ISAAC NEWTON: CONTAINS AWE

Working with students and teachers in period costume

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ABSTRACT

This workshop will be delivered by me, dressed as Sir Isaac Newton in period costume. It describes the work I have done with various groups of students and teachers over the past three years based on Newton's work. I explain how the use of digital technology has helped with some groups. The groups of students that have participated in sessions with Newton cover ages 13/14 years, 14/15 years and 16-18 years. The teachers who have attended sessions range from those in initial teacher education, newly qualified teachers and heads of mathematics in the UK. The work includes a thought provoking geometry problem from a piece of artwork featuring Newton that is accessible to both students and teachers and a brief history of Newton. It covers demonstrations of some of Newton's laws and provides some classroom materials on developing the binomial theorem with young people. Mention is made of Newton's work on optics which leads on to developing participants' mental geometry skills. Participants will get the opportunity to make themselves a set of proportional dividers and use some ink made according to Newton's own recipe using his original manuscript. All the classroom materials will be put on a CD-ROM and given to participants so they can use the materials with students and teachers at a later date.

1 The inspiration

I incorporate events from history into mathematics lessons because I find it very interesting to see the practical applications of mathematics set into the period when it was used. I chose to write about this episode since it has been developed over the past three years and is very rich in mathematical history.

Tzanakis & Thomaidis (2011) classify the arguments and methodological schemes for integrating history in mathematics education and my episode fits into the two-way table mainly as History-as-a-tool and Heritage though there are overlaps into the History-as-a-tool and History cell. The over-riding concept in my work is History-as-a-tool.

The reliability of this work in the sense of reproducibility by someone else is impossible to quantify, since teachers use such episodes in different ways with different students and probably not in costume! Every session I do with students or teachers is different according to local conditions and the knowledge people bring to the sessions, so nothing stays the same: that is one of the great joys of working with them. It is important that teachers recognise this so that they are flexible in their approach with how they present materials to students.

2 The evolution of the episode

This work came about after an invitation to do a 90-minute talk to around 250 students aged 16-18 and staff at local schools in the county of Somerset. I had accepted the invitation and being something for such a large group of people it meant I would not have enough workshop equipment to do what for me is the normal way I work with students. Therefore, I had to combine a lecture-style presentation with some paper-and-pen based activity, rather than a more active session. I did not want to do one of the sessions I have developed over the past decade, as it was time to do something new. In what follows I describe what has happened with all the people with whom I have worked: teachers and students aged 12-13, 13-14 and 16-18

2.1 Images of Newton

Since I had been using the British Library in the past year I had noticed that a modern statue of Newton had appeared on site (see Figure 1) and this was the inspiration I needed.

Figure 1. Statue of Newton by Sir Eduardo Luigi Paolozzi (1995)

It is based on a picture by William Blake (1757-1821: a seminal figure in the history of the poetry and visual arts of the Romantic Age) of 1795 (see Figure 2) and that features an interesting mathematical diagram that I have used ever since. If you examine Figure 2 you will see Newton working on a diagram in the bottom right corner.

Figure 2. Newton by William Blake (1795) (see Fauvel, Shortland, & Wilson, 1989, p. 226)

This diagram appears to be a segment of a circle in an equilateral triangle and I set participants the problem of finding exactly what fraction of the triangle is covered by the segment assuming that two sides of the triangles are tangent to the arc of the circle. Now this puts students and (many) mathematics teachers in an unfamiliar situation as there seems to be a complete lack of information and it is interesting to see how people progress with this problem. (Some of teachers are not very confident with their mathematics as they may be starting in the profession or lacking in subject knowledge. Unfortunately, in the UK there is a shortage of mathematics teachers and some people have had to retrain from other subjects to fill vacancies.) Some teachers just seem to sit back in resignation with comments such as 'I was never any good at geometry', or 'I don't know what to do'. Others seize on the problem with enthusiasm, working on their own or with one or two others: again, it is fascinating to see the dynamics of social and mathematical interaction as people remember some long-forgotten facts about circle geometry or the exact value of sin 60°. I offer a prize of a £1 note featuring Sir Isaac Newton (see Figure 3) for the first correct solution as an incentive to keep people going as well as to raise awareness of our country's currency heritage as that is also an important part of the history of mathematics. These notes were in circulation between 1978 and 1988 and are no longer legal tender.

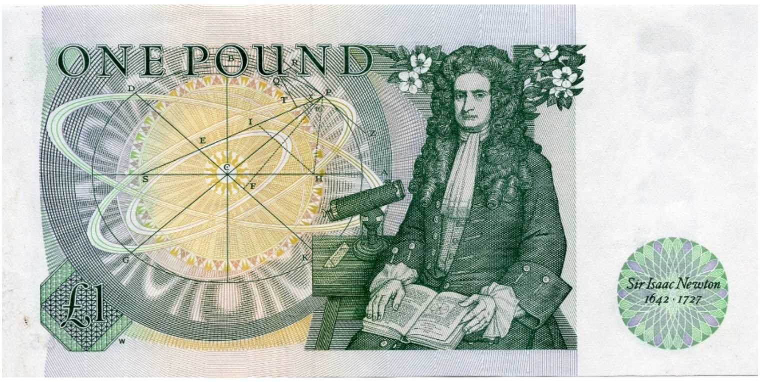


Figure 3. £1 note featuring Sir Isaac Newton

Students and adults seem pleased to be faced with the possibility of winning this piece of history, possibly to some because it brings back memories of the time before we had £1 coins. Now with younger students who may not have met much geometry or surd manipulation I tend to use Texas Instruments-Nspire CX wireless hand-held technology as this allows them an empirical approach by drawing the diagram and capturing data dynamically. Figure 4 shows a series of screen shots of what the students draw and the data capture.

In the first screen shot the student has drawn an equilateral triangle and using the fact that two sides of the triangle are both tangential to the arc of the circle, draws perpendiculars at the two vertices. The centre of the circle is therefore where these two perpendiculars intersect. Then the circle can be drawn as its centre is known as well as points through which it passes. The properties of the diagram are examined and students come to realise that if the original triangle is reflected in its base line then its top vertex will lie on the circumference of the circle as shown. Using the measurement tools on the hand-held device, variables such as the radius of the circle (rad), the area of the circle (cir) and the area of the triangle (tri) are determined and captured in a spreadsheet on a new page (shown on the second screen shot). By grabbing and moving one of the triangles vertices the values of these variables are recorded and many items of data collected. The third screen shot shows the area of the circle plotted against the area of the triangle and the last screen shot has the linear regression line superimposed from one of the menus available. Since all measurement is approximate there is a very small intercept on the y-axis due to rounding errors. The area of the segment can be found as there are three congruent segments (I expect the students to give an explanation of why they are congruent), so subtracting the area of the triangle from the area of the circle and then dividing by 3 gives the area of the segment. Students can then explore other relationships between these areas. It never ceases to amaze me how swiftly students cope with this technology: the session with 30 13/14 year olds had not used this technology before, yet were exploring the relationships as if it was second nature (which it probably is to these digital natives).

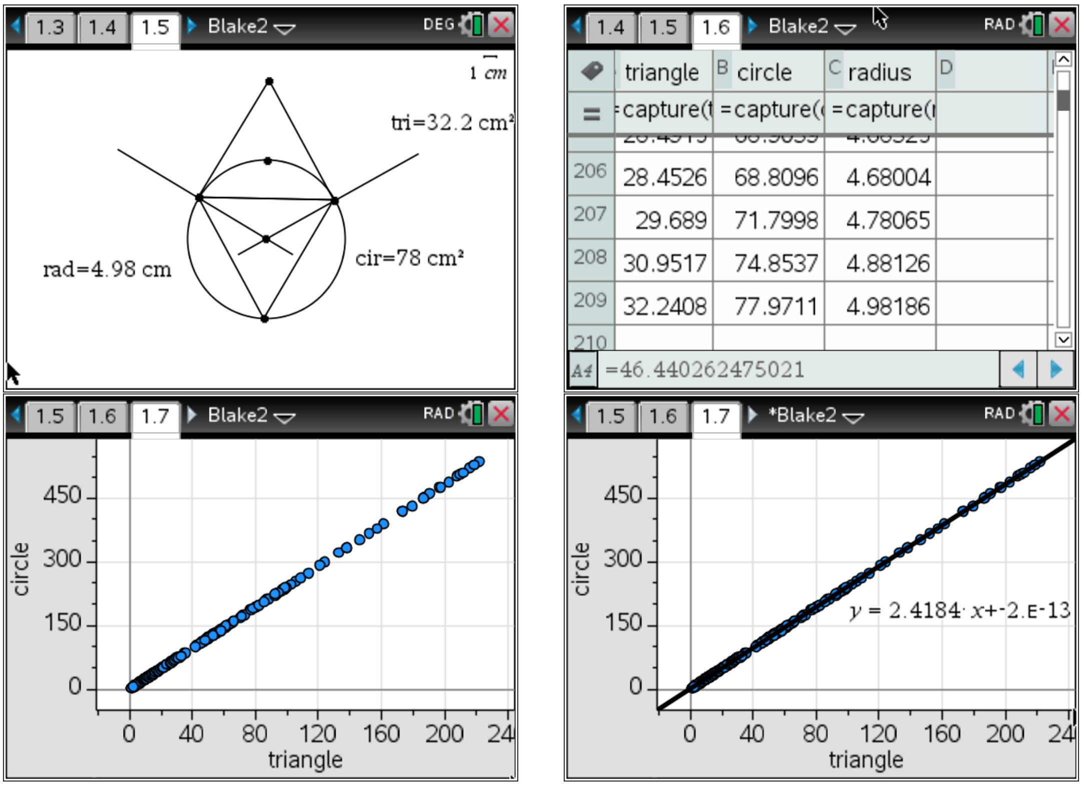


Figure 4. Screen shots from a student’s use of hand-held technology

2.2 Introducing the history

After this introductory exercise I deal with some of Newton’s history and his discoveries. In my teaching career spanning nearly four decades I have always found it good pedagogy to actively engage learners before talking for any length of time and Newton deserves a lot of time considering all that he achieved. In a two-and-a-half-hour masterclass however I tend to talk about some of the interesting facts so that students get a feel for some of his works and hopefully follow this up by reading more about him in their own time. After a brief five-minute talk about Newton’s early years I mention how he discovered the generalised binomial theorem in 1665 and this forms the basis of the next piece of work for students, no matter what their age. Rather than dealing with the expansion of $(x + a)^n$ we work with $(x + 1)^n$ as this fits in well with the students’ concrete experience of long multiplication. The pedagogical advantage here is that students will move from their concrete knowledge of multiplying whole numbers to the abstraction of algebraic multiplication of polynomials. Figure 5 shows the worksheet I use with students (and teachers who do not feel confident about algebraic manipulation).

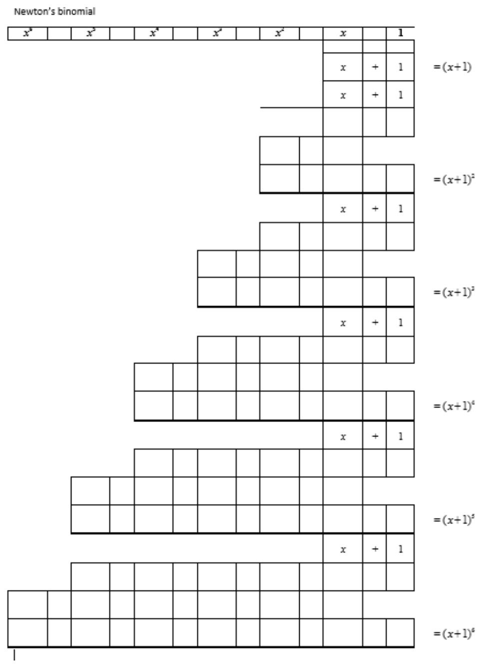


Figure 5. Worksheet used to expand $(x + 1)^n$

Here we use column headings of 1, x, x^2, x^3, … rather than **U**nits (10^0), **T**ens (10^1), **H**undreds (10^2), **Th**ousands (10^3) etc. which are used when multiplying whole numbers. Figure 6 shows what the first two results look like when complete.

x^6		x^5		x^4		x^3		x^2		x		1	
										x	+	1	$= (x+1)$
										x	+	1	
										x	+	1	
								x^2	+	x			
								x^2	+	$2x$	+	1	$= (x+1)^2$
										x	+	1	
								x^2	+	$2x$	+	1	
						x^3	+	$2x^2$	+	x			
						x^3	+	$3x^2$	+	$3x$	+	1	$= (x+1)^3$

Figure 6. Partly completed worksheet

It takes a little while for students to feel confident filling in the cells, but once they have completed a few rows they swiftly get a sense of rhythm and satisfaction. I generally have them working in pairs so that they help each other, which gives me time to work with those who need some support. The students are encouraged to write down what they notice and some who have met Pascal's triangle in the past remember that. There are many opportunities here for extension work and teachers who have been observing this session often remark on how well the students have worked and how impressed they are with some of the student discoveries. I am keen to point out that this is not a normal classroom situation however: these students are here because they come voluntarily and I am unknown to them. Students react differently when working with strangers rather than their regular teachers, so that needs to be taken into account.

2.3 Newton's laws of motion

After the mental geometry activity, it is time for some more talk and a demonstration. I show a picture of *Principia* (see Figure 10) and talk about Newton's first law of motion: An object either is at rest or moves at a constant velocity, unless acted upon by an external force. This is demonstrated by balancing an egg on a stand which rests on a horizontal card on a glass of water. We discuss the forces acting on the egg (its weight and the normal reaction) and then I knock the card away and we watch the resulting motion.

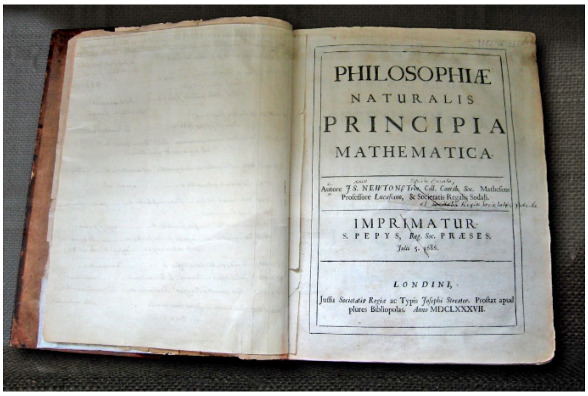


Figure 7. Newton's Principia, first published on 5 May 1687

I mention that this is just one example of Newton's laws, but there is never enough time to discuss or demonstrate the others. Anyway, that can be left to the physics teachers. It has always been my intention to whet students' appetites with some excitement of an experiment that may not work out exactly as planned and to realise that mathematics is not just a purely theoretical subject but that it involves experiments and practical work.

2.4 Newton's sundials

Since 1991 I have been obsessed with using sundials in school mathematics and it is an absolute delight to know that Newton made sundials from an early age (Ransom, 1998, (pp. 44-49). One of these (see Figure 8) was taken out of the wall of Woolsthorpe Manor, Newton's home, in 1844 and presented to the Museum of the Royal Society, where it is carefully preserved.

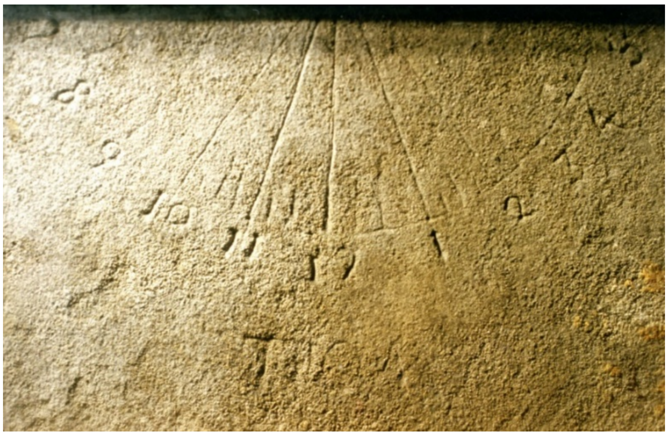


Figure 8. Newton's sundial in the Royal Society, London

Another (see Figure 9) is in the church at Colsterworth, a village near Newton's home. The Rev. John Mirehouse thought he would make a search at Woolsthorpe Manor and see if the second dial which Newton was known to have carved could be found. His effort was rewarded with success. The old stone was found in its original position on the south wall, covered up by a small coal house, and the relic was given by the owner of Woolsthorpe to the church. The disc is 11 inches wide at the top, and nearly 6 inches deep; it has been enclosed in a frame of alabaster and placed on the north wall of the Newton Chapel, with the following inscription:

> Newton: aged 9 years, cut with his penknife this dial: The stone was given by C. Turner, Esq., and placed here at the cost of the Rt. Hon: Sir William Erle, a collateral descendant of Newton, 1877

Figure 9. Newton's sundial in Colsterworth Church

I believe that in teaching mathematics we should show our rich mathematical heritage to students so they are aware of the impact it has on so many lives in many ways. Unfortunately, there is never the time in these sessions to delve deeply into sundials, but I have covered that in the first European Summer University on History and Epistemology in Mathematical Education at Montpellier back in July1993 (Ransom, 1995).

2.5 The kissing problem

In 1694, a famous discussion between two of the leading scientists of the day - Isaac Newton and David Gregory - took place on the campus of Cambridge University. They wanted to know how many identical spheres can kiss (that is 'touch') the one in the centre?

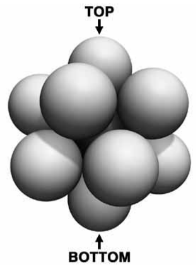
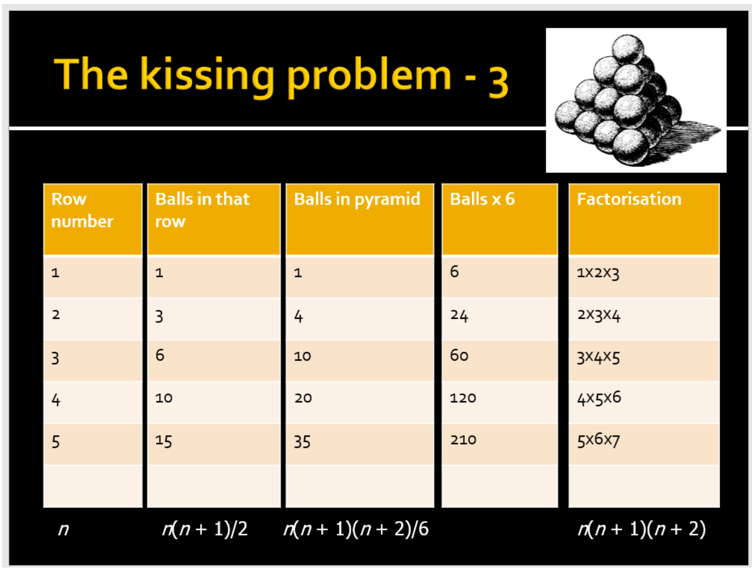


Figure 10. Kissing spheres

You can see (Figure 10) that 12 is possible, but in fact there is space for nearly 15 spheres! It took until 1953 for a proof that 12 is the limit. I give students a worksheet and small spheres to pile them up into a triangular based pyramid to investigate a similar problem, this time looking for a formula that links the number of layers with the total number of spheres in the pyramid. Figure 11 shows the results and formulae that students find and then we extend this by looking at the patterns in the formulae and extrapolate that to four dimensions, five dimensions and then to *n*-dimensions. Again, it is important for students to work with manipulatives before moving to the abstraction of algebra.



The kissing problem - 3

Row number	Balls in that row	Balls in pyramid	Balls x 6	Factorisation
1	1	1	6	1x2x3
2	3	4	24	2x3x4
3	6	10	60	3x4x5
4	10	20	120	4x5x6
5	15	35	210	5x6x7
n	$n(n+1)/2$	$n(n+1)(n+2)/6$		$n(n+1)(n+2)$

Figure 11. Results and formulae found when piling spheres

Conclusion

It is impossible to get through all these activities in two and a half hours (which includes a break) and as mentioned earlier, I select the activities to suit the audience and what I hope to achieve in the time allocated (which with teachers can be just one hour). Presenting in period costume (and here I must pay tribute to my mother, Mrs Joyce Ransom, who researched and made the costume at the age of 86) is not necessary, but for some students it adds an extra interest I believe as it helps bring our subject to life in ways that can appeal to not just the future mathematician, but mathematical historian. Teacher participants always receive a CD-ROM with all the classroom materials and PowerPoint presentations ready to use with a host of other relevant historical materials from other episodes I have presented to students and teachers over the past two decades. It is my hope that you, dear reader, will try at least one of these activities when the time is right.

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