ANALYSIS OF PUPILS TASKS IN THE USE OF HISTORY IN MATHEMATICS TEACHING

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ABSTRACT

After several years of video acquiring and analysis, this paper gives a synthetic view on the different tasks that appear in sessions blending history and mathematics. Based on a typology summarized by the acronym SaMaH (Specific, a-Mathematical and a-Historical tasks), this first reading grid enables the identification of the respective contributions of mathematics and history in a given school activity. This framework also makes visible the particular specific tasks in which both fields are involved. Then, the analysis of different forms of specific tasks will lead to a new a gradation between H-weak and M-weak sub-domains. This paper mainly gives a quick view on the most important theoretical results at this time only available in author's French publications.

1 Context and issues

Facing to the difficulties in interesting pupils at science and particularly at mathematics, many new pedagogical approaches have been engaged and the use of history in science teaching is one of them. Nowadays, the potential benefit of history of mathematics in secondary school teaching is not anymore a topic of debates. As for the acquiring of a humanist culture as for the fine understanding of scientific results, numerous authors have raised the interest of the historical perspective in teaching and in teacher training. In France, this movement is hardly linked to the national history of education. Through some major changes in the institutional demand since 1970 (modern mathematics, new teaching programs) and by the creation of new structures (creation and extension of the IREM, Research Institutes on Mathematics Teaching), this kind of practice, at the beginning developed by passionate teachers, tends to extend to any educator at any teaching level. Worldwide, many countries have followed the same history and lots of them have now introduced an institutional request for the use of history in science teaching and in teacher training. All local situations are slightly different but many books like History of Mathematics in Education – The ICMI Study (Fauvel & van Maanen, 2002), Recent Developments on Introducing a Historical Dimension in Mathematics Education (Katz & Tzanakis, 2011) or the last special issue of Science&Education 23-1 (2014) give a very good view on the importance of this question as a research topic. Back to the specific French context, man can notice that a massive institutional demand appears in secondary teaching programs as well as in students and teachers curricula. For instance, the teaching program of secondary school mentions at multiples levels the need of "coherence" in mathematics learning by the use "some problems based on historical elements" (Bulletin Officiel spécial n°6, 2008), or the reading of texts in order to "understand the genesis and the evolution of certain concepts (Bulletin Officiel Hors Série n°7 annexe, 2000, Bulletin Officiel spécial n°9, 2010). Each times, the aim is to give pupils a "scientific culture" in order to help them to get into the contemporary mathematics knowledge and become, for part a them, the new tomorrow scientists. In order to apply these recommendations, the teacher training program has been changed as well and mentions now the fact that any teacher should be able to "situate his topics in its history and in its epistemological issues" (Bulletin Officiel n° 30, 2013). A first exploratory study has been led during two years (june 2012 – june 2014) with

the support of a regional grant ("Programmes émergents", Région Nord Pas-de-Calais, grant amount 10k€). This first program has helped in the creation of a video corpus that has enabled the elaboration of a new theoretical framework for the didactic analysis of such pedagogical practice. A large part of the results (short videos and comments) is available on the website of the program[1]. Taking into account both levels of this introduction of an historical perspective, the present paper aims to analyse the didactic effects of such a pedagogical approach in secondary school practice and their consequences for teacher training situations.

At the international level, specialists in the use of history in mathematics teaching agree on the following double observation: the literature gives lots of good examples of historical approaches but what makes them pertinent rarely goes beyond the subjective considerations of the author himself. For instance, in one of his papers in 2009 (Relime vol.12 n°1), after a large review of what had been published on this subject, the Danish researcher U.T. Jankvist explains that "the literature does, however, offers a variety of arguments on why and how to use history in mathematics education" and M-K. Siu and C. Tzanakis (proceedings ICME 10, 2004) write as well that "what is needed now are empirical investigations on the effectiveness of using history." The analysis of sessions blending history and mathematics does not easily enter the traditional didactics theoretical frameworks that have been mainly developed for a single topic. Whatever is the chosen entry point, such studies are blocked by numerous obstacles. Thus, a study on the use of history of mathematics in teaching incites to create and adapt new didactic concepts. According to the results of the previous step of the program, the analysis of sessions in which to fields are at stake requires the taking into account of each of them in order to point out the real *in situ* relationship between them. Until now, the researches on this topic have been centered either on mathematics learning (mathematics didactic studies) or on the choice of a good historical resource (epistemological and historical studies). In considering right away both fields, this research program deals mainly on their interactions during the teaching process and the way they can help pupils in understanding abstract concepts.

The main theoretical choice made in the proposed framework is to consider simultaneously history and mathematics and try to explain their interactions in specific student tasks. It should seem to be obvious that, in a session in which history and mathematics are involved, both have to be taken into account, but it is not from a theoretical point of view (Guillemette, 2011). History and mathematics are largely distinct in their epistemological bases. Thus, the teaching aims different and they could not be summarized to the application of one field to an other (de Vittori, 2012). As said previously, except in some recent works, the literature is mainly related to the analysis of one domain and not both. The most advanced project appears probably the last Mosvold article in which he tries to situate the interest of history within the mathematics teaching contents (Mosvold&al, 2014). As the analysis in this paper will lead to a theoretical framework at the student tasks level, it is complementary to these studies focused on the teacher level. Before entering into the details of such a model, it is important to say that this short paper will not deal with the way an activity blending history and mathematics has been chosen or created. The main reason is that, in France, lots of resources are available, as much as in specific publications as in a major part of primary or secondary school textbooks. Some examples will appear below.

[1] Website, http://eduhm.univ-artois.fr An identification is required: login: region-npdc, password: region-npdc

2 Tasks identification: the SaMaH typology

By the co-presence of two fields in a same session, each one of them produces its own tasks and some of them are new when man compares them to a more usual mathematics teaching situation. As a session blending history and mathematics is focused partly one this second field, it is easy to identify tasks that are quite the same in a more traditional mathematics course. For those, mathematics didactics have given lots of useful theoretical frameworks. Each of them, depending the topic, the age, the tools, etc. are totally pertinent when the research is pointing its interest only on mathematics learning. Could these mathematical tasks been ignored? Of course not and the main reason is that these elements anchor the session in a school context. As said before, the use of history in mathematics teaching is a pedagogical approach. It means that parts of the purposes are related to the usual mathematics teaching programs and they could not be suppressed. Even if mathematics are deeply linked to their history, some tasks that do not especially involve history can be identified. They will be called a-Historical tasks (aH). Lots of examples are available in literature and, within them, these tasks are obviously the easiest to identify. For instance, here is a part of the chinese ancient book *Neuf chapitres de l'art mathématiques* from which a secondary school teacher asks his students to solve the problem the way they want (Figure 1).

« Une ville dont on ne connait pas la taille est entourée d'un mur de forme carrée. Chaque côté du mur mesure un nombre entier de pas.
Au milieu de chaque côté il y a une porte. A vingt pas de la porte Nord se trouve un arbre. Si on sort par la porte Sud, qu'après quatorze pas on tourne à l'ouest, il faut faire 1 775 pas pour voir cet arbre.
On demande combien de pas fait le côté de la ville. »

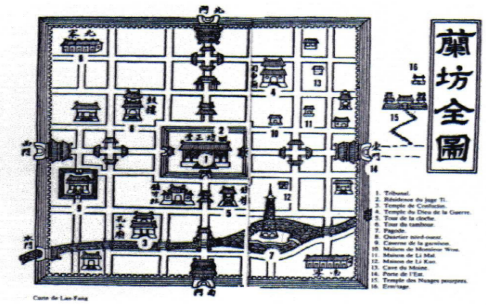


Illustration tirée du roman « les enquêtes du juge Ti : le fantôme du temple » de Robert Van Gulik, Editions 10/18.

Figure 1.

As the teacher does not give any indication, the pupils enter into the task only by the means of their previous mathematical knowledge (de Vittori, 2015). Temporarily, the historical content and context disappear in order to let the task become purely mathematical in a modern meaning. In the corpus analysed during our study, almost all activities propose some aH tasks in which pupils are mainly supposed to simply do some mathematics. For instance, in the textbook *Math 3e* (collection Prisme, Hachette, 2008, year 10 or 9th grade), two short exercises can quickly exemplify this phenomenon. In the first one (n°81 p.76), after a few words on Léon Foucault's pendulum, pupils have to calculate its period T by the means of the given formula and the length of the rope. The same way, in a second exercise (activity "Pour les curieux" p.99), pupils have to check the general formula of some Pythagorean triples by an algebraic calculus far away from the Greek mathematics. All these activities and exercises comprise a mathematical part that can be taken into account by the aH type which is one of the three categories in the SaMaH typology. The limits of such a pedagogical choice

will be discussed later. Let's first have a look at the two others.

In parallel of these mathematical tasks, the history present in the session gives its own things to do. Contrary to mathematics in which objects tends to be pure abstraction, historical contents have to be situated in their real context. In order to achieve such an historical situation, some time and cultural markers should be given. For the students, these historical tasks can be done through the reading of a text, the listening of a story, a film, a webquest, and so on. There is a lot of possibilities in which the teacher implication varies but in each case, the task is only related to history and/or historical methods. As the mathematical content temporarily disappears, such tasks will be called a-mathematical task (aM). A name, a date, a biography… can be totally ignored by pupils if no work is engaged in the history field. Any task inspired by the work of professional historian helps the history to take its place in the session. One of the most common examples of such a practice is the use of a short introducing text about the author or the context. For instance, in the beginning of an activity on second degree equation, a short text about Al-Khwarizmi is given (Figure 2).

Al Khwarizmi *est un des premiers mathématiciens et astronomes du monde arabe.*
Il vit entre 780 et 850 environ à Bagdad dans une époque très brillante.
Il fut le premier à répertorier de façon systématique des méthodes de résolution d'équations en classant celles-ci.

Figure 2.

Pupil's task simply consists in reading the text and talking about it with the teacher. It is important to say that the role of the teacher is crucial at this time. For instance, in an activity about the Rhind mathematical papyrus (Odyssée 6e, Hatier 2014, p.101, year 7 or 6th grade), two third of the page is used by a comics which gives the story of two young Egyptians talking about one of the problems discovered in the ancient document. A short paragraph (100 words) explains where and when the Rhind papyrus has been found and, then, are given the questions. The first one (« À l'aide d'une recherche sur internet et auprès de ton professeur d'histoire, trouve quel pouvait être le pharaon au temps d'Ahmès ? ») proposes an inter-disciplinary webquest helped by the History teacher. The purpose of such a task is to clarify the epistemological and historical background of the topic. Nonetheless, without a discussion, some mistakes on the value of the historical elements shown can be done. All of this requires a little expertise from the teacher in order to make the contextualization tasks works correctly. This point is related to teacher training which have been discussed by many authors as said in the introduction of this paper.

The consideration of the first two mono-disciplinary aspects (aH and aM) of a mathematics session using history enables the identification of the coexistence of both fields (history and mathematics) but it gives only a few information on the way they interact. As mentioned in the beginning of this text, one of the main specificities of the EDU-HM program is to take into account, simultaneously, both fields in order to identify the place where they interact together. Considering this, the analysis of the corpus (about twenty sessions) has shown a third type of task which is totally specific (S) to an activity blending history and mathematics. Neither purely mathematical nor historical, these tasks create a link between both fields in which the epistemology of each one enters into an effective conjunction. Deeply analysed in a previous paper (Barrier & al, 2012), an example of an activity based on Indian

Sulbasutras in which pupils use a rope in order to draw figures in the playground shows the way the mathematical contents (student's knowledge about the circle) and the historical context (Indians use ropes) interact. It is obvious that in order to be efficient in its synthetic dimension, the S task has to anchor itself in both fields. Each one of them appears by the means of its own tasks (aH and aM) and together they make an S task possible. One can see such a situation in the following activity given at multiple levels (secondary school, age 14 and 15, university L3). The main topic in this session is the extraction of a cubic root by the means of a rule and two sets square (Figure 3. This activity is deeply explained in de Vittori, 2015).

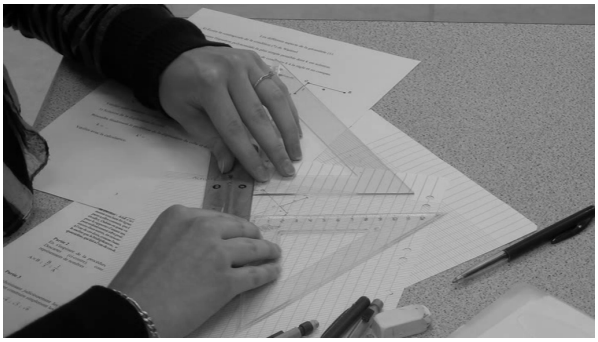
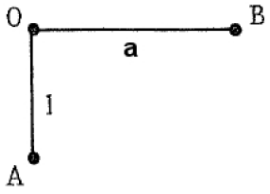


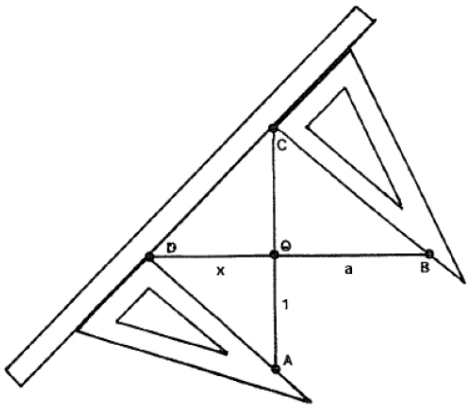
Figure 3.

Depending the level, the mathematical aim differs. In secondary school the activity is mainly related to integer powers and geometric construction programs, whereas in university it has been done in a pre-service teaching course about numbers that are geometrically constructible or not. Whatever the level, part of the session includes an aH time where the mathematics contents is involved and an aM time about the historical context. In this case, the method is a way to duplicate a cube; a very well-known class of problems since the Antiquity. What are the students supposed to do? Here is the situation (Figure 4):

Au IV^{e} siècle, un mathématicien grec propose la méthode suivante :

- Soit a le nombre dont on cherche la racine cubique.
- Tracer un segment [OA] de longueur 1 cm.
- Tracer un segment [OB] perpendiculaire à [OA] de longueur a.
- Placer les instruments « correctement » et on aura alors $x = \sqrt[3]{a}$.





Vérifier cette méthode en cherchant la racine cubique de 8.

Figure 4.

In order to extract the cubic root following this Eutocius', or more recently Lamy's, method (Hebert, 2004), students have to draw the segments [OA] (length 1) and [OB] (length *a*, perpendicular to the first one) and then they have to place a rule and two sets square into the good configuration. The cubic root of *a* is the length of the segment [OD]. The tools are well known but the way they are used is new. Indeed, in order to find the final position, the three tools have to be moved together by some rotations and variations of the gap. It is funny to notice that, as the use is unusual and a bit difficult, some students simply conclude that "it is impossible!". As in the activity with the rope in the playground, the co-presence of history and mathematics creates a new game on the constraints in the use of geometry tools. In such a context, the aH and the aM tasks both contribute to the justification of the S task and render actual the double anchoring.

Examples of specific tasks are at least as varied as the first two genres. Nonetheless, they generally are not part of the usual toolbox of the teacher, even if he has acquired some knowledge in history. Many times during interviews with teachers or master students, reluctance to engage in this kind of practice refers to the difficulty in conceiving the links with usual learning goals. Indeed, unless one considers them only as aH type, that is to say emptied of their historical content, the S tasks appear almost never outside this form of education. The deepest reason probably lies in the relationship between S and aM. Indeed, while some constructions or methods may seem usable beyond a historical context, the work of historians warns us against some too modern readings. For example, according to many students, an ancient figure from Euclid's time is the same as ours but, in fact, a line in the *Elements* is not this modern continuous object given in an abstract space. Similarly, in some equations, the unknown, the *thing* in medieval Arab mathematics, is not our *x* that can be multiplied by itself as many times as one wants. These epistemological distinctions are subtle and often beyond the reach of students, but they are real and they give its substance to the S tasks.

The identification of the different tasks helps clarify the issues of a session blending mathematics and history. In such an activity, part of the time is devoted to mathematics without any explicit link with history (aH), another focuses on pure historical knowledge without the use of mathematical content (aM). These two dimensions are complemented by specific tasks where the two fields explicitly join (S). In almost all sessions analysed in the EDU-HM corpus, the three aspects coexist in a whole that can be summarized by the acronym SaMaH.

3 From practise analysis to didactics engineering

Based on practice analysis, SaMaH typology allows at first to account for the richness and complexity of the various sessions studied. In the ordinary experiments and practices that have been observed, the three types of tasks are present in varying proportions. Thus, one can question the triptych S, aM, aH when an element tends to appear weak or misses. From the three types, three sectors can be defined: SaM, SaH, and aMaH (see figure 5 below).

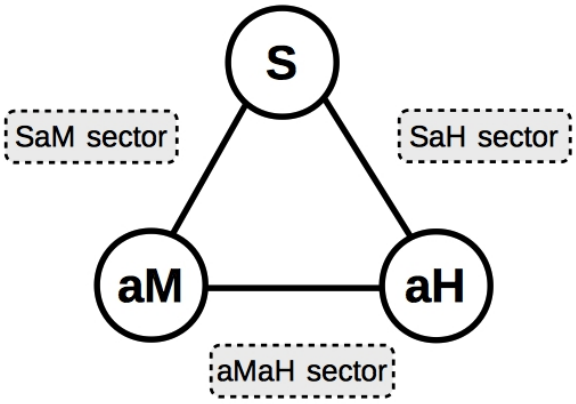


Figure 5.

What are the theoretical implications of the lack of one of the poles of SaMaH? In the case of SaM activity, the disappearance of the aH type carries with it the disappearance of the link with its anchor domain, namely mathematics. The result is an activity in which students have to deal with historical knowledge and do some specific tasks but all of this is unrelated to mathematics contents. It is therefore a disconnected session from school issues whose relevance could be justified in the context of extracurricular events (Science Festival for instance) but which cannot meet the current programs in a true educational context. Here, the S part is separated from its mathematical dimension that gives it its meaning. Moreover, if it can exist without its second pillar, the S task may be impossible to carry out due to difficulty in engaging some mathematics knowledge. Somewhat symmetrically, the SaH sector can account from another form of historically incomplete activity. In this second case, the aM pole is missing, namely the link with history. The activity is decontextualized and historical contributions are absent. As for the SaM case, the S type tasks are seriously compromised in this situation. As the advantage of the S task is to enlighten an ancient mathematical content by historical knowledge, it can be difficult or even impossible to do if the aM pole is not engaged. The last sector in this analysis regards the use of only the two poles aM and aH. There is no need to look far for examples of activities of this type because there are many situations in which historical elements are simply juxtaposed to a mathematics exercise. In

these cases, when, for instance, the portrait of an author or a copy of an ancient manuscript just illustrates a chapter, the history appears as a varnish without any work. Through the juxtaposition of the two fields, the aim is to make a nice layout or to evoke an ancient culture. Without being unfounded, this approach does not enter the type of sessions effectively combining mathematics and history.

4 Developed version of the SaMaH model: the S task as a conjunction point of two epistemologies

In their modern definition history and mathematics have too little in common to create anything by simply taking place together. So how the S tasks enable a link between history and mathematics? In a recent paper (de Vittori, 2015), S tasks in an activity based on Arabic medieval mathematics have been deeply analysed in order to find a first answer to this question. Here is a quick view. In a secondary school textbook, a problem uses a text from Al-Khwarizmi in which one has to find the length of the side of a square inscribed in an isosceles triangle (base 12, side 10). The exercise proposes three questions. In the first one, using only the ancient text and the figure, pupils have to explain the meaning of the words “la chose” (thing or root) and “le bien” (wealth or property) which refer to the geometrical objects segment and square. In the second question, pupils have to solve the problem using only these historical “concepts”. In those two first questions, the main point is that pupils are not supposed to use modern mathematics: they work in a specific old epistemological context. The modern mathematics reappear only in the third question in which the students have to associate a letter “l” to the “thing”, calculate and find the solution another way. It is important to notice that pupils are not used to manipulate algebraic quantities (year 7, 8th grade). This activity is designed to create betterment through an historical based thinking. With S tasks, the connection is made at an *in situ* epistemological level. The use of history of mathematics in the school context justifies the extraction of an epistemological part of history in order to develop a new pedagogical approach. This one creates a local modification of the epistemology of mathematics facing the students. Qualitatively, the magnitude of this local epistemological change varies from an activity to another. Thus, an ancient text in which the word “carré” is simply written “quarré” but where everything else follows a style very conform to contemporary mathematics involves a small introduction of history in the field of mathematics. Named H-weak thereafter, this type of approach produces only a little inflexion in modern epistemology implemented by students. In a quite symmetrical way, the application of a division algorithm in which one just allows the use of modern numbers notation creates a small modification of the old epistemology underlying. Such S tasks will be called M-weak. At an extreme point, if a task does not enable any change in the manipulated concepts nor by a historical contribution, nor by modern mathematics supplements, it will refer to the aH, respectively aM, tasks mentioned above. The diagram below (Figure 6) gives the overall organization of the various sub-domains. Referring to M-weak and H-weak types, the tasks which perform a full effective implementation of a double content is called S-strong.

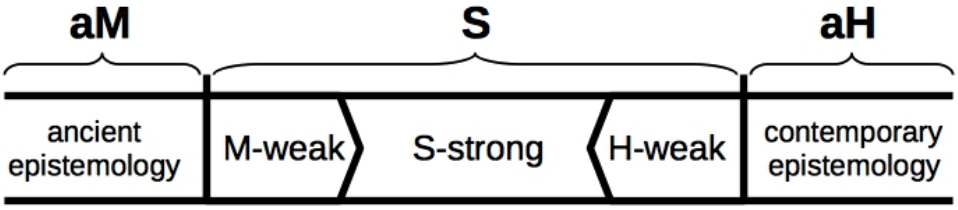


Figure 6.

At this point, one can notice that the SaMaH model enables too the localization of the potential contributions of different forms of historical research. It is clear that the local epistemological change required in S-strong tasks refers to a history of mathematics focused on the changes in the way mathematicians think about their objects. However, this does not imply the rejection of knowledge about the social development of science, on the biography of some scholars, or on the links with the political or economic history. These fields are far away from the mathematics contents, but they can predominantly appear in the aM tasks in which they take a true value in context explanation. Like mathematics, history is multiple and this quality cannot be a handicap.

5 Conclusion

The structure that has been presented here does not claim to completeness but strives to explicit the potential wealth of interactions between mathematics and history in a school context. Some examples have been given in order to illustrate certain patterns that have been observed in the corpus which has led to the development of the SaMaH model. This typology in three types (and sub-types) has been developed in order to create a framework for both the *a priori* and the *a posteriori* analysis. In an *a priori* analysis, some specific exercises or even some large textbooks collection can be explored in order to raise the most crucial points teachers should be aware of. Once some experiments have been done, the *a posteriori* use of the SaMaH framework can give a focus on the way the activity "works or not". The main point is that the characterisation in aH, aM and S tasks helps the identification of the place where new things happen. Mathematics' didactics is a well-recognized field in which the aH tasks can be analysed. On the other side, the aH tasks are deeply linked to the development of the research in history of science. As said previously, the S tasks, that make such pedagogical practice so special, have not been so much studied. The work on a didactic engineering specifically aiming the use of history in mathematics education is still in its beginning. The second part of the EDU-HM program mainly deals with the elaboration of activities where S tasks are clearly identified. Some experiments are already engaged and will be, hopefully, give new results soon.

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