Pedagogical value of the Russian abacus and its use in teaching and learning arithmetic in the 19th – early 20th century

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Abstract

The paper is devoted to the history of use of various forms of the beads abacus in mathematics classrooms. The authors briefly present the history of the Russian beads abacus (счёты / schyoty), discuss its transmission to Europe, and focus on the didactical applications of various types of the beads abacus in mathematics classrooms in Western Europe and North America in the 19th and early 20th century. They claim that while counting instruments designed as arrays of sliding beads were relatively well known in Western Europe even before the 19th century, the introduction of manipulations with beads abacus in mathematical curricula was a rather complex process related to the emergence of new educational theories and practices.

Keywords: teaching and learning arithmetic, bead abacus, pedagogical innovations, 19th century Europe, abacus in Russian and North American schools

Introduction

Under different forms and names, the abacus remained a useful computational tool in many countries and cultures over the history of humanity and recently has gained popularity in the context of school reforms. Historians of science are well aware of the fact that abaci of various types appeared in numerous cultures (Greece, Roman Empire, Medieval Europe, China, Japan, Russia, etc.) during certain historical periods; these instruments were used to perform a variety of calculations, starting from the simplest ones, such as adding up costs of items sold on the market, to quite complex and sophisticated ones, for instance, extraction of square and cube roots. However, the didactical value and implications of the use of these abaci for mathematics instruction still remains underexplored, even though it is obvious that the history of the abacus is closely related to the history of didactical approaches devised to teach basic arithmetic operations. Indeed, during certain periods, instruments of various types appeared as the mandatory teaching and learning tools in various countries, sometimes only to completely disappear later being replaced by other tools.

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The variety of counting devices is too large to be treated here, this is why we chose to focus only on one type of the instrument, namely, on the so-called 'Russian abacus' and its modifications that became popular among mathematics educators in Western Europe, North America, and in Russia/USSR in the 19th and early 20th century. This short paper, however, cannot provide a detailed account of the history of the instrument and of its applications in all the mentioned didactical contexts. This is why we decided to limit ourselves to a short discussion of a relatively small selection of cases and provide their detailed discussion in future publications.

Context and rationale of the present study

The present preliminary study of the Russian abacus is related to our ongoing investigation of the history of development of mathematics education in Russia. It is well known that the so-called Russian abacus ('schyoty') was widely used in Russia no later than the 17th century (Volkov, 2018 forthcoming); however, when working on L.F. Magnitskii's (1669-1739) *Arithmetic* (1703), the first Russian printed arithmetic textbook, we found no references to this instrument. Instead, the author focused on written computations with clear reference to the Western arithmetical manuals of the 16-17th centuries (Freiman and Volkov, 2012; 2014; 2015). While working on a forthcoming book on the history of computing and computational devices (Freiman and Volkov, 2018, forthcoming), we clearly realized the important role that the bead abacus played in the mathematics instruction of Russia and Western Europe in the 17th-19th century (Karp, 2018, forthcoming; Volkov, 2018, forthcoming). This prompted our special interest in the history of the device and of its use in mathematics classroom.

Some authors suggested that the massive introduction of the bead abacus (also known as 'numeral frame' or 'boulier') in (mainly) elementary school arithmetic classrooms in Western Europe and North America started with Jean-Victor Poncelet (1788-1867) who supposedly brought its Russian prototype to France (Guzevitch and Guzevitch, 1998). This hypothesis, which is directly related to the history of mathematics education as well as to the present-day debates concerning the role of counting instruments in the mathematics classroom, still deserves a further investigation (Volkov, 2018, in press).

The history of the didactical use of counting instruments and, in particular, of various forms of bead abacus in Europe and North America in the 19th and early 20th century, arguably, remains underexplored. This history is associated with the names of Johann Heinrich Pestalozzi (1746-1827), Samuel Wilderspin (1791–1866), and Marie Pape-Carpantier (1815–1878), who, among others, suggested various didactical applications of counting instruments and elaborated their own types of didactical devices (Kidwell et al, 2000; Régnier, 2003; Bjarnadóttir, 2014). Russian connections were explicitly mentioned by numerous authors (for one of the earliest publications mentioning this point see Curie (1846)), yet it has not been often noticed that the very idea of the use of the bead abacus as 'didactical tool' was brought to Russian educators *from* Western Europe in the 19^{th} and early 20^{th} century. The concluding part of the present chapter deals with this particular element of the history of Russian abacus.

The origin of the Russian abacus

A definitive history of the Russian abacus (счёты; hereafter, 'schyoty') still has to be written. The earliest available materials in Russian language describing the construction and operations with the device are dated of the mid-17^{th} century (Spasskiĭ, 1952). The first mentions of this instrument appeared in publications of West-European travelers who visited Russia in the late 17^{th} century. One of them was Nicolaas (or Nicolaes) Witsen (1641–1717), a Dutch traveler who visited Russia in 1664 and in 1692 published his book titled *Noord en Oost Tartarye* [North and East Tartary] containing a very brief mention of the schyoty.[1] The origin of the instrument suggested by Witsen (the Golden Horde) was later identified by a number of authors with the Mongol Empire, and, subsequently, with China. This hypothesis about the origin of the Russian instrument was energetically rejected by Spasskiĭ (1952).[2]

The instrument described in the Russian manuscript mathematical manuals of the 17^{th} century differed considerably from the one used in Russia starting from the early 18^{th} century till the late 20^{th} century. Unlike its later descendant, it contained two counting fields; the lower sections of each of these two fields were subdivided into two parts used for operations with common fractions of two types. The beads in the left section of the lower part represented fractions with denominators 2^n, $n = 2,\ldots,7$, while the beads in the right section represented fractions with denominators $3\cdot2^n$, $n = 0,\ldots,5$; see Figure 1.

The aforementioned manuscript manuals contained a table of identities that allowed conversion of fractions with denominators 2^n and $3\cdot2^n$ represented with the schyoty. The table was followed by a list of identities that allowed the operator reduce combinations of fractions of these two kinds to units or to relatively simple fractions, or combination of both; each identity was accompanied with a picture representing the respective configuration of beads on the schyoty. For example, the

1 Witsen (1692, p. 472). We would like to express our gratitude to Professor Jan van Maanen who kindly helped one of the authors (A. Volkov) obtain access to the 1692 edition of Witsen's book preserved in the Library of Utrecht University.

2 For a detailed discussion of Witsen's statement and the possible origin of the Russian instrument see (Volkov, 2018 forthcoming).

configuration shown in Figure 2 corresponded to the identity $^1/_4 + ^1/_8 + ^1/_{16} + ^1/_{32} + ^1/_6 + ^1/_{12} + ^1/_{48} + ^1/_{96} = 1 - ^1/_4$.

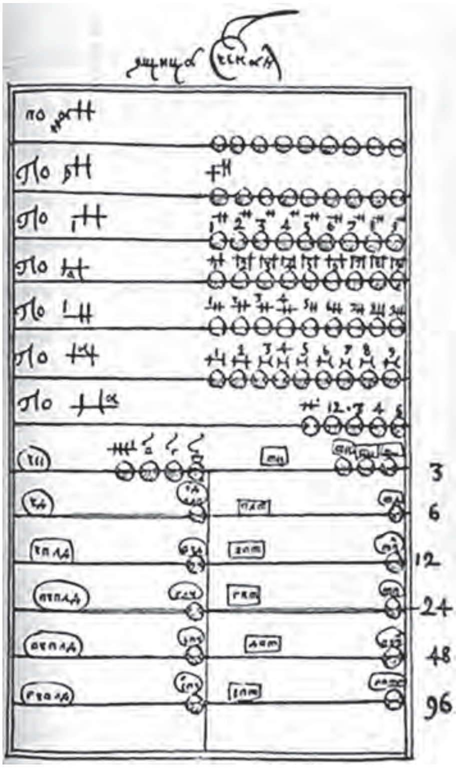

Fig. 1. Russian "Counting Board" ("Дщица счётная") of the 17th century (Spasskiĭ, 1952, p. 325).

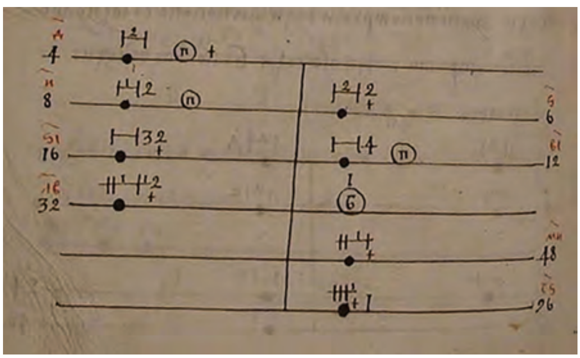

Fig. 2. Identity 1/4 + 1/8 + 1/16 + 1/32 + 1/6 + 1/12 + 1/48 + 1/96 = 1 - 1/4 represented with the Russian abacus ([Anon.], 1865, p. 110v.).

Poncelet and the introduction of the Russian schyoty to France

Michel Chasles (1793–1880) conjectured that a specimen of the Russian schyoty was brought to France by the French mathematician Jean-Victor Poncelet from his captivity in Russia that started during Napoleon's invasion in Russia (1812) and lasted till 1814 (Chasles, 1843). According to Chasles, the Russian instrument was used for mathematics instruction in the schools of Poncelet's hometown, Metz, after his return from Russia and later became popular in numerous schools across France. Modern historians did not find supportive evidence for Chasles' statement; however, Gouzevitch and Gouzevitch (1998) argued that this hypothesis can be safely accepted since Poncelet most likely knew about the publication of Chasles of 1843 and yet never contested the latter's conjecture. Even if Chasles is right, the Russian abacus was known in Europe well before the captivity of Poncelet: besides the abovementioned report of Witsen, there existed other rather detailed descriptions of the instruments, the earliest of which, provided by Jacob (Iacobus) Reutenfels in his *De Rebus Moschoviticis*, was published in 1680. This and other descriptions of the Russian schyoty and of the Chinese 'suanpan' widely circulated in continental Europe and in the United Kingdom prior to the early 19th century; see, for instance, the description of suanpan by John Barrow (1764–1848) (1804, pp. 295-297). These publications suggest that the beads abaci of Russian and Chinese type were known in France even before Poncelet's captivity. It remains unknown how exactly the French teachers introduced this new teaching tool in their classrooms, and what was the role played by Poncelet in bringing the Russian abacus to French schools; we hope to return to these topics in a later publication. As far as the beads abacus is concerned, such an instrument was indeed used for instruction in the first half of the 19th century in some European countries and in North America (see below), yet it remains unclear how much the use of beads abaci in schools were influenced by the specimen supposedly brought by Poncelet. In this respect, possible connections of the use of abacus in schools to the pedagogical innovations introduced by the Swiss educator Johann Heinrich Pestalozzi are particularly interesting. Indeed, Jules Thurmann (1804-1855), in his *Principes de pédagogie* (1842), refers to the beads abacus (Fr. 'boulier') as "Pestalozzi's abacus" ([boulier de Pestalozzi]) and states that it had been used in German and French educational institutions called "salles d'asiles" to provide young children with basic arithmetic instruction. The claim about Pestalozzi's use of the abacus (or of *an* abacus) thus certainly deserves a further investigation.

Pestalozzi's tables and the Russian abacus

The name of Pestalozzi is often cited when talking about didactical ideas of using particular ways to visualise numbers that may help grasping the concept of number in intuitive way. However, to the best of our knowledge Pestalozzi himself never mentioned the Russian or Chinese abaci in his work; instead, for teaching arithmetic he used tables. Not a mathematics teacher himself, he conceptualized some (apparently, believed to be innovative at that time) teaching approaches; among them was the use of table of units supposed to enhance the development of mental faculties based on visualization (Anschauung) emphasizing sense-impression, observation, perception, and intuition (Bjarnadóttir et al., 2013, p. 30).

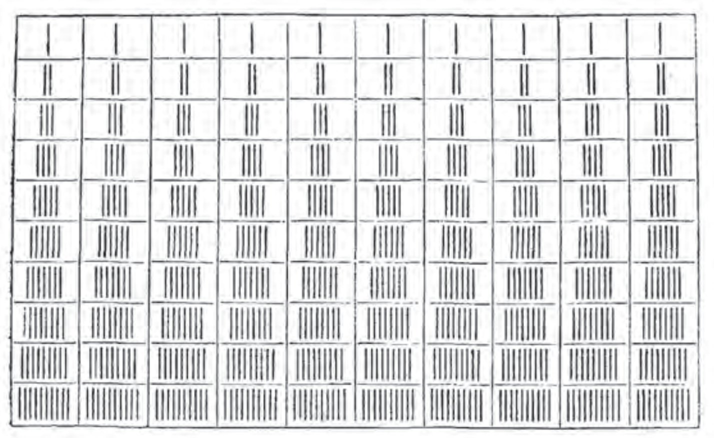

Fig. 3. Pestalozzi's table representing natural numbers from 1 to 10 in 10 columns (Galanin, 1912, p. 9, Figure 1).

The table shown in Figure 3 is divided into ten rows; each row contains ten cells with vertical line segments, all of the same length. The first row has ten cells each containing one vertical line, the second has ten cells with two vertical lines, and so on. Daniel-Alexandre Chavannes (1765–1846) who visited Pestalozzi's school, describes how the instructors taught students to count lines following, for example, the first row: one, two times one, three times one, etc. For the second row it would be one time two, two times two, three times two, and so on. The same way of counting was used for the rest of the rows. Further on, the table was used to help students visualize how many fives are contained, for example, in 37: the student would examine the fifth row to see that seven times five would be equal to 35; this product plus two units would altogether result in 37 (Chavannes, 1805, p. 32).

Beads abacus in French 'salles d'asile'

The use of abacus in French schools for infants [salles d'asile] is another interesting case. During the 19th century special schools for children of 2-6 years of age were created in France. Their development was guided by increasing emphasis on the importance of education and schooling, which included basics of arithmetic. A large-size abacus was a part of mandatory equipment of every classroom and was supposed to be used to perform demonstrations of manipulations and operations with (integer) numbers. The earliest instruction of the Ministry of Education concerning the use of this instrument that we were able to find is found in an official letter sent to all the directors of the 'salles d'asile' in April 1836 and titled *Instruction relative à l'établissement et à l'organisation des salles d'asile* [Instruction concerning the establishment and organization of the schools for infants]. It prescribed that each classroom should have a beads abacus "un boulier-compteur ayant dix rangées de dix boules chacune" [a beads calculator having ten rows of ten balls each] (Duruy 1865, p. 440). Later, "des exercices avec le boulier-compteur" [exercises with beads calculator] were mentioned in the program of examinations of the applicants for position of "directrice de salle d'asile" of 1856 (Duruy 1867, p. 22). The quality of the exercises with the abacus was one of the evaluation items of the candidate (ibid., p. 23). The abacus is again mentioned in one of the documents of 1899 as one of the classroom accessories (Duruy 1902, p. 698).

The abacus of Marie Pape-Carpantier

Marie Pape-Carpantier is known as the founder of a system of kindergartens whose curriculum included arithmetic (Cosnier, 2003); she claimed that she invented her own abacus (which she called "boulier numérateur") in 1868 and recommended it to be used to support teaching and learning of arithmetic (Pape-Carpantier, 1878, pp. 17-18; Régnier, 2003). The instrument was supposed to be used in classroom; it consisted of a large vertical wooden frame with bars and sliding beads. The main difference between Pape-Carpantier's abacus and its precursors consisted of the presence of 'broken' bars, that is, bars having horizontal and vertical parts; see Figure 4.

The *Manuel de l'Institutrice* [Manuel of the Educator], co-authored by Pape-Carpantier, provides a description of the instrument (Pape-Carpantier, Delon, & Delon, 1869, pp. 199-200); the instrument apparently was a modification of the bead abacus [boulier-compteur] that had been used for some time in France (see the previous section). The modified 'boulier' invented by Pape-Carpantier consisted of a wooden frame 70 cm of height and 60 cm of width crossed by 11 wires, two of which (on the top) were horizontal (and contained, according to Figure 4, 15 beads each), while nine other bars comprised vertical and horizontal parts. To set a digit, the operator was supposed to place the given number of beads on the vertical section of the respective bar (Pape-Carpantier, Delon & Delon, 1869, p. 199).

The extant specimens of Pape-Carpantier that we inspected had the 'broken' bars organized in three groups of three bars each; the color and size of beads in each group was different from those of the beads in two other groups. The difference of sizes was insisted upon by Pape-Carpantier in her description; it was supposed to inform the learner about the relationships between units (that is, powers of 10) in different positions. Designed to introduce the basics of decimal system and elementary operations, the abacus could have been used to show, for example, that 9 units on the first wire on the right could not form ten in other way than by using one bead on the second wire representing tens, and so on. However, while Pape-Carpantier and her co-authors mention that while being an excellent tool to introduce counting and number system to beginners in a visual and exact way ("boulier … parle aux yeux et ne permet ni incertitude, ni erreur", that is, "the abacus… speaks to eyes, and does not allow either uncertainty or error"), the abacus *per se* does not guarantee an understanding of the structure of numbers and could become just a tool for counting beads from 1 to 9 without grasping the concept of decimal position. Hence its use, according to them, should be combined with other counting tools to count different types of objects.

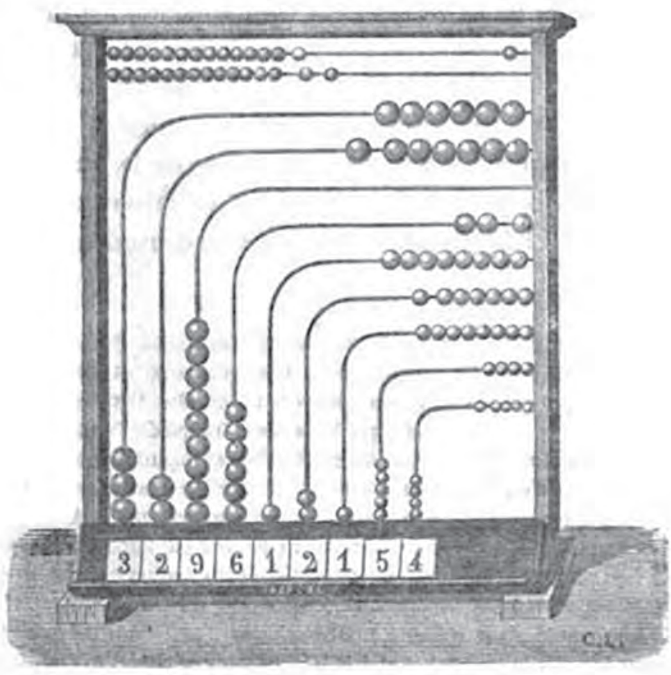

Fig. 4. "Boulier-numerateur" of Marie Pape-Carpantier (Pape-Carpantier, Delon & Delon, 1869, p. 199).

Interestingly enough, Pape-Carpantier did not know that the abacus that had been used in France originated from the Russian schyoty and believed instead that it came from Ancient Greece:

> L'instrument employé au début dans les Salles d'Asile françaises pour l'enseignement de la numération est le boulier-compteur, imité de l'abaque des Grecs. On en a imaginé un plus en rapport avec la numération décimale qui n'existait jamais dans l'antiquité. L'ancien boulier, comme l'abaque, offre à l'enfant des boules enfilées par des tiges horizontales, et ne peut servir que pour la numération par unités. En disposant les tiges verticalement, et en y plaçant des boules de grosseurs graduées comme les ordres d'unités dans la numération décimale, l'instrument se prête à toutes les opérations et démonstrations du calcul […]. (Pape-Carpantier et al., 1887, p. 211)

Abaci in the United Kingdom

In the late 18th and early 19th century descriptions of beads abaci of various types (esp. of the Chinese suanpan) widely circulated not only in continental Europe but also in the United Kingdom; see, for instance, the description of suanpan by John Barrow (1764 - 1848):

> Their [i.e., Chinese. –V. F. & A.V.] arithmetic is mechanical. To find the aggregate of numbers, a machine is in universal use, from the man of letters, to the meanest shopman behind his counter. By this machine, which is called a Swan-pan arithmetical operations are rendered palpable. […] [A detailed description of the Chinese suanpan follows. – V. F. & A. V.] This is clearly a system of decimal arithmetic, which, for the ease, simplicity, and convenience of its operations, it were to be wished was generally adopted in Europe, instead of the endless ways in which the integer is differently divided in different countries, and in the different provinces of the same country. The Swan-pan would be no bad instrument for teaching to a blind person the operations of arithmetic. (Barrow, 1804, pp. 295-297)

It is important to stress that Barrow does not mention education of young children and instead suggests that the abacus may be used for teaching blind persons. The idea that a tangible representation of numbers may have been used for such teaching was advanced by other authors before him; see, for example, the writings of Nicholas Saunderson (1682–1739).[3] In English it was called "palpable arithmetic" (this name was given by John Colson, 1680–1760). The instrument of Saunderson was quite enthusiastically presented in the textbook of Christian Wolff (1679–1754).[4]

In 1819 Samuel Wilderspin, whose fundamental principle was to introduce arithmetic to children on the basis of experience with material objects prior to introducing the abstract concept of a number, experimented with buttons moving along a screen to help children with counting. Spending years in inventing a numeral

3 See Saunderson, 1741, pp. xx-xxvi. See also Tattersall, 1992, p. 358.

4 Wolf refers to Saunderson's *Elements of Algebra*, published in 1741; see Wolf, 1747, vol. 1, pp. 71-79; 1777, vol. 1, pp. 145-154.

frame, he designed two original versions of beads abaci to be used in the "infant schools" for children of one to seven years of age established across the United Kingdom. The first device called "Transposition frame" was designed as a beads abacus with 12 bars with 1, 2, …, 12 beads of the same color; the second device called "Arithmeticon" was designed as a large size vertical wooden frame with twelve horizontal wires each having 12 black and white sliding beads.

Pestalozzi's work drew the attention of English educators of the mid-19th century. For instance, F. Curie (1845; 1846) analyzes methods of teaching the whole numbers by comparing Tables of Units (from Pestalozzi's treatise *Exercises of Arithmetic for Elementary Schools*) with what he calls "Russian Balls Frame" (that is, a beads abacus resembling the Russian schyoty) and which, according to him was "little known to this country" (Curie, 1845, p. 131). While comparing it to Pestalozzi's table, the author finds that the device is "still more mechanical and tangible apparatus;" yet, without diminishing Pestalozzi's merits, the printed textbooks and diagrams accompanying them would hardly "arrest and engage children's attention for any length of time" whereas the infant's mind would be "immediately interested and impressed" with the "Ball Frame" and its coloured balls and wires. The work with the abacus thus would help the instructor to introduce to the learners "any collection of units, tens, hundreds, or thousands" as well as to rapidly and clearly show "the relations and various factors in multiplications." Curie's abacus, the "Russian Ball Frame," is shown in Figure 5. The author mentions the possibility of placing the frame vertically or horizontally, while in his figure the instrument is placed in such a way that the bars are vertical; this orientation of the instrument is similar to that of the Chinese abacus and not of the original Russian one. With nine balls on each wire, the nine digits in each position can be represented. (In Figure 5, position *a* is that of units, *b* of tens, etc.; wire *f* thus represents hundreds of thousands whereas the wires to the right of wire *a* represent the decimal fractions 10^{-1}, 10^{-2}, etc.).

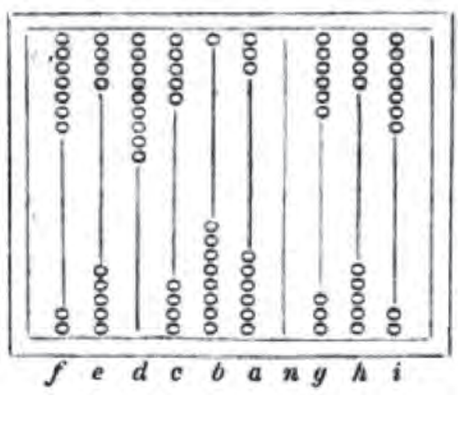

Fig. 5. "Russian Ball Frame" according to Curie (1845, p. 132).

North American experience

In the United States, William S. Phiquepal (1779–1855), a follower of Pestalozzi, used an abacus with 10 wires and 10 beads on each wire in 1825. Some authors noticed differences between his abacus and the instruments designed by British educators (Wilderspin and Wilson): the numbers of the wires, of the beads, and the shape of the beads were all different from those of the British instrument. It was suggested that Phiquepal brought the instrument from France (Kidwell, Ackerberg-Hastings, and Roberts, 2008), but it is unknown whether he knew about the Russian abacus (or its modifications) used in France. According to L. Dunton (1828–1899),

> the best kind [of abacus. – V.F. & A.V.] is composed of a wooden frame about four feet long and two feet wide, in which, running horizontally from end to end, are fastened ten brass or steel rods; on each of these rods are ten easily moved wooden balls about an inch and a half in diameter. The whole is supported at a convenient height by means of upright standards attached to bars running crosswise at the bottom. It is well to have the balls painted different colors, say three red ones at the left on each wire, then three yellow ones, and four green ones at the right ; or, two and two, black, red, yellow, green, and white. One-half the frame should be covered with a board, so as to conceal all the balls that are not used in any example. (Dunton, 1888, p. 10)

As particular advantages of the tool, Dunton listed (ibid.):

- Balls can be easily seen across the classroom;
- Different colors used for the balls help student (even the farthest) see their number;
- It can be used in introducing first steps of counting, and perhaps later on during exercises (on arithmetic operations).

Although the author mentioned that different (and more sophisticated) forms of bead abacus had been invented, he argued that the simplest ones were the best to assist teaching.

Schyoty as computing device in Russia in the 19th and 20th century

It appears plausible to conjecture that the interest of Western authors in the Russian abacus (schyoty) expressed in their publications of the early 19th century soon became known to Russian readers and drew their attention to the instrument. An important role was played by a retired General of Russian Army, Fedor Mikhaĭlovich Svobodskoĭ (b. in 1780s or early 1790s; d. 1829). The instrument of Svobodskoĭ was a combination of several "counting fields" (usually 12, but sometimes even 30) each of which was a set of horizontal bars with beads, that is, a specimen of traditional schyoty. Two "fields" were used to set the initial data, and the other fields were used for intermediate results. A special commission of the Russian Academy of Sciences explored the instrument of Svobodskoĭ and approved its use for educational purposes in all Russian universities. Numerous manuals, for instance Orlitskiĭ (1829a; 1829b), Tikhomirov (1830), explained the operations performed with the instrument of Svobodskoĭ (Spasskiĭ, 1952). However, in the present paper we shall not discuss the use of the beads abacus and its modification in Russia and the Soviet Union, since the instrument was used primarily for actual computations and not for educational purposes (hence the university courses devoted to its use); the old traditions of operations with it were presumably followed by the authors of the manuals of the 19th and early 20th century. There exist numerous pieces of evidence suggesting that the manipulations with schyoty were studied by merchants, accountants, salesmen, clerks and office workers before and after the Socialist Revolution of October 1917. Despite the innovations of Svobodskoĭ, the instrument used the most was the traditional schyoty with only one counting field.

Teaching tools in Russian schools: a possible German influence?

While the bead abacus was long time used in Russia in everyday life, the history of its didactical application for supporting teaching and learning arithmetic had not been explored till the beginning of the 20th century. Dmitriĭ D. Galanin in his history of the use of visual manipulatives in teaching and learning arithmetic in Russian schools (1912) suggested the attention of Russian educators of the 19th century to didactical applications of the beads abacus was drawn by Russian educators under a clear influence of their German counterparts; however, it remained unclear how and when the abacus should have been used in Russian classroom (Galanin, 1912, p. 23).

Galanin claims that the Russian beads abacus (schyoty) was brought to Germany in 1812 by Russian soldiers (and not by Poncelet mentioned above),[5] and that later on, German educators understood their didactical value as visual support to be set vertically in the classroom, so all students could see it (1912, pp. 22-23). On the contrary, Galanin found it meaningful to return to the primary (i.e. directly related to the real-life computations) role of the bead abacus: according to him, it was the device for counting and calculations to be put "in the hands of every student" (1912, p. 25). However, despite all the presumed usefulness of the bead abacus for teaching arithmetic, Russian educators saw it as only one of many other tools invented or used abroad, in particular in Germany where, according to Wilhelm August Lay

5 This claim of Galanin is not supported by any evidence and, most likely, is purely conjectural. Another conjecture of Galanin (1912, p. 22), namely, that a form of beads abacus (a board with wires and rings sliding on them) was mentioned as early as 1522 in a work of Adam Riese (1492 or 1493–1559), is equally questionable: the instrument mentioned by Galanin is apparently the "counting on lines", the well-known medieval counting device using round tokens placed on a counting board (or any flat surface) with drawn parallel lines.

(1862-1926) cited in (Galanin, 1912, p. 13), more than 200 different visual manipulatives were used. Among the tools most frequently mentioned in the literature devoted to the use of manipulatives in arithmetic teaching, Galanin (1912, pp. 19-20) mentions the invention of Ernst Tillich (1780–1867) called "arithmetic box," as well as graphical figures 'Zalenbilders' introduced by the German mathematician and physicist Friedrich Gottlieb von Busse (1756-1835) in 1786 (Smith, 1903, p. 77) and further developed by other educators in the 19th century (Figure 6).

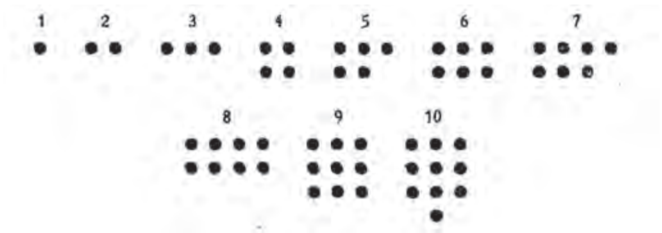

Fig. 6. Arithmetical figures of F. G. von Busse (from Galanin 1912, p. 25).

Galanin also analyses in details Pestalozzi's tables which were apparently not very efficient in doing calculations mentally without visual support; these difficulties, according to Galanin, led the Pestalozzi's followers (among them Johann Joseph Schmid, 1785–1851, a pupil and later a collaborator of Pestalozzi) to look for other visual tools to help students more efficiently. The use of manipulatives was also discussed by a renown Russian educator P.S. Gur'ev (1807–1887) whose didactical ideas are worth mentioning. In the context of Russian schools of the 19th century, the use of visual manipulatives in general, and of the bead abacus in particular, were subjects of numerous debates among educators. It remains not clear what was the place of the bead abacus in the portray of school arithmetic in the beginning of the 20th century and more detailed investigation is therefore needed.

Conclusions

The fact that the abacus was enthusiastically embraced by the educators of Western Europe and North America in the first half of the 19th century leads us to a deeper look into this case from the didactical perspective which was not necessarily related to the simplicity and efficiency of the instrument itself but rather to the new (at that time) didactical theories that stressed the importance of manipulations with physical objects in learning basic mathematical concepts and operations. Some of these theories, in particular, those found in the works of Pestalozzi, especially his concept of object-teaching method based on the intuitive, visual and hands-on pedagogical perspective had (and still has) a great influence on educational systems worldwide.

We would like to stress that the interest of Western educators in the abaci of Russian and Chinese type and their attempts to use them in schools in the 19th century were not related to the discovery of these instruments which, as we have shown in the opening sections of this chapter, had been described and were relatively well known much earlier. We argue that their reappearance (this time, in educational context) was related to certain shifts in didactical theories which implied development of new practices, in particular, manipulations with tangible (and not abstract) objects. At that moment the old and relatively well-known abaci (Russian, Roman, Chinese) came into focus and started to be explored and used by the educators. Interestingly, this was done not in the countries of origin of these instruments (Russia and China) where they were still used as mere tools for calculations, but in the countries where they were considered 'exotic' and 'unusual'. It is also worth noting that not only the functions of the instruments were modified (from computing device to educational tool) but also the very operations with them were redesigned: the original operations with the instruments used in Russia and China most likely remained unknown to the educators who had to develop their own 'software' mainly dealing with the most elementary operation to be performed on the abacus.

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