Problems in the teaching of arithmetic: records in French school notebooks (1870-1914)

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Abstract

This study analyzes how problems were present in French primary school based on what was recorded in school notebooks between 1870 and 1914. The corpus of French documents used in this investigation is composed of 13 notebooks from the Fonds Histoire de L'Education collection of the University of Limoges. The analysis shows that both daily-lesson and monthly-lesson notebooks from that period prioritized "type-problems" that involved the idea of proportionality regarding situations, to be solved through cross-multiplication. The existence of "type-problems" also involved a model for spatial organization and solution procedures composed of a sentence in order to explain what each operation referred to and was followed by the referred operation, which was represented horizontally. These models can be observed in all notebooks.

Keywords: history of mathematics education; problems; primary school; school notebooks

Introduction

Problems gained prominence in French legislation at the end of the 19th century, with proposals that involved knowledge useful for social life and development of moral values (Sarrazy, 2003). The French studies that identified the space taken up by problems, like Sarrazy's (2003) and D'Enfert's (2003), approached legislation and problems, documents in which Teaching Regulations and Guidelines were exposed in the form of prescriptions provided by the government. However, how were the guidelines and regulations consumed'? How were proposals for the use of problems in the teaching of arithmetic present in primary school classrooms? School notebooks constitute a possibility to search for answers to these questions.

For this reason, this study aims to analyze how problems were present in French primary schools, based on what was recorded in school notebooks between 1870 and 1914.

¹ Consumption is understood as presented by Certeau (2014).

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School notebooks as source material for research in a historical perspective

In general, notebooks have different uses and purposes. A notebook can be used for personal notes, accounting or as a diary, among many other possible applications. In some cases, the determination of how it will be used depends solely on its owner, whereas in others it will depend on the person who will read the records contained in it. As a kind of material to be used for students who follow their teacher's instructions, school notebooks may also have had different purposes and ways of use over time, but they differ from those used outside school. For children to be able to use school notebooks, they have to learn specific sets of rules (Viñao, 2008; Santos, 2008; Lopes; 2008), which concern the school universe. Hence, notebooks constitute a product of school culture.

This perspective has consequences for the process of doing research in which notebooks are used as documents, because it is important that the rules which generate the activities contained in the notebooks be "known so that researchers understand and interpret the resulting school product" (Viñao, 2008, p. 26). Besides, when a notebook is used as a historical source, it is also important to combine its analysis with analyses of other historical sources (Viñao, 2008, p. 27).

Gvirtz and Larrondo (2008, p. 45) broaden the previously presented discussion by defending that the notebook is not a neutral source and

must be understood both as a product and a producer of school culture, as a generator of specific speeches and specific effects.

The authors state that it is possible to build your own chronology through questioning the notebooks used as sources. These chronologies may or not coincide with cycles of educational policies or with the evolution of scientific subjects.

The same authors (2008, p. 35) also highlight that notebooks are a privileged source for educational research because

the school notebook -a space of interaction between teacher and student - allows the effects of this interaction to be seen, that is, schoolwork itself.

This privileged access to schoolwork might offer the researcher clues concerning the activities that were set as priorities to be recorded in the notebook, as well as clues to school time organization. It also allows the researcher to observe rhythms, sequences and moments through the identification of daily, weekly, monthly and yearly uses (Viñao, 2008) as far as schoolwork is concerned.

The time frame and the corpus for the research

We decided to study the time period 1870-1914 due to D'Enfert's studies (2003) on official documents related to the teaching of mathematics in French schools, in which the author identified this period as a time of educational renewal.

According to D'Enfert (2003), the 1870s represented a movement of ideological renewal. Such movement led to the nomination of Jules Ferry² for the Ministry of Public Instruction and to the passing of important education laws in the 1880's that recommended the application of the intuitive method. In general, the proposal was to respect children's 'natural' predisposition to start learning from what was known, and therefore easy, then proceed to the unknown and difficult with great emphasis on observation and use of concrete objects. Such guidelines resulted, according to D'Enfert (2003), in some recommendations for the teaching of mathematics, like: the use of the abacus; the teaching of the metric system through observation or manipulation of conventional measurements; mental calculation not to exercise memory, but as an initiation to arithmetic and to develop intuition skills in children; the teaching of geometry through observation and concrete objects and thinking. Besides, in that period, there was the proposal for "concentric" education: the same program should be studied in each of the three levels of primary school (elementary, intermediate and superior³) and reviewed in more depth as levels progressed (D'Enfert, 2003).

The 'corpus' of French documents used in this investigation is composed of notebooks from the *Fonds Histoire de L'Education* collection of the University of Limoges. At present, this library is developing a project⁴ that involves the cataloging and digitalization of school notebooks used between 1878 and 1987. The collection holds 408 cataloged notebooks in the digitalization stage; online availability of this collection has been started.⁵

Thirteen notebooks from this collection were selected as sources for this study, because they were originally used in the time period we chose for this study and they contain mathematics classes' records. Out of the 13 notebooks, 7 are monthly-lesson

² Jules Ferry (1832-1893), a rich and cultured lawyer, was an enthusiastic advocate for the Republic. He was nominated Minister of Public Instruction in February 4th. 1879. The 1882 law, which established secular and compulsory primary school, is one of the laws that were passed during his term as a Minister (Albertini, 2006).

³ According to the first article of the decree on educational organization and study plans for public primary schools of July 27th, 1882, primary school in the French public educational system was divided in three courses: elementary course (*cours* élémentaire) from 7 to 9 years old; intermediate course (*cours moyen*) from 9 to 11 years old; superior course (*cours supérieur*) from 11 to 13 years old.

⁴ Further information on the Project can be accessed on the library's website: http://www.unilim. fr/scd/2016/06/06/fonds-de-leducation/

⁵ Available online: http://www.unilim.fr/histoire-education/

notebooks and 6 are daily-lesson notebooks. The daily-lesson notebooks contained the records of different subjects studied on each day, whereas the cover of each monthly-lesson notebook contained instructions on how they should be used, as shown below:

Every student, when entering school, will receive a special notebook that they have to keep throughout their school life. The first lesson of every month, of every subject, will be written in the notebook by the student, in the classroom and without any help by others, in such a way that the group of lessons allows to follow the sequence of exercises and appreciate the student's progress year by year. This notebook will be deposited in school. (in free translation)

We understand that 13 notebooks can be a relevant number in a study that has a historical perspective because the analysis promotes discussions between the records found in the notebooks and data from other historical sources, as well as from other previous studies. This perspective is supported by results of educational research conducted on notebooks, such as the results presented in Mignot (2008). Such results show that notebooks can contribute to educational research, even if they provide only small samples, for the tasks performed in the classroom enable us to take a closer look at the teacher-student interaction.

In general, despite being of different types, the notebooks present a type of organization that is similar in some aspects: all records kept by children were written with the same ink color - black or blue - except for drawing activities, which were recorded in pencil; there were no personal remarks made by the children, only the lessons; they offered indicators of date with day, month and year and also titles for each activity like arithmetic, grammar, moral, etc.; teacher's marks were in different colors, mostly red, and were made in the left margin of the notebook or over the student's writing. The marks over the student's writing were corrections, whereas the marks in the left margin were related to assessing the child's work, sometimes presenting a grade from zero to five, other times presenting indicators like "good", "bad" or "exact".

Problems in French school notebooks

When analyzing a notebook involves an observation of how problems were present in them, it is necessary to start with identifying the problems. In a historical analysis, it is important that this identification be based on the understanding that people had at that time of what a problem would be.

So, by analyzing texts by Leyssenne (1888) and Serrazy (2003), which approach the use of problems for the teaching of mathematics during the historical period studied in this research, it is possible to observe that the activities identified as problems, in the notebooks, were the ones related to the teaching of arithmetic that explicitly had the nomenclature "problem" or the ones that involved real life situations or moral values.

Monthly-lesson notebooks

In the monthly-lesson notebooks, students recorded activities throughout their school life, so they did not refer to a specific learning level because they were used throughout the years. Six out of the seven notebooks analyzed in this study belonged to students who shared a common teacher (Table 1).

Student	Period of use	Teacher
René	1897-1900	Alain
Pierre	1909-1910	Alain
Armand	1910-1912	Alain
Louis	1910-1912	Alain
Jean	1912-1913	Alain
Sophie	1913-1917	Alain
Lorran	1902-1903	Nicollas

Table 1. Monthly-lesson notebooks⁶

It was possible to identify that Pierre, Armand, Louis and Jean were in the same classroom throughout the years since the notebooks used in the same school years had the same activities recorded on the same dates. Thus, those four notebooks allow us to observe the activities presented by teacher Alain from 1909 to 1913.

Table 2. Period in which each notebook was used.

	1909	1910	1911	1912	1913
Pierre					
Armand					
Louis					
Jean					

Using this kind of notebook as determined by law, according to Hébrard (2001), allowed a process of evaluation, that is, of checking students' progress because all tests taken throughout a student's school life are gathered in the same document. In that sense, it was possible to notice that problems were a favorite activity to assess

⁶ All names used to identify students and teachers are fictitious.

children's performances in arithmetic. Activities like writing, dictation, drawing and others were performed on different days every month. The activities related to the teaching of arithmetic were always performed around the 20th of each month and involved the solving of two problems. Activities like number writing and calculations with no relation to problems were identified two times, at most, in the same notebook, which implies a very low representativeness rate considering the fact that the notebooks were used for more than one year.

In most cases, problems were presented under the title "Calculation". The option of using two problems in each monthly test and the use of the term "Calculation" seem to indicate a connection with the study certificate exam, which contained, at that time, two arithmetic questions referring to the use of calculations and the metric system with a justified solution.

The way solutions were recorded also presented a close relation to the instruction given for the study certificate because it highlighted the record of the justified solution. All children applied the same method to record the solution. In his analyses of French school notebooks from the 19th and 20th centuries, Hébrard (2001) drew our attention to the existence of a spatialization of language in the notebooks, which is revealed through a set of graphical demands for the recording of each activity in models of space management of the notebook. According to the author, this was also observed regarding arithmetic problems

First, and all over the line, the wording of the problem; then there is a division of the page in two unequal columns (one-third, two-thirds). In the narrow column, identified by the title "Operations", additions, subtractions or multiplications were copied in the form of "calculations"; in the wide column, dedicated to the "Solution" (sometimes called "Development" or, to please more demanding teachers, "Developed Solution"), stereotyped formulas in written language explain the operations contained in the other column. [...] in the last line of the "Solution" column, or sometimes all along the page extension, the "answer" is explicitly formulated according to the terms presented in the wording. (Hébrard, 2001, pp. 131-132)

The same spatial organization was observed in the notebooks analyzed in this study. There was also, in the records, the teacher's correction mark in a score that ranged from zero to five. The act of grading the student's work is justified because this notebook was destined to contain the children's assessments.

Un boucher achiete 18 moutons à 77 fr. piece Combien doit _ il P Raisonmement Li un moutons vaut 27 frances 18 moutons Liun moutons vant vaudront 18 fois plus ou 97 X 18 = 486 frances Réponse ce boucher doit 2 86 hours

Fig. 1. Problem solved by student Louis on January 19th 1912.

A butcher has bought 18 sheep for 27 fr. (francs) each. How much does he owe? Reasoning If one sheep costs 27 francs, 18 sheep cost 18 times more, that is 27 x 18 = 486 francs Answer: the butcher owes 486 francs.

Un boucher achèle 18 montons à 2 the pièces. Combien doit-on il?

30 hun mouton vour 37 fr. les 13 moutons vaudrout 18 fois plus ou: 24 × 18 - 4 867 Réponse il tot doit : 486 fr.

Fig. 2. Problem solved by student Armand on January 19th 1912.

A butcher has bought 18 sheep for 27 fr. (francs) each. How much does he owe?

If one sheep costs 27 fr., 18 sheep cost 18 times more, that is: $27 \ge 18 = 486$ f Answer: he owes 486 fr.

Calul 235 Un hectolitre de blé coûte 23 frances Com bien conteront so hectolitres. Si un hectolitre de bli conte 23 francs, s 1 hedditres conteront s 7 fois plus, c'est a dire 23 francs répétes 5 % fois,ou 235/14=1311

Fig. 3. Problem solved by student Lorran on May 28th 1902.

A hectoliter of wheat costs 23 francs. How much will 57 hectoliters cost? If one hectoliter of wheat costs 23 francs, 57 hectoliters will cost 57 times more, that is 23 francs repeated 57 times, or 23f x 57 = 1311f Answer: 1311 francs. *

In addition to a model of graphic organization for the recording of problems, there was a model regarding the way solution procedures were written down in teacher Alain's class. The sentences used by students to express the "Reasoning" and the "Answers" were pretty similar, sometimes even identical.

The existence of a solution model was obvious when we analyzed the same problems solved by different children (Figures 1 and 2) and also when we observed the same student solving problems over time or students' notebooks of different teachers (Figures 2 and 3).

As years went by at school, problems sometimes needed more complex solutions (solutions that involved a larger number of operations). However, spatial organization and solution-writing models remained. The "solution" was always composed of a sentence that aimed to explain what each operation referred to and was followed by the referred operation, which was represented horizontally.

The existence of a model for problem-solving that involved not only spatial organization, but also solution procedures allowed us to recognize that what was

presented in these notebooks is related to what Sarrazy (2003) identified in school documents: the teaching of rules for the solution of "type-problems", which indicates that the teaching of problems was based on memorization and repetition, according to the author.

Another aspect that is noticeable in the solution to most problems in the analyzed notebooks is that the indication of the unit species that was being calculated was written out in full in the sentences presented in the "Solution" and represented with one letter (first letter) or word next to the number in the indication of the operations, both in horizontal organization and also in a vertical calculation organization. For example, the letter "f" to express an amount in francs (Figure 3), the letter "m" to express quantity in meters or "d" for decimeter.

In that historical period, teaching guidelines involved specific emphasis on observation and utilization of concrete objects (D'Enfert, 2003) and problems related to social life – commerce, industry, agriculture, domestic management (Sarrazy, 2003). The need to somehow indicate what was being calculated seems to be a way of guaranteeing – in all solution procedures – the connection between the calculated quantities and social-life situations. In solutions, concrete numbers gained prominence, as defined by Minet and Patin (1913, pp. 5) in their manual for the intermediate-course: "a number is considered **concrete** if the species unit is mentioned. Example: forty soldiers."⁷. The value of this kind of indication is confirmed by the teachers' corrections, who complemented the information when it was not provided by students.

Single daily-lesson notebooks

Out of the six single daily-lesson notebooks, three are from intermediate-course students (IC) and three are from the superior course (SC) – the latter are from the same student (Table 3). None of the notebooks identified the teacher.

Student	Period of use	Class
Anne	1895 (March, April)	IC
Simon	1895 (April, May)	IC
Yves	1903 (October, November)	IC
Guy	1914 (May, June)	SC
Guy	1914 (June, July)	SC
Guy	1914 (July)	SC

Table 3: Daily-lesson notebooks (1870-1914)

⁷ Original French text: "Un nombre est concret, s'il désigne l'espèce d'unité. Exemple: quarante soldats."

In the single daily-lesson notebook students recorded activities done over the same day. In all of them there was first the date and then the activities separated by "titles", which categorized them according to content, subject or kind of activity. For example: moral, grammar, writing, dictation, science, history, calculations, problems, demonstrations, geometry, drawing, algebra and others.

According to Hébrard (2001, pp. 135), this kind of notebook "is irrefutable evidence of all the work done", which would not have been possible if a different notebook had been used for each subject. The author argues that the single notebook allowed the family to supervise students' work and inspectors to supervise teachers' work, so it became a way of controlling the "work done by the teacher over each student's piece of work".

As in monthly-lesson notebooks, problems held a prominent place in single daily-lesson notebooks likewise. There were around two problems on virtually every day on which activities were registered, which cannot be observed in any other activity related to mathematics. As far as spatial organization is concerned, it is the same as Hébrard's observations (2001) and as our observations in this study.

Still regarding problem-solving structures, student Guy's notebook was the only one that contained illustrated records. In this case, the drawings were always related to problems that involved calculating surfaces (rectangle and trapezium) and volume (parallelepiped). The pictorial record in these cases was accompanied by notes of the measurements of the figures' sides; these records did not seem to be mere illustrations, but a way of organizing the problem data that would be used in the explanation of the procedures adopted for the solution. In two of the notebooks there were problems with area and volume calculations. However, pictorial records were not used in any of them.

Although problems had a prominent place in activities involving contents related to mathematics, the notebooks presented other activities that allowed us to search for an establishment of relationships between the contents approached in problems and in other activities.

In student Anne's notebook, which was used in March and April 1895, in addition to problems, for every day on which the student kept records, there were also "Calculations". The calculations referred to operations performed in vertical form and, in this notebook, they involved mainly multiplication and division with decimal numbers.

It is interesting to observe that all problems proposed in this notebook involved the calculation of a surface of a rectangle or calculations involving proportionality in situations to be solved using cross-multiplication. Therefore, all problems required the use of multiplication or division with decimal numbers. Repeated occurrences of a similar kind of problem all along the notebook and the order in which activities were recorded are indicators of a desire to teach students to solve specific "type-problems". Moreover, there was the notion that "calculations" offered students opportunities to "practice" the operations, prioritizing those which were used to solve problems – multiplication and division were the most frequent ones in the activity "Calculations" and also the ones that were harnessed in problems involving proportionality and calculation of surfaces.

In Simon's notebook, from the intermediate course, which was used in April and May 1895, the idea of proportionality was present in all problems. It was not possible to identify a relation to other activities because they were not recorded in the notebook; there was only the indication of titles about "principles related to multiplication and division" and "general properties of fractions" without the recording of the activities.

Despite approaching the notion of proportionality, Simon's notebook contains problems with more data and more relations among the different pieces of information contained in the data, which demands a bigger number of operations (up to nine) for the solution than what is recorded in Anne's notebook.

Yet, Matthew, an intermediate-course student, used a notebook in the months of October and November 1906, in which, besides problems, there were activities (under the title "Exercise") related to the metric system: square meters, measurement conversion, surface measurements. In the notebook there were some problems that harnessed the use of measurement conversion, but it did not happen in most examples.

In this notebook, we have also identified problems related to moral values: one of them questions how much bread one could buy for his family if he had saved the money he had spent at the cabaret; the other involved money one would save if he stopped buying cigarettes. Sarrazy (2003) and D'Enfert (2006) highlighted, in their studies based on official documents, that problems involved necessary knowledge for adult life in that historical period, as well as moral values. In the notebooks analyzed in this study, we observed that domestic economy was a very popular theme and it was only related to moral values – like spending on cigarettes and cabarets – in one notebook: Matthew's.

In all three Guy's notebooks from the superior course, which were used between April and May 1914, it was possible to detect some relations between problems and other activities recorded in the notebooks. It happened regarding contents explored in activities under the title "Algebra", like reduction to the same denominator, systems of equations and first-degree equations, which were harnessed in problems four days later. It was also possible to notice a solving procedure which was only used in dailylesson notebooks in the superior course: the use of algebraic solutions to problems, which appeared with arithmetic solutions. Both solutions were separated and individually named.

The algebraic solutions involved the representation of the data of the problems in equations or systems of equations, and the answer to the problems was found with the solving of those equations or systems of equations. One problem in Guy's notebook is used as an example:

A manufacturer sold 225 meters of canvas and 240 meters of cotton cloth in a first negotiation for 1098 francs. In a second negotiation, he sold 180 m of canvas and 375 meters of cotton cloth for the same price. Find the price of each material.

He started the algebraic solution by identifying the price of canvas as X and the price of cotton cloth as Y. He wrote the equalities in a system of equations, which he later solved:

225 x + 240 y = 1098180 x + 375 y = 1098

Other contents explored under the title "Algebra" (special binomial products and second-degree equations), as well as the content recorded under the title "Geometry" (point and line relationships, geometric constructions of triangles), were not harnessed in any of the problems recorded in the notebook.

It is important to highlight that activities involving a specific content were recorded before that content was harnessed in problem-solving. This is an indicator that problems made use of what had been previously studied. However, in all notebooks, most problems did not have a relation to other activities and, in addition, it was possible to notice that many problems repeated the same format, the same proposal, the same "type" of problem with some variations regarding themes and data. Thus, it seems that problems were applied to teach students to solve "typeproblems" (Sarrazy, 2003), which were of the same "type" as those observed in monthly-lesson notebooks.

Some Considerations

Both in daily-lesson and monthly-lesson notebooks, there was the prioritization of "type-problems" that involved the idea of proportionality regarding situations to be solved through cross-multiplication.

The themes used in the problems both in daily-lesson and monthly-lesson notebooks also presented a "type" that was mostly used in all notebooks: the relation to men's adult work life. Situations involving money were the most common (buying, selling, profit, amount received for a job, amount saved). The ones who bought, sold, got paid, saved and profited were workers, shopkeepers, family men, butchers, farmers, gardeners, tailors, potters, etc. The few situations that mentioned women's adult life approached their role as mothers or dressmakers. It is also important to highlight the great representativeness of problems that, in these themes, involved data referring to the use of the metric system (length measurements).

Although some problems involved only two pieces of information that needed to be used to perform an operation to solve the problem and others involved a higher complexity of relations with the presentation of different data that needed to be related in many operations so that the problem's question could be answered, the themes involved in the narratives contained in the wordings were very similar.

Another common aspect in problems contained in monthly-lesson and dailylesson notebooks was related to the magnitude of the numbers used. Mostly they involved numbers up to the first digit of thousands, that is, up to nine thousand. Numbers of greater magnitude were more common in problems that were solved with addition and subtraction, while numbers of lower magnitude appeared more in problems that involved multiplication and division.

Here we resume the relations between this study and research studies by D'Enfert (2003), Sarrazy (2003) and Hébrard (2001). D'Enfert (2003) and Sarrazy (2003) identified in legislation and in textbooks the indication of problems in the teaching of arithmetic as a way of bringing social life into primary school from 1880 on. In the notebooks, just like Hébrard (2001) had observed before, such problems gained centrality because many times they were the only recorded activity in relation to arithmetic.

In his analyses, Hébrard (2001) pointed out the graphical demands for the use of the notebooks, which resulted in a specific spatial organization for problem solving. In this study, we observed the same spatial organization, but in addition to a spatial organization model, we detected a model for the solution: a model for writing the solving procedures.

The occurrence of "type-problems" whose solution would be learned by students based on memorization and repetition (Sarrazy 2003) was also observed in the notebooks that we analyzed. Here the discussion is expanded through exploring the relation between problems and all other arithmetic activities. In the notebooks, the recording of activities related to some arithmetic contents happened before those contents were used in problems, which indicates that those other activities could be used as tools in problem-solving. Another possible contribution from the analysis of school notebooks is the observation of 'how' some orientations were used, "consumed". In the historical period approached in this study, educational guidelines demanded a connection between the teaching of arithmetic and concrete objects (D'Enfert, 2003) and to social life situations (D'Enfert, 2003; Sarrazy, 2003). In the notebooks, the use of "concrete" numbers with the identification of species units in operations seems to be one way of meeting those guidelines. Moreover, a link to social life was present in the problems recorded in the notebooks through themes related to adult life and to the monetary system.

The analysis enabled us to identify a specific type of problem which was mostly used in arithmetic classes between 1870 and 1914. This kind of problem involved: the idea of proportionality; themes related to social life and to men's adult life; models for spatial organization and solution procedures. It is important to highlight that the analyzed corpus does not represent teaching in French in its entirety. However, in this paper we discuss important aspects of the use of problems in the teaching of arithmetic in French primary schools. The results that we present here will soon be linked with other results, as the research continues with records of arithmetic classes in later periods in order to observe how this proposal can be characterized over time.

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