Differential calculus in a journal for Dutch school teachers (1754-1764)

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Abstract

From 1754-1769, a monthly mathematics journal for teachers was published in the Netherlands. The journal was a combination of mathematics and news; the news sheet contained items on vacancies, comparative exams for new posts and other items which were of relevance to teachers in Dutch schools. The mathematics section contained some theory, many mathematical problems and their solutions, sent by readers and published a few months after the problems and discussions on some of the solutions. The readers were encouraged to send mathematical problems, in the form of questions. The content provides insight in the mathematical knowledge of teachers in the 18th century. Most of the questions were on arithmetic, algebra and geometry combined with algebra, however, there was also a small number of problems solved with fluxions, differential calculus, using Newton's dot notation. In this paper the use of differential calculus by the teachers in the 18th century is analysed and discussed.

Keywords: history of mathematics education; mathematics journal for teachers; knowledge of differentiation; Dutch teachers; 18th century

Introduction

In recent years several authors have published on mathematical journals for teachers, all journals from the 19th century. Preveraud (2015) wrote about American mathematical journals in the 19th century and their connection with the French textbooks, written for the Ecole Polytechnique. Furinghetti (2017) wrote about mathematical journals for teachers in Italy and the development of mathematics teachers' professional identity, also in the 19th century. Pizzarelli (2017) researched mathematics in educational journals in Turin in the second half of the 19th century. Oller (2017) published an article on the mathematics section in *El Progreso Matematico*, a journal for teachers in primary education in Spain at the end of the 19th century. As both Oller and Furinghetti remark, during the 19th century the number of mathematics journals grew continuously, including mathematics journals for teachers. That was no doubt related to the development of the profession of mathematics teachers, through teacher education and teacher associations (Furinghetti, 2017; Schubring, 2015). However in the Netherlands already in the mid-18th century, during about 16 years a mathematics journal for teachers flourished, based on private initiative and more than half a century before the start of a modest system of teacher education.

Bjarnadóttir, K., Furinghetti, F., Krüger, J., Prytz, J., Schubring, G. & Smid, H. J.(Eds.) (2019). "*Dig where you stand" 5. Proceedings of the fifth International Conference on the History of Mathematics Education*. Utrecht: Freudenthal Institute.

In 1754 in Purmerend, a small town in the north west of the Netherlands, a new type of journal was published, different from the many periodicals which were available during that period in the Republic. This new journal was meant for a specific group of people. *Mathematische Liefhebberye* [Mathematical Pastimes] was a journal on mathematics, for teachers in primary schools, and for teachers in private schools who offered lessons in mathematical subjects (Krüger, 2017). A large part of the content consisted of problems, entered by the editor or by readers, and the solutions of those problems, sent by the readers. The published solutions mostly showed a sound knowledge of and competency in mathematical methods that spread among users of mathematics during the 17th century, such as solving systems of linear equations and of higher order polynomial equations, use of trigonometry and of spherical trigonometry, etc. However, a small number of solutions, often new solutions of older problems from well-known authors, showed the use of a new technique: differential calculus, mostly in the dot notation of Newton.

The development of calculus and its use in mathematical applications in the 18th century is discussed by many authors, e.g. Bos (1993), van Maanen (2006) and Struik (1995). Not much attention has yet been paid to the circulation of knowledge about calculus amongst the 'minor' users of mathematics, the practitioners (van Maanen, 2006). One such group was formed by teachers: primary school teachers who wished to improve their basic skills in mathematics, teachers who worked in and perhaps owned a private school (called a 'French' school) and teachers who wished to attract private students for instruction in more advanced mathematics.

This unexpected use of differentiation to present a new method of solving old problems gives rise to some questions.

- How common was the knowledge and use of differential calculus among those teachers?
- Which problems in the journal were solved with help of differentiation?
- What was the position of calculus in the 'body of knowledge' of this group of teachers?

Mathematics in Dutch education in the 18th century

As described before (Krüger, 2014; Krüger, 2017) at the start of the 18th century most villages had a primary school; in towns there were usually more schools. Children could learn basic arithmetic, usually after learning to read and to write. In many towns there were privately owned primary schools, the French schools, which offered comprehensive primary education, with more subjects and often more mathematical topics. Thus more advanced arithmetic and other mathematical subjects, e.g. accounting, geometry, algebra, trigonometry and navigation, could be learned through private tuition, in specialized mathematics institutes and through self-instruction.

At universities, for example in Leiden, Utrecht and Franeker, mathematics was part of the undergraduate courses. Sometimes also lectures on architecture, hydraulics and other applied mathematical topics were offered. Some mathematical subjects were offered in Dutch language, such as practical geometry (surveying techniques) and fortification at the universities of Leiden (Duytsche Mathematique or Dutch Mathematics) (Krüger, 2010) and Franeker (Dijkstra, 2012), navigation and fortification at some of the 'illustere scholen'[1].

Professor Willem Jacob 's Gravesande (1688-1742), from 1717 professor in astronomy, mathematics and philosophy at Leiden University, was an admirer of Newton and did much to propagate ideas of Newton on the Continent. He taught experimental physics, combined explicitly mathematics and physics and emphasized the practical usefulness of mathematics (van Dijk, 2011). It is likely that 's Gravesande as well as his colleagues and successors, Petrus van Musschenbroek (1692-1761) and Johannes Lulofs (1711-1768) gave lectures in differential and integral calculus (van Dijk, 2011). However, not many teachers in primary education would have had the opportunity to follow lectures at a university.

There was a demand for mathematics teaching, especially mathematics that could be applied in practical situations. Throughout the 17th and the 18th century knowledge of practical mathematics gained more and more relevance, for example for navigation, civilian and military architecture, surveying, water management, military engineering and the fine arts (Krüger, 2012; van Maanen, 2006). The demand for teaching of mathematical subjects naturally led to a demand for teachers of mathematics. As in 16th century England (Rogers, 2012), mathematics education was very much a matter of 'grass roots' activities, with an emphasis on useful mathematics.

However, there was no national system of secondary education, nor were there institutes for teacher education. Whoever wished to become a teacher had to find a post as help-teacher in a school. After some years a help-teacher could apply for a teaching position of his own in a primary school. In the 18th century it was expected in certain regions to sit a comparative examination for such a teaching position (Krüger, 2017). These examinations included more and more often mathematics, sometimes consisting only of a few arithmetic problems; but it happened also that the mathematics exam consisted of four to five subjects and took many hours. If one wished to start a private (French) school or to teach in a French school a broad knowledge of mathematics definitely was an asset. So for teachers, knowledge of

1 These institutes offered university type courses, without right of promotion. They were situated in the larger towns, such as Amsterdam, Rotterdam, Middelburg and Deventer.

mathematics was important for their career options, however, they had to acquire the knowledge by themselves.

Mathematische Liefhebberye, a mathematics journal for teachers

Mathematische Liefhebberye was a monthly, published from April 1754 until December 1769 by the librarian Pieter Jordaan[2] (Krüger, 2018). It consisted of two parts: the *Nieuws* [News], for teachers of Dutch and French schools and *Mathematische Liefhebberye* [Mathematical Pastimes]. Important items in the section with news were vacancies and the mathematics questions of the comparative examinations for a teaching position, especially in the west of the country. The mathematics section focused initially on arithmetic and algebra, also solutions of the questions of examinations were published regularly. It was the first journal of this kind in the Netherlands. Mathematical topics were sometimes discussed in more general journals, such as *Boekzaal* and the *Journal litéraire de la Haye*, of which 's Gravesande was one of the editors (Jorink & Zuidervaart, 2012). However, these were general journals, for an erudite audience, while *Mathematische Liefhebberye* was aimed at teachers in general and teachers of mathematics.

We will discuss a few characteristics of the journal which are relevant to the topic of this paper. For a more extensive description of content and contributors, see (Krüger, 2017; Krüger, 2018). Though the name suggests a recreational journal, the aims stated by Jordaan and by the longest serving editor, teacher and mathematician Jacob Oostwoud (1714-1784), indicate that it was a journal meant to improve the mathematical knowledge of teachers and to enable teachers skilled in mathematics to share their knowledge with a larger group of colleagues. Readers were encouraged to send problems and solutions; there were also occasionally discussions about published solutions. Several teachers contributed regularly by sending problems; a much larger group contributed by sending solutions. Oostwoud and his successor, Louis Schut, published the names of the contributors of problems, the names of the readers who solved the problems and also which problems each one solved and for many problems the source, often from Dutch mathematics books, but also from German and English books and journals.

The first editors, the teachers P. Karman and P. Molenaar, treated series extensively. They also published some other problems on algebra, arithmetic and probability. Oostwoud, mathematics editor from November 1754 until July 1765, started with the treatment of arithmetic and with exercises which consisted of both simple and more advanced problems. His aim was to give opportunity to improve mathematical skills for different levels; for those teachers who were starting to learn

2 The digitalized version of all year volumes is available at the site of the library of the University of Amsterdam (http://uba.uva.nl/home)

mathematics and for those who already had mathematical skills in varying degree. A method to improve the skills of those who were not advanced in mathematics was explanation of a topic, followed by some exemplary problems and problems for the readers to solve. Thus Oostwoud explained several rules in arithmetic, the procedures in modern algebra, equations, cossic algebra, geometric, arithmetic and harmonic series, fractions, simple calculations in probability, special numbers, logarithms, some topics in spherical trigonometry, etc.

Besides these clarifications there were always many problems to solve, varying from rather simple to more complex. Oostwoud selected from existing publications and published problems sent by readers; these also often came from other publications.

Examples of the type of problems are

- simple questions on arithmetic and algebra;
- modelling of a situation, resulting in (systems of) linear and quadratic equations;
- problems to do with trade;
- calculations with series;
- simple questions on probabilities;
- problems solved with proportionality, also in mechanics;
- questions about mixtures of substances;
- questions about distances travelled;
- problems in plane and solid geometry;
- calculation of a maximum or minimum.

In November 1758 the first thousand problems had been published, and on that occasion Oostwoud gave a categorization of these problems, which differs very much from the topics mentioned above. He gave again a categorization after the second thousand of problems was published, that number was reached in March 1763. The third thousand questions were complete in July 1769. See table 1.

Table 1 Categorization of three times thousand problems

Category by Oostwoud	**1-1000**	**-2000**	**-3000**	**Total**
	'54-'58	'59-'63	'63-'69	
Interesting problems	97	247	139	483
Problems on history or other important matters	11	13	9	33
Proofs	5	7	6	18
Probability	10	11	--	21

Calculation of most and least (maximum and minimum)	58	15	2	75
Mechanics	5	--	--	5
Finding (a) word(s) as solution	5	14	22	41

There is some overlap between the first category and the others. In the solutions of problems on 'most and least' quite often differential calculus was used. Oostwoud did not mention problems on algebra, arithmetic, series, geometry or other common mathematical subjects. He seemed to mention mainly the topics which were a bit out of the ordinary and those problems which were a bit more demanding.

Use of differential calculus

Already in the solution of one of the first problems, April 1754, when Molenaar and Karman were editors, differential calculus was used. The last time this method was used occurred in a problem of February 1764. In the next paragraph some examples are shown. All together 70 of the 75 problems on maximum/minimum were solved with help of differential calculus; in some cases solutions with and without differential calculus were published. The use of differentiation reached a peak in 1756 and 1757 and then diminished. See also Figure 1. During the time Schut was editor, no differential calculus was shown.

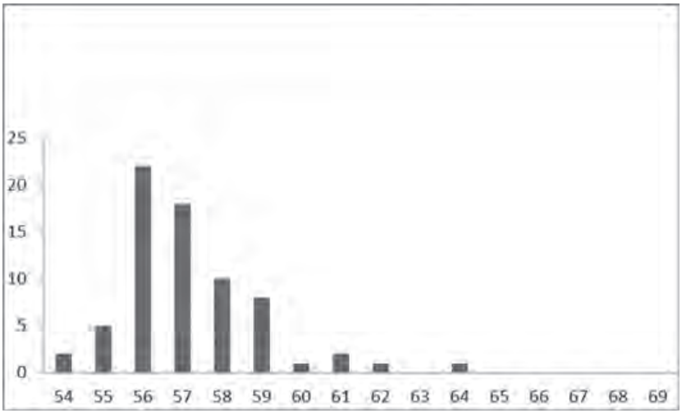


Fig. 1. The frequency of solutions with differential calculus, per year.

The large increase in solutions by differentiation in 1756 and 1757 was entirely due to Oostwoud himself. In 1756 he published 19 (out of 22) of those problems, originating from *Mathematisches Sinnen-Confect* [Mathematical Delights] by Paul Halcke (1719). In 1757 also 13 of the 19 maximum/minimum problems were added by Oostwoud, again from Halcke. In 1767 he published a Dutch edition of Halcke's book, with many additions, referrals to Dutch mathematics authors and also application of differential calculus for maximum/minimum problems. Halcke himself, a 'Rechenmeister' and member of the Mathematical Society in Hamburg did not use calculus in his book, which treated arithmetic and algebra. All together 43 problems solved with differential calculus originated from Halcke's book.

Some examples[3]

1754: 33 (by Molenaar & Karman, no source mentioned)

> A measurer of seeds needs a new measuring vessel; he wishes to have it made of the least amount of wood possible. Which will be the ratio of the height to the width?

Solution by D.T.V.W and Jan van Leeuwen, translation of the first part of the solution, see also Figure 2[4]:

> Let the volume be *a*. The proportion of diameter and circumference equals 1 to *p* [p=π]. Let the radius be *x* and the height *y*. Then 1:*p* :: 2*x*:2*px* is the circumference. The area of the bottom = *pxx*. The volume = *pyxx*=*a*.
>
> The lateral surface equals 2*pxy*. The total area = 2*pxy*+*pxx* which should be a minimum.

Continued as follows: Which Fluxion equals (Welker Fluxie is, Figure 2)


Welker Fluxie is $2py\dot{x} + 2px\dot{y} + 2px\dot{x} = 0$

$2p$ ———

Of $y\dot{x} + x\dot{y} + x\dot{x} = 0$

En $pyxx=a$ Welker Fluxie is $= pxx\dot{y} + 2pyx\dot{x} = 0$

px ———

Of $x\dot{y} + 2y\dot{x} = 0$

Dies $y\dot{x} + x\dot{y} + x\dot{x} = 0$ } Sub.

En $2y\dot{x} + x\dot{y} = 0$ }

$x\dot{x} - y\dot{x} = 0$

$\dot{x}$ ———

Ergo $x - y = 0$ En dus $x = y$

Dat is, de hoogte gelyk aan de halve wyte.

D. T. V. W.

NB. Is mede Beantwoord door *Jan van Leeuwen* te Rotterdam.

34 VOOR-

Fig. 2. The last part of the solution of problem 33 (1754), by two readers.

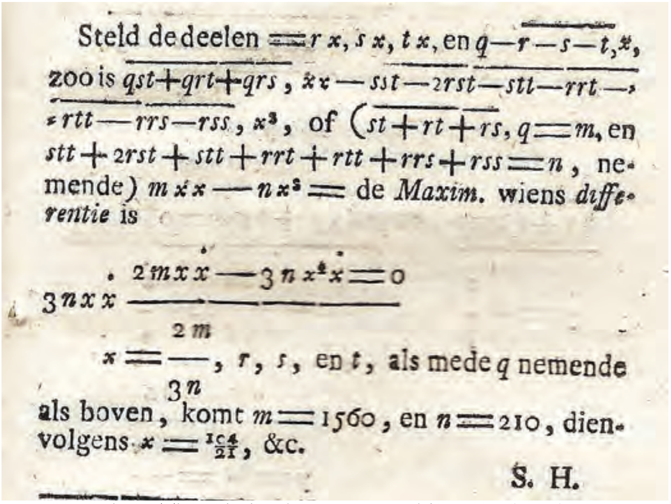
3 All transcriptions and translations of the exercises are by the author.

4 The letter symbols were all written in italics.

A typical problem from Halcke (1719), provided by Oostwoud, is number 337 in 1756 (Halcke nr. 371)

> Divide a given number *q* in 4 parts, of which *a:b:c=r:s:t*. The sum of the four products *abc, bcd, cda* and *dab* is the maximum. What are the parts?

Solutions were sent by 5 readers, out of 22 who sent solutions that month.


Steld de deelen = rx, sx, tx, en $\overline{q-r-s-t},x$, zoo is $\overline{qst+qrt+qrs}$, xx — $\overline{sst-2rst-stt-rrt-rtt-rrs-rss}$, x^3, of ($\overline{st+rt+rs}, q=m$, en $stt+2rst+stt+rrt+rtt+rrs+rss=n$, nemende) $mxx-nx^3$ = de *Maxim.* wiens *differentie* is

$$2mx\dot{x}-3nx^2\dot{x}=0$$

$$3nx\dot{x} \quad \overline{2m}$$

$x=\frac{2m}{3n}$, r, s, en t, als mede q nemende als boven, komt $m=1560$, en $n=210$, dienvolgens $x=\frac{104}{21}$, &c.

S. H.

Fig. 3. The solution by S.H. to problem 337 (1756).

Translation:

Let the parts = *rx, sx, tx* and $q-(r+s+t)x$, then $(qst+qrt+qrs)\ xx-(sst+2rst+stt+rrt+rtt+rrs+rss)\ x^3$ or

(let $(st+rt+rs)q=m$ and $sst+2rst+stt+rrt+rtt+rrs+rss = n$)

$mxx - nx^3$ = the maxim(um), of which the differential is[5]

$2mx\mathrm{d}x-3nx^2\mathrm{d}x=0$ Divide by $3nx\mathrm{d}x$

Etc.

Without the use of calculus an expression for *x* would be found by means of lengthy calculations of products of polynomials, summing those products, finding a suitable factor for division and so diminishing the degree of the polynomial. See for an example Halcke (1719, p. 300) or the translation by Oostwoud (1767, p. 422), both available on-line[6].

5 For technical reasons the dot-notation is replaced by d*x*, d*y*, etc. in transcription

6 Halcke (1719), *Deliciae Mathematicae* in Google books en zie ook http://echo.mpiwg-berlin.mpg.de/MPIWG:6XTQN17A. Oostwoud (1767), Mathemaisch Zinnenconfect zie Google books.

A different problem, contributed by a reader, G. de Gooyer, in 1757 is 555. No source was provided.

> Two straight roads, both East-West, at a distance of 135 rods. At DE Daphne is running West, at a velocity of ½ rod/second. At AO runs het pursuer Apollo, at a velocity of 36 rod/minute. However, on the rough land between the roads his velocity is only 27 rods/minute. Which course should Apollo choose to reach his mistress in the shortest time and how much time will he need?

Solutions were sent by 3 people, out of 15 who sent solutions that month. In Figure 4 is shown part of the solution sent by J. Kok and J. Kooyman, both teachers at Texel.

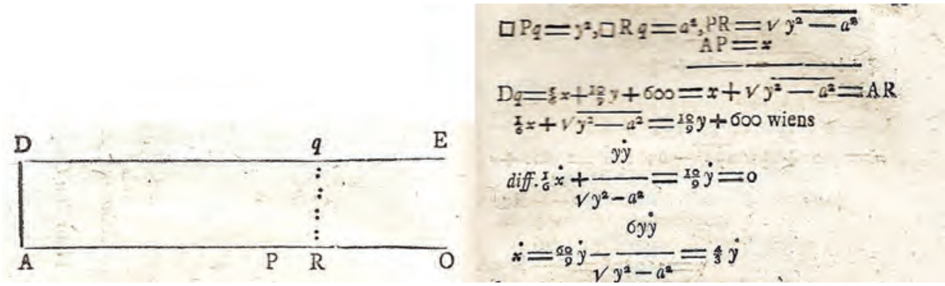


Fig. 4. Part of the solution to problem 555 (1757), by Kok and Kooyman.

The position of differential calculus in the journal

The problems sent by the readers were published if they met the criteria of the editor(s). Between 1754 and December 1758 about 90 persons sent problems which were published; and between January 1759 and December 1769 about 100 persons saw their questions published. Of these contributors about ten persons sent problems which were solved with help of differential calculus. All those had to do with maximum or minimum; the sources mentioned were Paul Halcke (1719), Laurens Praalder (1753), Gerard Kinckhuysen (1723), Frans van Schooten jr. (1659, 1660) or problems previously published by Molenaar and Karman, but without a published solution. For all those problems more traditional solutions were known.

All together a few hundred people participated in sending solutions to all type of problems. About 27 of them used differential calculus in solutions, sometimes only once or twice, others such as G. de Gooyer, J. Kok and J. Kooyman, very regularly. So it was a relatively small group that used this method. However, it looks as if gradually more readers started using differentiation, perhaps because some became more aware of its practical use. Besides the relative low number of contributions two matters are remarkable.

Firstly: there was no explanation of the procedure of differentiation or the mathematical background. Oostwoud and others regularly gave an explanation, of virtually every topic in the journal; however, there was no explanation at all on differential calculus. Not how to use it, nor when to use it.

Secondly: problems on maximum/minimum were the only problems for which differentiation was used in the solution. The notation was nearly always Newtonian, the words used were both fluxion ('fluxie') and differentiate ('differentie').

So only a minority of the contributors used calculus, they only used it for one type of problem and there was no information on the why and how of the use. It looks as if a small group of people knew how to use it and that some were interested to learn this method to solve a well-defined type of problem. But somehow it seemed not to be part of 'official' mathematics. Since calculus was not asked in the comparative examinations for teachers, a large group would not be bothered, perhaps only those who were more curious. Also the topic might be taught to private students by skilled teachers in mathematics, such as Oostwoud at Oost-Zaandam and R. van Vreeden at Arnhem.

From where did the knowledge about use of differential calculus come? Mathematical knowledge could be dispersed through books, private lessons or lectures, written manuscripts such as dictations from private lessons and journals. Most teachers had a simple background; in the 18th century most of them would not have the means to attend university lectures or to take private lessons from a professor, such as 's Gravesande or Lulofs. Manuscripts of simple people such as teachers or artisans are extremely rare, they usually aren't preserved. So that leaves books and articles in journals, available to the Dutch teachers of mathematics.

Books and journals as sources of knowledge about calculus

Oostwoud mentioned more than 120 authors as source for published problems and for different types of solutions. Though he mentioned no publications in relation to the use of differential calculus, some of the books and journals mentioned in relation to other topics might contain something about calculus as well. We will first look at some books, published before 1750, which had possibly relevant information and were available to Dutch teachers.

Books

Let us start with a well-known and respected mathematics teacher and prolific author of mathematics books: Abraham de Graaf (1635->1717). He published on arithmetic, accounting, geometry, algebra, trigonometry, navigation, etc. and was often referred to by Oostwoud and others. It is very likely that books by De Graaf and also the publications by Struyck and by Hellingwerf (see below) were a source of knowledge for Oostwoud and others who were interested in the new technique of differentiation. In *Analysis of stelkunstige ontknoping in de meetkunstige werkstukken* [Analysis or algebraic solutions in geometrical problems] (1706) De Graaf dealt with maximum and minimum, tangents to curves, etc. It is a theoretical treatise building on his previous work and using Cartesian geometry. De Graaf mentioned Descartes, Huygens, Slusius, Hudde, Frans van Schooten and also Von Tschirnhaus[7], but not Leibniz or Newton. De Graaf based his treatment of tangent lines, maximum and minimum etc. on the work of these 17th century mathematicians from the northern and southern Netherlands. Their work and also *Analysis* by De Graaf may be considered a forerunner of the calculus of Leibniz and Newton (see also Jahnke, 2009). For example Proposition XVIII states:

> If in a curve one has two points infinitely close together then the extended chord is tangent to both points.
>
> Consequences
>
> 1- If a line CK touches the curve in C and on the curve is another point D, infinitely close to C, then CK also touches the curve in D.

De Graaf used specific symbols; next to *x, y,* z for variables, he used *a*, *b*, *c* for parameters, *f*, *g*, *h* for infinitely small and *m*, *n*, *o* for infinitely large. In the chapter on tangents (how to find tangents on curves) one finds the following example Figure 5).

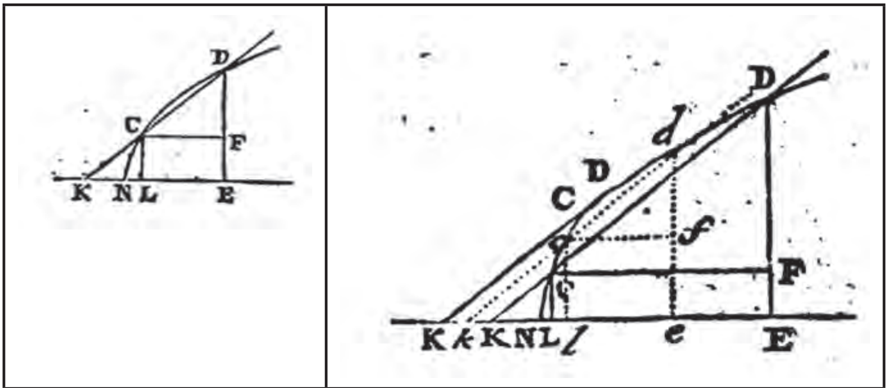


Fig. 5. A. de Graaf, to find the tangent to a point of a curve LCD with known equation. (*Analysis*, pp 14-18)

Compilation of text by De Graaf (p.15)

7 Christiaan Huygens (1629-1695), Jan Hudde (1628-1704), Frans van Schooten Jr. (1615-1660), René François de Sluse (also Sluze, Slusius) (1622-1685) were mathematicians from the Northern and Southern (Sluse) Netherlands, Ehrenfried Walther von Tschirnhaus (1651-1708) was a Saxon mathematician and philosopher.

NL=x, LC=y, CF=f, DF=g, KL=z and $z:y=f:g$, thus $f=(gz)/y$

Let the equation of the curve be $y^3=rrx$

NE=$(x+f)$, ED=(y+g)

$y^3+3yyg+3ygg+g^3=rrx+rrf$; subtraction of $y^3=rrx$

$3yyg+3ygg+g^3=rrf=(rrgz)/y$; divide by g and multiply by y

$3y^3+3yyg+ygg=rrz$

If $f=0$ then also $g=0$

Thus $3y^3=3rrx=rrz$ or $3x=z$ = KL

One could say that De Graaf dispersed through his *Analysis* the concepts of infinitely small distances, laying the ground for calculus.

In 1718 Pieter Hellingwerf, a mathematician from Hoorn, published *Wiskonstige Oeffening* [Mathematical Practice]. This book contains chapters on mechanics, music, sundials, specific gravity and mathematical miscellaneous, more or less an overview of existing knowledge, with mathematics applied to physics and reports of experiments by the author. In the last chapter, on p.421, Hellingwerf presented in a short text on "The flow of magnitudes" a definition of differentiation and integration. He mentioned differentiating ('differentiëren'), flow ('vloeyen'), reduction on the infinitely small ('reductie op het oneyndig kleyn') etc. He mentioned De Graaf and his symbols, but gave an example using dot notation, similar to the notation in use in *Mathematische Liefhebberye* (Figure 6).

This Hellingwerf does not figure among the authors mentioned by Oostwoud, however the style of this text in combination with the overall character of the publication (overview of existing knowledge) suggest that the procedure of 'flow' (differentiation) was a relatively new, but known procedure.

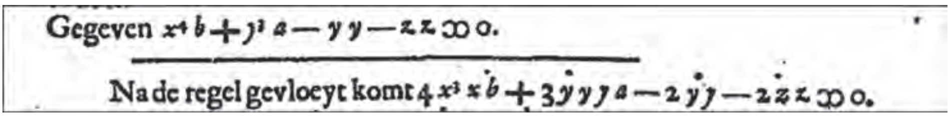

Gegeven $x^4b+y^3a-yy-zz \infty 0$.
Na de regel gevloeyt komt $4x^3\dot{x}b+3\dot{y}yya-2\dot{y}y-2\dot{z}z \infty 0$.

Fig. 6. Hellingwerf (1717), p.421. De Graaf used the reversed symbol instead of =.

This Hellingwerf does not figure among the authors mentioned by Oostwoud, however the style of this text in combination with the overall character of the publication (overview of existing knowledge) suggest that the procedure of 'flow' (differentiation) was a relatively new, but not unusual procedure.

In 1738 Joan Christoffel van Sprögel published the first volume of *Grondbeginzelen van alle de mathematische weetenschappen* [Principles of all mathematical sciences], a translation of *Anfangsgründe aller mathematischen Wissenschaften* by Christian Wolff (1710). This work contained clear definitions and descriptions of differential calculus, with all rules for differentiation of algebraic expressions explained in a lucid way, using the notation of Leibniz. Applications mentioned are tangents to curves, but not problems with maximum/minimum. Wolff was quite popular in the Netherlands, Oostwoud also mentioned him a few times.

In 1740 Nicholaas Struyck (1686-1769), a mathematician, who wrote about probability and several other mathematical topics, published *Inleyding tot de algemeene geographie met eenige andere natuur- en sterrekundige verhandelingen* [Introduction to general geography with some treatises on Physics and Astronomy]. Struyck was an admirer of Newton and mentioned him several times. In the second part (Physics and astronomy) he used differentiation of algebraic expressions a few times, always mentioning 'flux' and always with dot-notation, see for example p.168. Struyck was a member of the Royal Society in London and correspondent of the Académy des Sciences in Paris. Oostwoud referred to Struyck regularly, for example in 1757, in the solution of exercise 275.

In 1749 Symon Panser, a teacher of mathematics, astronomy and navigation in Embden[8] published *Mathematische Rariteit-kamer* [Mathematical Repository]. The book was mainly about algebra; it contained in four volumes many exercises and their solutions on basic algebra, polynomial equations up to degree 6, arithmetic, geometric and harmonic series and applications, rules to find the largest and the smallest (maximum and minimum), logarithms, etc. As origin of many exercises Panser mentioned Meissner, Halcke, De Graaf and Kinckhuysen. On p. 441 treatment of maximum and minimum starts. Panser mentioned first the use of arithmetic progression, referring to De Graaf and Kinckhuysen, followed by differential calculus and the names of Leibniz and Wolff. There are definitions, rules for differentiating algebraic expressions and applications in finding maximum and minimum, using problems from Halcke's *Mathematisches Sinnen-confect* as source. He used the notation of Leibniz. Panser was very well known to Oostwoud and his readers and more than 30 times one of his exercises or solutions were used in *Mathematische Liefhebberye*. Besides the notation used, this book is very similar to what Oostwoud wrote. Both Oostwoud and Panser used Paul Halcke as a major source for exercises and both were a member of the Mathematical Society in Hamburg.

In 1750 Johan Lulofs, professor in mathematics, astronomy and philosophy at Leiden University, published *Inleiding tot eene natuur- en wiskundige beschouwing des aardkloots* [Introduction to a physical and mathematical view of the Earth], in which

8 Embden is situated in Saxony (Germany), very close to Groningen (Netherlands).

he combined geography with physics and mathematics. Lulofs used occasionally differential calculus, he mentioned amongst others Newton and Keill, a follower of Newton. However, he used the Leibniz notation. Lulofs' work was known to Oostwoud, he mentioned him occasionally.

The lists of authors, which were provided by Oostwoud and later Schut, also contained many authors from France, Germany and England; sometimes but not always translated into Dutch. Regarding these authors there seems to be no link with calculus.

Journals

Sometimes Oostwoud mentioned a journal as a source for published problems. Examples were *De maandelykse Berlynsche schatkamer der wijsheid* [The Monthly Treasure Trove of Wisdom from Berlin], het *Europeis Magazyn der byzondere zaken* [European Magazine of Extraordinary Matters], the *Ladies Diary* (also *Woman's Almanach*), the *Gentleman's Diary* and the *Imperial Magazine*. The English journals were mentioned from 1759 onwards, and provided some examples of differentiation (fluxion). The first two journals (Dutch) were also mentioned before 1758, but appeared to offer no relevant information about differentiation.

Discussion

In this mathematics journal, solutions with differential calculus were limited to one type of problem and a relatively small group of readers. There are indications that the group of users increased, possibly through the examples in the journal. Though by the mid-eighteenth century even in Leiden, the centre of promotion of Newton's idea, the Leibniz-notation had become more common in mathematical publications; Newton's dot-notation prevailed in *Mathematische Liefhebberye*.

The body of knowledge of teachers in Dutch and French primary schools was defined by the practical use of mathematics. In the first place arithmetic, a quite extensive topic. Other important topics were algebra, with polynomial equations, and of course geometry. Algebra in geometry, linked with the name of Descartes, had become respectable and quite common. Potential private students were those who might want to learn surveying (lessons in geometry, trigonometry, logarithms and algebra), navigation (geometry, spherical trigonometry, astronomy, algebra and logarithms) or commerce (arithmetic, algebra and accounting). Differential calculus was finding its applications, for instance in geography, astronomy and for mathematics, in the determination of tangents to curves and in finding maxima and minima of polynomials.

Teachers of mathematics did not develop new mathematical knowledge; they were instrumental in dispersing the existing knowledge; most often the knowledge about the practical use of mathematics. Finding a maximum or minimum (volume, area, costs, etc.) evidently belonged to their domain, tangents to curves far less so.

This explains to some extent why the use of differentiation was limited to this one type of problems and also why only a limited group of these teachers could use or was willing to learn differentiation. On the other hand, the use of this new method by a growing group of teachers shows that there was interest in new developments and new methods and that among those teachers there were quite a few with considerable knowledge of mathematics. For one thing mathematical competency enabled one to improve one's income, by acquiring better paid positions and through attracting private students. There was not yet a structure with secondary education to attract mathematics teachers, so in primary education one would find a large variety of mathematical competency among teachers.

Vermij (2003) discusses the introduction and dispersal of Newton's ideas in a network of science and mathematics amateurs in Amsterdam in the 1690's by David Gregory. This network, in which Adriaan Verwer, a merchant from Rotterdam and Joannes Makreel, a broker and amateur mathematician, were central figures, probably formed the first group of admirers of Newton in the Netherlands. Makreel also had been part of a network with Ewald von Tschirnhausen and Abraham de Graaf, in the 1680's and he had close connections with Bernard Nieuwentijt (1658-1719), a physician and regent in Purmerend. Van Nieuwentijt published during 1694-1695 two tracts on calculus, in which he showed himself a defender of Newton and criticized Leibniz. He was a very popular and influential author in the Netherlands and elsewhere through his physico-theologian work from 1715, *Het regt gebruik der wereldbeschouwing* [The Religious Philosopher: Or, the Right Use of Contemplating the Works of the Creator] (Jorink & Zuidervaart, 2012). So there was an early group of non-academic mathematical amateurs and authors of textbooks whose members knew about and were admirers of Newton's mathematical work, in Amsterdam and vicinity. Through influential authors such as De Graaf, Nieuwentijt and Struyck, it is likely that other teachers of mathematics in the area learned about fluxions and differentiation.

The lack of discussion of the procedure and of the background of differential calculus indicate that as far as these teachers were concerned calculus was not yet part of the established body of knowledge. It had the character of a method which worked well in practice, but the mathematical foundation was not too clear. An instructive book in Dutch language on calculus was only published in 1775 by Johannes Arent Fas, assistant professor at Leiden University. He wrote the book for his students at the course for Duytsche Mathematique at Leiden University.

It is intriguing that the Newton dot-notation persistently was used, with few exceptions, in *Mathematische Liefhebberye*. By 1754 the Leibniz notation or at least a notation d*x*, d*y* was at least as usual, if not more common. One reason may be typographical. If the fraction notation is used, one needs more than one line unless the fraction is printed very small. In *Mathematische Liefhebberye* the fractions were printed very small, which made reading more difficult. A dot-notation was relatively quick to write and not too complicated in printing.

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