] (Year III, February 1921. No. III, pp. 1-2), which was published in Buenos Aires addressed to expatriates. The eloquent title of that note was *Contra la Corriente* [Against the Grain] and, since it was published only one year before his death, we may consider it as some kind of pedagogical testament.

As such, this short text contains many of Benejam's main ideas regarding teaching. For example, he says: "the education that comes from a book turns out to be a dead education and we need an alive education", and related to this: "in the school we deal with dead ideas […] Boredom is nearly unavoidable" and also: "facts, real facts and not the imaginary ones are those to be preferred".

These ideas, quite similar to those of the Institución Libre de Enseñanza [Free Educational Institution], were rather general, but we have seen them exemplified in his journal and, particularly, in the arithmetic section. Thus, the use of particular situations as a starting point can be related to the search of an alive education, while exercises like the mental calculations may have been conceived to avoid boredom

and to engage the students. Finally, most of the problems included in the section were stated in a real context.

Benejam's 1921 note includes also a very short and simple statement that, nevertheless, encloses a very strong idea: "it is easy to teach, but it is difficult to educate". The main goal of Benejam was always to educate his pupils and not only to teach them and this could also be achieved while doing arithmetic. The following excerpt comes from Issue 10 where the peseta (the Spanish currency at the time) was being introduced in the first degree

> There are gold and silver coins which are more worthy than a peseta: you don't have them, do you? You only have a few copper coins. Nevertheless, do not spend them in vain because some day you may need them […]. Well, each time you have a coin, think over for a while before spending it (p. 118)[16]

Even though it belongs to the arithmetic section, there is no mathematics in the previous fragment. However, it contains an important piece of advice for the students and it shows that we can contribute to the education of our students at the same time that we teach them mathematics.

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16 Monedas hay de plata y oro que valen más que una peseta: vosotros no las poseéis, ¿verdad? Sólo disponéis de algunas piezas de cobre. Mas no por esto las gastéis inútilmente, porque alguna vez tendréis necesidad de ellas […] Pues bien, cada vez que poseáis alguna moneda, reflexionad un poco antes de gastarla. (Translation by the author).

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