

SPACE, STRUCTURING, FIGURE

A prehistoric legacy

Olivier KELLER

Irem de Toulouse, France

autolykos@orange.fr

Before Euclid's *Elements* (circa 300 BCE), there were the Babylonian tablets and the Egyptian papyri of the first half of the second millennium before our era. But what about before that? How did the notions of figure, line, point, etc. – which are implicitly presented as self-evident in Middle Eastern documents¹ – develop and take shape? With the help of a few insights, I hope to convince the reader that certain fundamental geometric concepts underwent a veritable *gestation* period in prehistory, from the time when the human race first emerged 2.5 million years ago. Traces of this gestation survive today, in “inert” form in archaeology, and in “living” form in the ethnography of illiterate peoples.²

But, first of all, what might “geometry” have meant in a prehistoric context? Certainly not surveying, and still less a deductive system with definitions, axioms and propositions. Here geometry is taken to mean the general capacity to isolate, within a surrounding environment, a *space* (for example, an initial pebble, a fragment of a cave wall). From this space one then “extracts” a preconceived object, the *figure*, through a planned, methodical process, or plan of work, that is the *structuring* of this space:

Initial object	Plan of work	End product
Space	Structuring	Figure

The gestation of geometry, from the simple debitage of the early days of the *Homo* species,³ to Euclid's *Elements*, consisted in the emergence and development of this fundamental blueprint. This means that, far from simply copying natural forms, or abstracting certain elements from them, humankind *imposed* figures on nature, starting with that most rebellious of materials: stone.

1 Geometry of the Palaeolithic stone industry

Early geometry, it seems, mainly consisted in the *debitage*⁴ of pebbles or stones (“*spaces*”) in order to obtain flakes with sharp edges. There is a clear distinction between a space, i.e. the

¹ See also <http://www.mathunion.org/fileadmin/ICMI/docs/HPM2004Proceedings.pdf>, pp. 83–98.

² Used correctly, the archaeological and ethnographic sources shed mutual light on one another. On the significant issues posed by this rapprochement, see Keller, O. (2004). *Aux origines de la géométrie. Le Paléolithique et le monde des chasseurs-cueilleurs* Vuibert, Chapter 1; and Keller, O. *L'invention du nombre. Des mythes de création aux Eléments d'Euclide*. Classiques Garnier (forthcoming), Annex B.

³ In reality, the creators of the first tools may well have been australopiths. But it is certain that no living animal species is capable of implementing the basic schema outlined above in its entirety. Even the humble beginnings of debitage are out of reach of our cousins the chimpanzees.

⁴ The term “debitage” is used when the flakes are the desired end product. If the desired tool is not the flake itself, but an altered nucleus achieved by removing flakes, the term used is “shaping”.

initial pebble or chunk of rock to be worked on (known as the “nucleus”), and the tool that is used to do this, i.e. the stone hammer. This deliberate set of gestures loosely structures the raw material, evidenced by the fact that the flakes were often removed “in layers”, as reassemblages have shown. The figure, in the sense of a preconceived finished product, is not the flake itself (because its form is not predetermined), but rather its – possibly retouched – edge.

The process is reversed when it comes to the fabrication of choppers, in that the edge is obtained by shaping the nucleus itself, by removing a few flakes from one or both side(s), thereby producing an edge. The removal of the flakes is itself a process of structuring, while the figure, in turn, is the part of the nucleus which is worked on.

Such was Oldowan industry⁵ in the Archaic Palaeolithic period. These were the beginnings, and, though certainly humble in comparison with later developments, they nevertheless represent a fundamental qualitative rupture when compared with the capacities of other animal species.

Oldowan industry was followed, in the Lower Palaeolithic, by Acheulean industry⁶, which was characterized by the *systematic shaping* of the nucleus to produce the era’s famous *handaxes* (Figure 1).

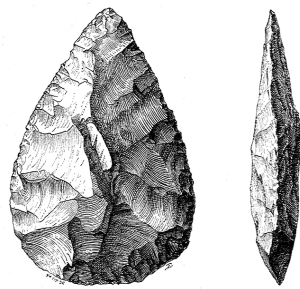


Figure 1. Handaxe found near Aurillac. Unspecified date.

The structuring consists in the removal of flakes to obtain a plane sharp edge. The volume is progressively reduced so that, when viewed in profile, it presents two symmetrical surfaces that provide a plane edge. The handaxe, when viewed front-wise, can have several possible (though probably standardized) forms, and, invariably, an axis of symmetry.⁷ In the case of the handaxe – and in contrast with the figures of earlier epochs – this figure represents an initial step towards the modern-day meaning of the term, by virtue of the decisive importance of its internal relationships, namely equal magnitudes (symmetries) and definite ratios of magnitudes (possible standardized forms).

⁵ From the Olduvai Gorge in Tanzania. Archaic Palaeolithic: in Africa, 2.5–1.5 million years BCE. These dates and those below should be taken only as an indicative order of magnitude.

⁶ From the site at Saint-Acheul in the Somme. Lower Palaeolithic: in Africa, 1.6–400,000 million years BCE; in Europe, from 600,000 BCE onwards.

⁷ For each period, I describe the typical tool in its completed form, leaving aside intermediary forms and the ongoing use of previous techniques. The production of handaxes obviously did not entail the abandonment of the fabrication of flake tools.

Oldowan industry was followed, in turn, by the Levalloisian⁸ and laminar industries of the Middle and Upper Palaeolithic. These were characterized by a return to debitage, but, more precisely, to *systematic debitage*. In Levalloisian debitage, the volume of the pebble or chunk of rock is worked on layer by layer in order to progressively extract one or several flake(s) (Figure 2), possibly in a predetermined shape, such as a triangular point.

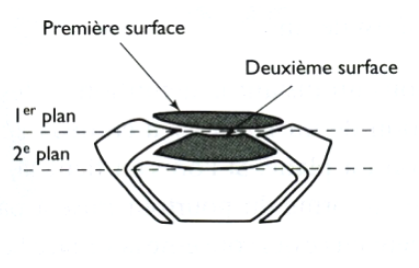


Figure 2. The concept of Levalloisian debitage.

If need be, the edge of each flake could then be retouched. In other words, the plan governing the production of a given flake was structured into three successive stages: the volume was prepared to produce a surface for debitage (in a sense, the structure supporting the flake); the flake was removed; and the edge was then retouched. Volume, *then* surface, *then* line. Furthermore, since the successive layers are parallel, it is clear that the overall plan of work reflects a *preliminary* and conscious structuration of the raw material, i.e. the initial pebble or chunk of rock, in accordance with its three dimensions. Geometrically speaking (in terms of the definition we are using here), the phenomenon is barely different from the debitage of blades during the Upper Palaeolithic and even beyond: the nucleus is prepared so that a series of blades can be extracted from it, each time with a single blow, and the edges of the blades are then retouched to produce a great variety of forms, depending on the function of the tool.

If this slow (over two million years!) appropriation of local space through work – the broad outlines of which we have described above – was certainly a form of geometry, in the sense of a spatial structuring to produce figures, it was nevertheless only an embryonic form of the future science. The vocabulary we have used (volume, surface, line, symmetry, plane, etc.) is linguistically convenient, but should not be misconstrued. In reality, it simply evokes the different states of the reworked stone in relation to physical movements. By becoming images and habits, and by separating the processes into distinct stages with a distinct product at each stage, as in the case of Levalloisian debitage, the states and movements in question were no doubt schematized in one way or another. But they were certainly not conceptualized. In the hunter-gatherer societies that produced stone artefacts, learning involved copying others and did not require an overarching vocabulary or technical description. If there was description, it was mythical, or possibly ritualistic, such as, for example, the incantations to the stone performed by certain aboriginal Australians prior to the debitage of blades. This is a far cry from the moment when surface, line, straight line, etc. would become not only ideas entirely disassociated from any material, but, above all, would be placed in abstract relation to one another (axiomatics), without reference to a technical process of any sort.

⁸ From the Levallois-Perret quarries.

2 Geometry of cave and mobiliary art

On becoming *Homo sapiens* some 200,000 years ago, humans began to draw, paint and engrave.⁹ They also sculptured statuettes and admirable bas-reliefs, and produced bodily ornaments. Here, however, we will limit our discussion to the most significant new development compared with earlier periods: two-dimensional representation, or, more exactly, what remains of this activity, given that all sorts of drawings (on the body, animal skins, bark, etc.) are obviously no longer extant.

The novelty was twofold. On the one hand, the object to be worked on is two-dimensional, i.e. a surface; on the other, as ethnography attests, the figure thereby obtained conveys thought: it is an active symbol, or sometimes even the very truth, of the original, and its function is entirely independent of its materiality as charcoal, ochre or engraving. This was a considerable revolution compared with the production of figures in the stone industry, which, in principle, were inseparable from their function as tools and therefore from their materiality. I say “in principle” because the beauty of certain handaxes (Figure 1), which show no signs of having been used, and the magnificent Solutrean leaf-shaped points, which were doubtless too fragile to be used as tools, suggest a momentary reversal of values in which the aesthetic function may have outweighed the technical function, and the pleasure of the object’s form may have prevailed over the constraints of its function.

Whatever the case may be, this revolution was no doubt the culmination of a long process of maturation, for if *Homo sapiens* emerged *circa* 200,000 BCE, the first drawings (Figure 3a) did not appear until *circa* 77,000 BCE (engraved ochre pieces, Blombos Cave, South Africa) and *circa* 60,000 BCE (hatchings on ostrich eggshells, Diepkloof Rock Shelter, South Africa).

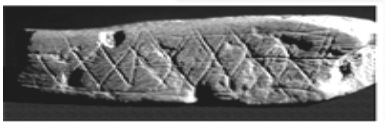
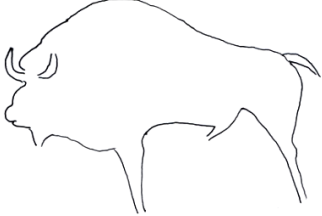
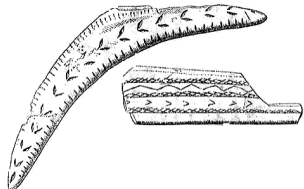
		
<p>3a. Engraved ochre baton. Blombos, South Africa, <i>c.</i>77,000 BCE.</p>	<p>3b. Engraved outline: absolute profile and twisted perspective of the horns. La Grèze Cave, Dordogne, <i>c.</i>18,000 BCE.</p>	<p>3c. Examples of friezes. Engraved reindeer antler, Laugerie-Basse, Périgord, <i>c.</i>13,000 BCE.</p>

Figure 3.

Homo sapiens, then, turned to surface as a new space on which to work. *Surface*. Really? How can one be sure that there was a genuine awareness of the idea of “that which has length and breadth only”?¹⁰ To answer this question, one can first adduce numerous ethnographic

⁹ A few rare traces of this phenomenon might suggest far older graphic activity. But the phenomenon becomes incontestable and massive only with the modern human, *Homo sapiens*.

¹⁰ Euclid’s *Elements*, definition 5. Of course, “length” and “breadth” should not be interpreted as measurements.

testimonies evincing the conception of the rock-face as a site of contact and transition between the real and supernatural worlds. For example:

According to the Aboriginal Elders, the landscape was formed by the actions of Ancestral Beings [...] At many locations the Beings entered directly into the landscape after their travels and creative acts, *often leaving their images behind on the rock surface*. [...] In western Arnhem Land, Aborigines made seasonal camps in the shelters located at the base of the escarpment and adorned the walls and ceilings with thousands of hand or hand-and-arm stencils, monochrome and polychrome paintings. By stencilling and painting they bonded more closely with specific sites and *tapped directly into the power of Ancestral Beings represented in animal, human or mythical form*.¹¹

Yet, as a site of *contact*, the rock-face clearly lacks breadth. Secondly, the idea of surface also exists in negative form, since in order to represent an object on it one has to remove one of its dimensions, hence the need for an authentic construction in the literal sense of the term. This can be a simple *section*, which gives an “absolute profile” (Figure 3b), or a *projection*, in which case one obtains a *trompe-l’œil*, with what appears to be a foreground and background. But the truly specific construction of prehistoric art, in my opinion, consists in the rabatment of elements that are judged to be indispensable, such as horns or the undersides of hooves. Abbé Breuil¹² called this phenomenon “twisted perspective” (Figure 3b) and, in later prehistoric periods, it can extend to “flattened perspective”, that is to say technical drawing of sorts, if you will forgive me the anachronism. In practice, the various modes of construction coexisted. Lastly, and thirdly, the idea of *a surface structured in accordance with its two dimensions* emerges very clearly in mobiliary art on bone, deer antler or ivory. This art is made up of friezes (Figures 3c and 4b), that is to say a motif running along the “length” of the object, possibly with a symmetry about the perpendicular direction. It has been mathematically demonstrated that there are seven types of friezes, and all of them can be observed in prehistoric mobiliary art.

This is the evidence in favour of the existence of the idea of surface – not, as we have seen, as an empty abstraction, but rather as a motivation for predetermined constructions, and as the site of these constructions: section, projection, rabatment and structuring, in two orthogonal directions. If we remove one further dimension of space, we now come to the issue of *line*.

Let’s take an easily recognizable figure, such as the representation of a bison. If it is complete, this figure structures the rock-face into two adjacent parts, the inside and the outside, and as a consequence determines a boundary between the two: a *line*. When the entire animal is painted, or when “negative handprints”¹³ are used, the line is certainly present, but only in our minds – imbibed as they are with contemporary topology. Its existence is not shown with a specific mark. In contrast, when the representation is reduced to an outline (Figure 3b), the latter

¹¹ Taçon, P. S. C. The power of stone: Symbolic aspects of stone use and tool development in western Arnhem Land, Australia. *Antiquity*, 65 (1991), 192–207. Italics mine.

¹² Abbé Henri Breuil (1877–1961), known in his lifetime as the “pope of prehistory”.

¹³ Obtained by leaning one’s hand against the rock-face and spit-spraying a pigment over it. These negative handprints can be found at a great many prehistoric sites around the world. In the case of modern-day hunter-gatherers, there is sometimes proof of an act of contact with the world of ancestral beings.

is, objectively, a *specific sign of line*, in that it serves as a boundary mark between two adjacent surfaces. Though it is possible to consider a fully painted representation using skilful nuances of colour and minimal trompe-l'œil as a talented copy of a spontaneous visual impression, the same cannot be said of representation using outline alone. No one ever “sees” an outline and, consequently, that outline can be only an interpretative code, an artificial sign of a boundary invented by our ancestors. We can therefore unequivocally conclude that the latter not only invented surface, but also line.

This analysis produces the following observation: we can be sure that a stroke is the sign of a line only if this stroke produces a *recognizable* outline, or fragment of an outline. But, to take the example of the strokes incised on the ochre baton from the Blombos Cave (Figure 3a), there is no reason to describe these strokes as lines, in the sense of “breadthless length” (*Elements*, definition 2). They may be nothing more than traces of meaningless entertainment, and this is perhaps also true of the very numerous meanders (“macaronis”) traced by one or several fingers in the damp clay of the rock-face.

By removing one further dimension, we finally arrive at the geometric existing nothingness, “that which has not part” (*Elements*, definition 1): the *point*. Alignments of points are common in decorated caves. We talk of alignments because by following them with our eyes, we scan a line. But is the word “line” appropriate? We talk of points because these are little spots of paint made using a stamp or fingertip. But is the word “point” appropriate? I would suggest the following: since, as we have established, the complete outline of a recognizable figure is a line, a dotted outline of this same figure (Figure 4a) serves to suggest this line through a series of its “constituent parts”. Yet, to my knowledge, these “constituent parts” are always the product of a *contact* with the rock-face, using either a stamp or a finger, and not dashes on the rock-face, in which case we would be dealing with fragments of line. The evidence therefore suggests that the “constituent parts” of the outline should be considered as *signs of the idea of a point* – yet another invention that we can credit to these ice-age artists.


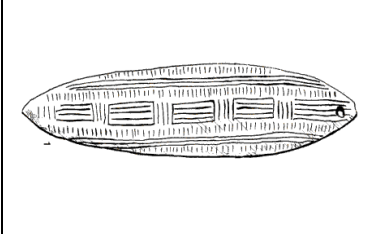

		
<p>4a. Ibex drawn with dotted lines. Lascaux Cave, c.15,000 BCE.</p>	<p>4b. Frieze of rectangles. Engraved reindeer antler, La Roche de Lalinde, Dordogne, c.11,000 BCE?</p>	<p>4c. Engraved and painted rectangles. Lascaux Cave.</p>

Figure 4.

Surface, figure, line, point. Each of these fundamental ideas can be detected in the signs invented at least 40,000 years ago, and which, over time, would morph into the concepts of a future science. Prehistorians, for their part, think “geometry” only when they find themselves

confronted with patterns whose meaning escapes us but whose form evokes classic figures: triangles, rectangles, circles, etc. This, admittedly, is a very superficial conception of geometry, yet it poses a substantial problem: did prehistoric peoples invent these classic figures? It is reasonable to use this term in at least one case, that of the rectangle (Figures 4b and 4c), and it is on this subject that we will conclude. The “reason” is the following: if we accept that, without a genuine idea of orthogonal symmetry, it would have been impossible for our *erectus* ancestors to produce handaxes; if we accept that our *sapiens* ancestors used the same idea to create friezes on bone, ivory or deer antler, and, moreover, structured the decorated surface in two perpendicular directions; if we accept all that, while also bearing in mind that symmetry can be verified by sight, by imagining a fold, then all the conditions are in place to make veritable rectangles. How? Simply imagine forming a rectangle by making successive folds in a sheet of paper of any given shape: starting from a given point, two suitable folds are all that is needed to obtain the four vertices – or rather, with four suitable folds, the fold axes produce the four sides. No theories or definitions are involved. Is it unthinkable that prehistoric peoples were able to mentally conceive of this construction (“in their head”), since all that was required was the practice of symmetry?

At the end of the Palaeolithic, geometry still had a long way to go before it would become a science. In the Neolithic, a new type of space emerged, as well as new two- and three-dimensional figures and combinations of figures.¹⁴ In the time of the first empires, the association of number and figure, enabled by the invention of systematic measurement,¹⁵ prompted a considerable leap forward and gave rise to the first treatises. Finally, the philosophical state of mind, in the sense of thought seeking its own premises, was at the origin of that astonishing mathematical introspection to which we owe Euclid’s *Elements*.

Translated from the French by Helen Tomlinson

¹⁴ For a detailed presentation, see <http://www.apmep.fr/IMG/pdf/P2-27-compte-rendu.pdf> (in French), and Keller, O. (2006). *Une archéologie de la géométrie*. Vuibert.

¹⁵ Measurement, in the sense of a *general system* associating number and magnitude, does not exist in societies without writing. At most one finds measurements associated with very specific needs, and which are very restricted in scope and unrelated to one another.