

# ***PLAYING WITH INFINITY OF RÓZSA PÉTER***

## **Problem series in a Hungarian tradition of mathematics education**

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### **ABSTRACT**

In this paper I present a workshop dedicated to in-service teachers about a special “Hungarian tradition” of mathematics education, focusing on Rózsa Péter’s *Playing with infinity*, a book popularising mathematics (Péter 1944/1961). The workshop in question is part of a three-days teacher training we offer every year since 2012 to secondary teachers of the Parisian region; the teacher training itself is related to an interdisciplinary research project in history of sciences and to a working group for teachers.

First I will present this complex project which offers the context of my workshop. I will detail the construction of the workshop, and then I resume its content: the historical context of 20<sup>th</sup> century Hungarian mathematics education, and some elements of the analysis of Péter’s text. In the conclusion, I will talk about recent perspectives of the presented workshop, namely about the work of a young French mathematics teacher inspired by the presented texts.

### **1 The context of the workshop: a complex project associating historical research, text edition, teacher education, and a working group for teachers**

In a previous HPM paper (Bernard & Gosztonyi, 2015), we presented a complex project including historical research, text editing, in-service teacher training and a working group for teachers. Every part of this project is focused on the study of some historical texts having the form of a collection of questions and answers. As we explained in the aforementioned paper, the term “problem” has to be understood in this case in a very broad sense, as referring to any kind of verbal challenge: this includes mathematical or scientific problems in the usual sense, but also riddles (*enigmata*) or questions, in general any kind of practical, pedagogical or intellectual “task”. The main originality of the research project consists in focusing not on individual ‘problems’ but on the principles, the characteristics and the possible roles of their collection in a certain order.

The research project entitled “Series of problems at the crossroad of cultures”<sup>1</sup>, developed with the collaboration of researchers from several, mostly Parisian research laboratories<sup>2</sup>, gathers around 15 researchers, including master and PhD students, from various disciplines: history, epistemology and anthropology of sciences and of literature, history of texts, cultural history, and educational studies. The “crossroad of cultures” can be understood here in several ways: it refers to the various origins of the examined texts (from Mesopotamia

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<sup>1</sup> For the research blog of the project see <http://problemata.hypotheses.org>.

<sup>2</sup> In the frame of the “laboratoire d’excellence” HASTEC. See <http://www.hesam.eu/labexhastec/partenaires/> (accessed 21 February 2016).

and Greek antiquity, through Middle Age and Renaissance, until 20th century's Hungary), to the diversity of scientific and professional cultures treated (from mathematics through marketing until theology), but also to the interdisciplinary character of our researches.

The project have recently led to a first collective publication, where each contributor presents the interest of studying series of problems from the point of view of his or her own research (Bernard 2015). In the following, we would like to proceed to the editing of a sourcebook about series of problems: the book would present extracts from the analysed texts with research commentaries. We also hope to add commentaries addressed to teachers (and also written by teachers in a part of the cases) which would make this book a useful source not only for research but also for education and for teacher training.

Since 2012, a three days in-service teacher training is associated to this research project that we organise every year in Paris. The public of these training sessions are in-service secondary teachers, most of them specialist of mathematics, but we also invite teachers of literature and history. The main aim of this training is not to give ready-to-use teaching material for the participants, rather to offer them cultural and historical knowledge and to stimulate reflections about their own teaching practice through the “meeting” with the texts.

Since 2014 autumn, we invite the participants of the training to continue their experience in the frame of a working group. The group, integrated in the structure of the IREM Paris Nord, basically follows the principle of the IREM workshops<sup>3</sup>: the collaboration of researchers and teachers at various levels (primary, secondary or university), looking for personal professional development, for the preparation of resources and the organisation of teacher trainings for colleagues. We encourage the participating teachers to develop projects answering to their own professional problematic, which projects are inspired by the texts discovered during the training session. At this moment, we follow the projects of three secondary teachers; one of them uses the documents of the Hungarian teaching tradition, presented below, to develop a problem-based, dialogical teaching practice in her classrooms.<sup>4</sup>

## **2 The construction of the in-service teacher training**

The three-day teacher training is constructed by six three-hour sessions. The first one or two consist in a general introduction into the notion of “series of problems” and into the principles and the reasons of their study. The later sessions are respectively led by different contributors of the “series of problems” research project, presenting in each case one or more texts, their historical context and some results of our researches about them, susceptible to interest in-service teachers. In each case, we intend to leave enough time to reading and discussion and look for the emergence of teachers’ professional reflection related to the analysed texts.

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<sup>3</sup> “IREM” is the acronym for “Institut de Recherche sur l’Enseignement des Mathématiques”. See <http://www.univ-irem.fr/spip.php?article6> (accessed 21 February 2016) for a presentation (in French) and (Fauvel & van Maanen 2002, pp. 96-97), in English.

<sup>4</sup> For more details about the construction and the principles of the project, see (Bernard & Gosztonyi, 2015)

However it is difficult to leave enough places, in three hours, to an adequate presentation of the elements of historical context allowing the interpretation of the text, *and also* to deep discussions between participants. Therefore we experiment with different forms of organisation each year. At the beginning, we introduced each workshop by lectures, before reading collectively the texts. Later we tried to start with guided readings and furnish elements of context during the discussion. The novelty of the 2016's session is the aforementioned publication, at the end of 2015, of the first collection of articles related to the “series of problems” research project: thus, we are trying to invite the training's participants to read extracts from the related articles before each workshop of the training.

### **3 The workshop about *Playing with infinity* and Hungarian teaching traditions**

The workshop I present every year in this frame is based on my PhD researches about the Hungarian “New Math” reform and its historical context (Gosztonyi 2015a). The Hungarian reform movement of mathematics education, led by T. Varga between 1963 and 1978, is closely related to the international New Math movement; at the same time, it is reputed in Hungary as an important representative of Hungarian traditions of mathematics education, focused on problem solving and on the discovery of mathematics. In my thesis, I compared Varga's reform to the French “mathématiques modernes” reform to show that, despite some common elements deriving from the common international context of the New Math movement, there are important differences between the two reforms. I argued that the epistemological background of the reforms is one of the key elements causing these differences: mathematicians play an important role in the conception of both reforms, but, while they represent a “bourbakian” kind of epistemology in the French case, mathematicians behind the Hungarian reform promote rather a “heuristic” epistemology of mathematics, close to the conception of Pólya or Lakatos.

The mathematicians in question wrote barely explicit philosophical text about the nature of mathematics. Nevertheless, their conception on the epistemology of mathematics can be reconstructed from their writings about mathematics education and especially from their books popularising mathematics. In the frame of the “series of problems” workshop, we read an extract from one of these books, namely the *Playing with infinity* of Rózsa Péter.

#### **3.1 The historical context**

Hungarian mathematical research culture passed through a spectacular development at the turn of the 19<sup>th</sup> and 20<sup>th</sup> century: until the last decades of the 19<sup>th</sup> century, hardly any significant mathematical research was led<sup>5</sup> in the country, but from this period, a number of internationally important mathematicians appear in Hungary<sup>6</sup>. 20<sup>th</sup> century Hungarian mathematical culture and mathematics education is reputed as focusing on problem solving and heuristic methods in mathematics.

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<sup>5</sup> The two Bolyais, Farkas and János represent rather an exception.

<sup>6</sup> About the possible reasons of this progress, see (Császár, 2005; Frank 2012; Hersch & John-Steiner, 1993).

In my research, I concentrate on Tamás Varga's reform movement of the 1960's and 1970's. Varga, although participating in the international discourse about the New Math movement, emphasises similar ideas as many other Hungarian mathematicians, concerning the importance of discovery processes in mathematics education. I attempted to show that a group of Hungarian mathematicians, namely L. Kalmár, R. Péter, A. Rényi among others, exercise important influence on the conception of Varga's reform. Not only did they actively support Varga's movement, but since the 1940's Varga participated with some of them in an interdisciplinary group discussing questions of education. The young I. Lakatos, future philosopher of sciences as well as Á. Szabó, future historian of mathematics, were also related to this group<sup>7</sup>.

### 3.2 The epistemological background

The above mentioned mathematicians express their ideas about the nature of mathematics and about its teaching in some books popularising mathematics and some lectures about mathematics education. An analysis of these texts shows that they represent a quite coherent conception about the epistemology of mathematics, with important consequences on teaching.

They present mathematics as a constantly developing and changing creation of the human mind, this development being guided by series of problems. According to them, the source of mathematics is intuition and experience; mathematical activity is basically dialogical and teaching mathematics is a joint activity of the students and of the teacher, the teacher acts as an aid in the students' rediscovery of mathematics. Excessive formalism is discouraged; formal language being also seen as a result of a development. They present mathematics as a creative activity closely related to playing and to the arts<sup>8</sup>.

One of the analysed texts is Rózsa Péter's *Playing with infinity*: this is what I chose to treat more in detail during the teacher training workshop, because of its accessibility (translated in French as well as in English, easy to read) and because of the palpable presentation of a teaching situation in the book.

### 3.3 *Playing with infinity* of Rózsa Péter: a modelling in terms of series of problems

*Playing with infinity* (1944/1961) is a book popularising mathematics written to the non-mathematicians, especially to the people "of literature, of arts, of the humanities" (p. v), to "pass on something of the feel of mathematics", of the "joy of mathematical creation" (pp. xii-xiii). The book is written in a literary style, in a quasi continuous form almost without formulae; however, it leads the reader from the simplest mathematical notions, through different themes of high school mathematics until the theorems of Gödel and Church, important contemporary results of mathematical logic, special research domain of the author.

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<sup>7</sup> For more details about the historical context of Varga's reform, including political, socio-economic, institutional and cultural aspects, see (Gosztonyi 2015a). About the relationship between Kalmár, Péter, Rényi, Lakatos, Szabó, Varga and the leader of the mentioned group, the pedagogue, psychologist and Calvinist pastor S. Karácsony, see (Gurka, 2001; Máté, 2006).

<sup>8</sup> For more details about this epistemological analysis, see (Gosztonyi, 2015; Gosztonyi, forthcoming).

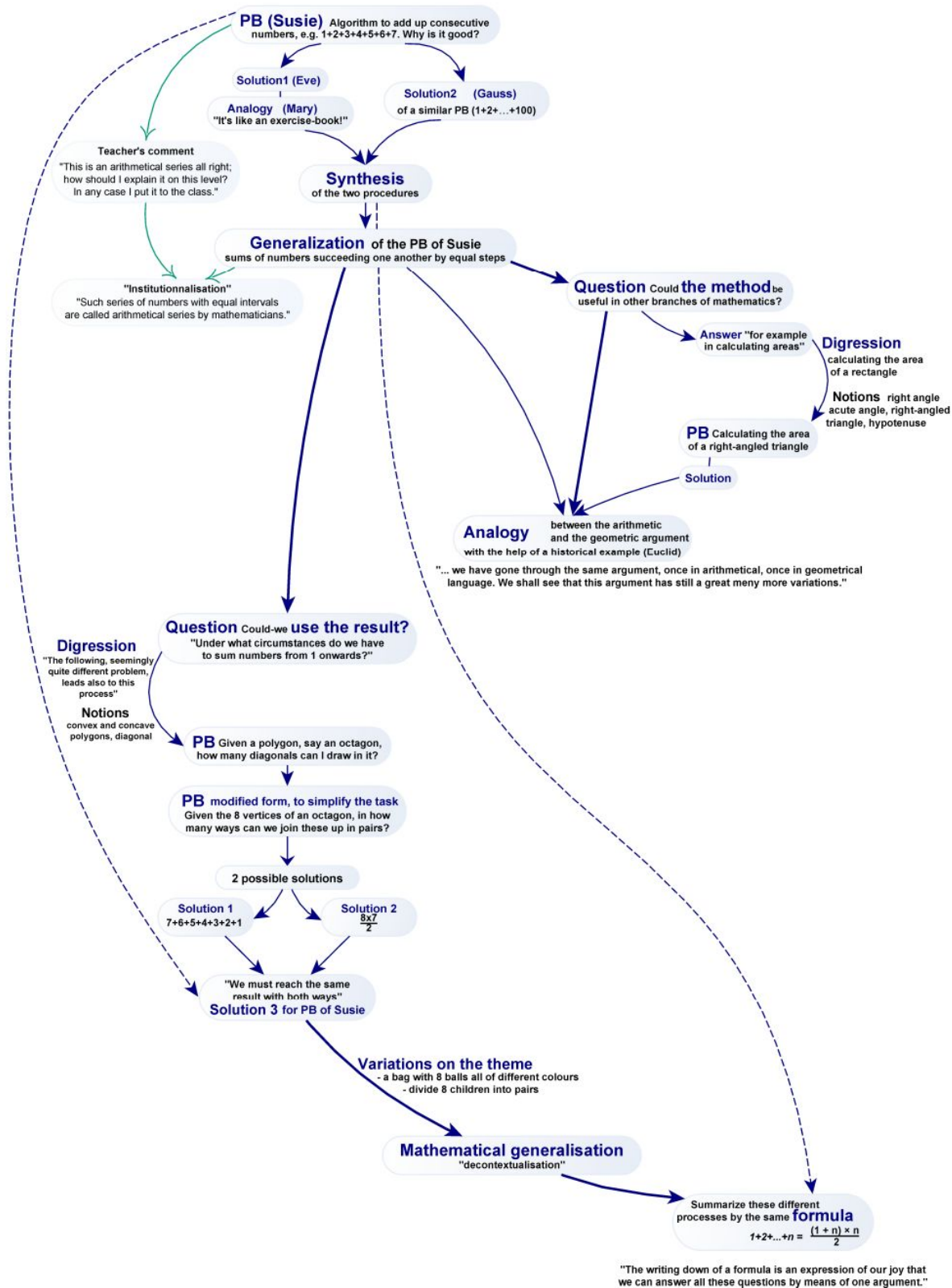


Figure 1. The modelisation of *Playing with infinity*, chapter 4 and 5, as a series of problems

During the workshop, we analyse two chapters of this book (chapter 4 and 5), describing a classroom experience of the author<sup>9</sup>. We look for a modelisation in terms of series of problems: I try to show that the two chapters are built on a sophisticatedly organised series of problems, and on the dialectic of questions, attempts at answers, and new questions emerging from these answers (see Figure 1). The described classroom experience takes the form of teacher-student dialogues where the participants are partners in a collective discovery project, and the teacher plays the role of an experienced guide during this research.

The analysis of series of problems helps to understand how this discovery project is organised. Problems are related by using similar methods or similar mathematical results; new problems are created by analogies or generalisation among others. Mathematical knowledge develops progressively by the synthesis, the successive generalisations of the treated problems and their solutions. When, at the end of chapter 5, we arrive to a formula summarizing the solutions of different problems treated, the author concludes: „The writing down of a formula is an expression of our joy that we can answer all these questions by means of one argument” (p. 33).<sup>10</sup>

#### 4 Conclusion

Teachers participating at our teacher training arrive with various motivations: some of them are looking for new tools, for example problems or texts that they can use in classrooms. Others are appealed to by the idea of the „crossroad of cultures”: they hope to find tools to reveal the interest of their multicultural classes<sup>11</sup>. Many of them come to enrich their personal culture concerning history of science, or to develop an interdisciplinary approach for the teaching of mathematics. Of course, they are quite often attracted to also by the question of teaching by problems.

The workshop I propose in the frame of this training lets them discover a mathematics teaching tradition relatively close in time and space to the French one, however quite different in their characteristics. The presentation of Péter’s book in its historical context and with its epistemological background leads to interesting discussions about the cultural and historical determination of mathematics education. The lecture and the analysis of the text often provokes vivid reactions: some teachers recognise in it teaching principles close to their own ideals, and find in the book a useful resource to develop a teaching practice based on problem solving.

One young teacher, a participant of the 2013/2014 session of the training continues to use this book in the frame of our working group. She tries to develop a dialogic teaching practice based on series of problems, and looks for inspiration in the Hungarian resources I presented. Our actual project concerning the teaching of Pythagoras’ theorem is principally

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<sup>9</sup> Although acknowledged researcher in mathematics, Rózsa Péter didn’t obtain any academic position until the end of the Second World War and therefore she worked in a middle-school.

<sup>10</sup> For more details about the modelling, see (Gosztonyi, 2015b).

<sup>11</sup> Most of the participating teachers work in schools of the Parisian *banlieue*, with students from various social and cultural origins.

inspired by Péter's book, by Varga's textbooks and teacher's handbooks. The project is then utilized, not only in her classes but also during this year's session of the teacher training: she just presented her project after my workshop in April, hoping to receive useful feedback to further development, but also to inspire some more teachers to investigate in similar projects in the future.

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