THE INFLUENCE OF MATHEMATICAL CLASSICS READING ON UNIVERSITY STUDENTS' MATHEMATICS BELIEFS

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ABSTRACT

We report the influences of an undergraduate-level liberal-arts course, in which students in a medical university were guided to read through important parts of Euclid's *Elements* and taught about related cultural issues including logic and some Greek philosophy. Class questionnaires and reflexions showed that parts of students' beliefs changed after the course. Concerning the values of mathematics, students were more prone to believe that mathematics had real-world applications and improved one's sensibilities to beauty; for the nature of mathematics, they were more prone to believe proof was the only way to justify mathematical knowledge. The last change was expected but also interesting, since in another course in the same university about mathematics culture, students were confused about the context of discovery and that of justification. The course reported here was more helpful since it also had cultural connections but did not similarly confuse students, which is meaningful in a society where problem-solving, not logic, is considered the most useful aspect of mathematics.

1 Introduction

History is useful in mathematics education, as demonstrated in many scholarly works (e.g., Katz & Tzanakis, 2011), and it, too, could directly or indirectly shape how modern people view different academic disciplines, such as medicine or mathematics. The case of Taiwan is an interesting example. Medical practitioners – doctors, dentists, pharmacists …etc. – usually enjoy high social statuses Fernando Pessoa in modern Taiwanese society. Besides the facts that they have relatively high income, and that they help save human lives, there are also reasons rooted in Taiwan's history. Taiwan was colonised by Japan for half a century, since 1895 to 1945, during which period different laws were enforced in Taiwan from the 'Home Islands'. Post-secondary education, for instance, was highly restricted. For many reasons there were much less Taiwanese studying humanities and social sciences than in other disciplines in Taipei Imperial University, and most of the Taiwanese elites sent their sons and daughters to study medicine.[1] As a result, many of the leaders of social movements during the colonial period, and in later times before the democratisation of Taiwan, were originally medical doctors. Even today, a significant

[1] For discussions about higher education systems and practices in Taiwan's Japanese colonial period, refer to, for instance, (Zheng, 2002).

portion of Taiwanese politicians and social leaders have medical backgrounds, and this is also why medical practitioners generally receive relative high respects, and their opinion towards different aspects of social life, including education, are taken seriously by Taiwanese society. What does this have to do with mathematics education?

Since the admission to medical schools is highly competitive, medical students in Taiwan must have extremely high scores in examinations of basically every subject, including mathematics. However, after they have entered medical schools from secondary schools (medical education in Taiwan is in the undergraduate level), they only have to study very little mathematics, consisting of basic calculus and statistics. So what message does this practice send to the medical students? It seems that they would see mathematics as one of the tools to enter university, and that they *do not need* mathematics in their professional training and practices. Indeed, medical practitioners do not apparently use very much mathematics in their clinical practices as engineers, natural scientists or economists in their professions, but the logical, critical and reflexive thinking that could be improved in mathematics learning are also important for any profession. Since the beliefs of medical practitioners could influence public opinions, and some medical doctors may in fact become politicians and even educational policy makers in Taiwan, it is meaningful to do research in the country about mathematics beliefs held by medical students and try to bring them more diversified views on the values and nature of mathematics.

One of the authors of this paper has been doing research and teaching in a medical university in Taiwan, so one natural question following the previous paragraphs is: What kind of mathematics courses can influence medical university students' mathematics beliefs, and what changes can be observed? One possible kind of courses are selective liberal-arts courses about mathematics, since the lecturer has more space to talk about the aspects of mathematics other than problem-solving, such as its cultural and social connections. In fact, the authors had previously done an exploratory study about the influences of liberal-arts mathematics course with an emphasis on culture and history (see Literature review). Another possible kind of courses that may bring students deeper into the cultural roots of mathematics is classics reading, since in each civilisation there are always classics that carry its philosophical reflexions about the universe and human creations such as mathematics, and those reflexions have stood the test of time. Under the rationale, this study examined how a course about mathematical classics reading changed mathematics beliefs held by medical university students in Taiwan.

2 Aim of study

The main aim of this study was to explore how medical university students' mathematics beliefs changed after taking a liberal-arts course about mathematical classics reading. The authors wanted to explore how this course changed mathematics beliefs held by medical university students in Taiwan, especially about their beliefs toward the nature and values of mathematics. In what follows, relevant literature is reviewed, and a study about this topic and its results shall be reported.

3 Literature review

3.1 Mathematics beliefs

Many studies have shown that students' understanding and beliefs about the nature of mathematics do influence their mathematics learning. Over the course the mathematics education research late in the last century, belief had been considered a hidden variable in mathematics education (Leder, Pehkonen, & Törner, 2002). Alan Schoenfeld's research tells us related facts. For instance, after observing the problem-solving behaviours of high school and university students, he concludes in his article that a person's mathematics beliefs shape his or her ways to do mathematics (Schoenfeld, 1985). Also, Schoenfeld (1992; 1994) finds that inappropriate interpretations towards the nature of mathematics not only cause students' conflicting epistemological beliefs about mathematics, but also influence negatively on their mathematics learning effects. Besides, Cifarelli and Goodson-Espy (2001) find that university students' epistemological beliefs about mathematics do affect their mathematical learning.

Op 't Eynde, De Corte, & Verschaffel (2002) points out that mathematics-related beliefs, or students' subjective conceptions about mathematics that are implicitly or explicitly held true, have significant influences on their mathematical behaviours (Op 't Eynde, De Corte, & Verschaffel, 2002). In particular, Hong (2009) tells an interesting phenomenon about gender issues among students in universities of technology, that mathematics teaching environments unsuitable for female students would lower their willingness to pursue mathematics-related professions. For students' mathematics-related beliefs, Op 't Eynde, De Corte, & Verschaffel (2002) propose a framework, in which mathematics-related beliefs are described in students' class context, and are divided into three categories: beliefs about mathematics education (mathematics a subject, learning and problem-solving, and teaching in general), beliefs about self (self-efficacy, control, task value, and goal-orientation), and beliefs about the social context (social norms in the mathematics class). The questionnaire used in this study was developed based on this framework (see below).

The studies mentioned above tell us that the role mathematics beliefs play receive more and more attentions in recent years. This study also hopes to enrich research in this area, through exploring the effects of a course about mathematical classics reading.

3.2 History of mathematics, mathematics learning and mathematics beliefs

It is generally accepted that history helps mathematics teaching and learning. For instance, Liu (2003) proposes five reasons to use history of mathematics in school curricula: (1) history helps increase motivation and develop a positive attitude towards learning; (2) past obstacles in the development of mathematics helps explain what today's students find difficult; (3) historical problems helps develop students' mathematical thinking; (4) history reveals the humanistic facets of mathematics; (5) history gives teachers a guide for teaching. Also, history stimulates students' interests in mathematics (e.g., Furinghetti & Paola, 2003). In particular, historians of mathematics have been trying to use history to help students reflect upon the nature of mathematics (e.g. Jankvist, 2011). Besides, teachers may learn to appreciate mathematics as a cultural phenomenon through history (Tzanakis and Arcavi, 2000). Moreover, Horng (2000) finds that students may learn to appreciate the evolution mathematical knowledge through different solutions to the same problem in different times and locations. In the university level, Kjeldsen (2011) shows that using history as a means to teach tertiary-level mathematics helps students understand the special nature of mathematical thinking.

Following the aforementioned studies, the authors of this paper also tried to explore how history influences students' mathematics learning, especially in the tertiary level, because not few studies had been focused on this topic. In (Ying, Huang & Su, 2015), the authors of this paper and a colleague of theirs describe a study about the influences of a liberal-arts mathematics course, with an emphasis on culture and history, on the mathematics beliefs of medical university students. The results show that after studying through the course, the students were more prone to believe that, among other items, 'sensibility to beauty' and 'creativity' were both important values of mathematics. However, the results also revealed that the course did not clarify the difference between the 'context of justification' and the 'context of discovery' for students. A possible cause for this is that the contents of that teaching experiment were focused on the discovery of mathematical methods, but it did not talk very much about philosophy and logic. The authors then tried to find other ways to teach liberal-arts mathematics courses, and the teaching experiment described here is the outcome.

4 Research Setting

The study had a single-group pretest-posttest design. Research tools for this study included a liberal-arts course about mathematical classics reading, a questionnaire administered in the pre-test and the post-test, and students' reflexions. The same group of students took the pre-test, participated through the teaching experiment, wrote reflexions about class discussions and their reading, and finally took the post-test. A total of 40 students took the course, of which one of the authors was the lecturer. In what follows we shall describe the research setting in detail.

4.1 Subjects

The subjects in this study were a group of medical university students in Taiwan. They took an elective liberal-arts course about mathematics in the first semester of Taiwan's 2014-15 school year (September 2014 to January 2015) that was designed to be the teaching experiment in this study. Students' majors included medicine, dentistry, pharmacy, nursing, health care administration, medical technology, respiratory therapy, dental technology, and gerontology. Most of the students in the course were in their first or second year in university, and only one was in her third year.

4.2 Research tools

The research tools of this study included a teaching experiment, students' reflexions, and a 20-question Likert-scale questionnaire. We shall first explain how the teaching experiment went, including students' free reflexions, and then describe the questionnaire.

The course used as the teaching experiment was taught in a sixteen-week semester, with two hours of class time each week. In this course of mathematical classics reading, the main text used was Euclid's *Elements* (its modern English and Chinese translations). During the course, students were guided to read through various parts of the *Elements*, including the entirety of Book I, the so-called "geometrical algebra" in Book II, important propositions about the circle and regular polygons in Book III, theory of similarity in Book VI, essential propositions of number theory in Book VII to IX, basics of incommensurability in Book X, and finally, propositions about solid geometry leading to the five regular polyhedra in Book XI to XIII. During teaching, the lecturer might also talk about relevant Greek philosophy and history if necessary, including the philosophy of the Pythagorean School, Aristotelian logic, Plato's dialogue *Meno*, and ideas about the five classical elements. Students were required to write reflexions about their reading and about contents taught in class. The lecturer also discussed in class with students about their reflexions, so they might go deeper into the questions they were pondering upon. The reflexions were later used to confirm their belief changes seen from the questionnaire analysis. Since the flow of the course depended as much on the syllabus as on students' reflexions and discussions, the lecturer had a flexible way of teaching of the course, letting multiple topics be discussed in the same week if necessary. Table 1 shows the topics discussed in each week of the teaching experiment.

Table 1. Topics in each week of the teaching experiment

Week	Topics
1	Introduction to the course and Greek mathematics.
2	The *Elements* Book I; Aristotelian logic.
3	The *Elements* Book I; neutral geometry.
4	The *Elements* Book I; compass-and-straightedge construction; Athenian democracy.
5	The Pythagorean School and the Pythagorean Theorem; structure of Book I.
6	Brief history of Euclidean and non-Euclidean geometry.
7	The *Elements* Book II to III; Plato's dialogue *Meno*.
8	The *Elements* Book II to IV; geometrical algebra; regular polygons.
9	Midterm break; self-reading and reflexion.
10	The *Elements* Book II to VI; the laws and sine and cosine; similarity.
11	The *Elements* Book VII to IX; number theory.
12	The *Elements* Book X; incommensurability; Pythagorean mysticism.
13	The *Elements* Book XI and XII; solid geometry.
14	The *Elements* Book XIII; regular polyhedra; classical elements.
15	Conic sections; three famous classical problems of construction of antiquity.
16	The influence of Euclid's *Elements* in the Western civilisation.

As for the questionnaire administered in the pre-test and post-test, the authors used the same one developed in their previous research (Ying, Huang & Su, 2015). It was first modified from the framework provided in (Op 't Eynde, De Corte, & Verschaffel, 2002), and was designed to explore the changes in students' beliefs towards the (epistemological) *nature* of mathematics and the (social and personal) *values* of mathematics, which can be considered as two independent dimensions of mathematics beliefs (e.g., Goldin, 2002). The questionnaire consists of 20 Likert-scale questions, each of which is a declarative sentence about mathematics. The responses from which students could choose, for each statement, are typical seven-level Likert-items: ‘strongly agree,’ ‘agree,’ ‘slightly agree,’ ‘neither agree nor disagree,’ ‘slightly disagree,’ ‘disagree,’ and ‘strongly disagree.’ After the administrations of the questionnaire, their responses were transformed into scores for analysis, with the highest score 7 for ‘strongly agree’ and lowest score 1 for ‘strongly disagree.’ The questions in the pre-test and the post-test were identical, so the changes in students’ beliefs could be revealed. Table 2 shows the structure of the questionnaire.

Table 2. Structure of the questionnaire.

Dimensions	Questions
Nature of mathematics	1. Mathematics is a discipline described with symbols. 3. Mathematics is a discipline composed of procedures and formulae. 5. Mathematics is a discipline that finds general principles from individual facts. 7. The general principles found in mathematical research can be applied to all kinds of situations that fit the conditions of the principles. 9. Mathematics is a discipline that constructs common models from complex phenomena in reality. 11. Results calculated with mathematics can accurately explain many phenomena. 13. Mathematics is a rigorous and logical discipline. 15. Proof is the only method that justifies mathematical knowledge. 17. If an assumption in mathematics fits most situations, then it is justified. 19. Mathematics is objective truth.
Values of mathematics	2. The research results of mathematics can help human beings describe phenomena of the real world. 4. The research results of mathematics can be used as tools to solve problems in other fields of study. 6. Learning mathematics can help us solve problems encountered in daily lives. 8. Learning mathematics can cultivate our logic and reasoning. 10. Learning mathematics can cultivate our creativity. 12. Learning mathematics can cultivate our observing ability. 14. Learning mathematics can increase our knowledge. 16. Learning mathematics can elevate our ability to judge things around us. 18. Learning mathematics can improve our sensibility to beauty. 20. Learning mathematics is helpful to our career development.

The questions were given an iterated order according to their dimensions (odd-numbered questions for the nature of mathematics and even-numbered ones for the values of mathematics) to reduce the disturbance that might be caused by similar wording of the statements in the same dimension (Ying, Huang & Su, 2015, pp.14-16). In the next section, we shall discuss our findings seen from the questionnaire and students’ reflexions.

5 Results and discussions

A total of 35 students went through the whole teaching experiment, that is, they took the pre-test, stayed in the course through the whole semester, wrote several reflexions, and finally took the post-test. Paired-Samples *T* Test ($\alpha = 0.05$) were used to compare the averages of two paired samples which contained these 35 students. Several interesting results were found and are discussed below. Analysis showed that students have significant changes in the scores of the following six questions: question 2 (from 5.00 to 5.89, $p = 0.002$), question 6 (from 5.24 to 5.92, $p = 0.000$), question 11 (from 5.13 to 5.70, $p = 0.045$), question 15 (from 3.74 to 4.73, $p = 0.011$), question 18 (from 4.11 to 5.32, $p = 0.000$), and question 20 (from 4.84 to 5.51).

For the values of mathematics, statistics show that answers for four of the questions have significant changes. After taking the course, students were more prone to believe that mathematics could help human beings describe phenomena of the real world (q.2), that learning mathematics could help us solve problems encountered in daily lives (q.6), that learning mathematics could improve our sensibility to beauty (q.18), and that learning mathematics could help their career development (q.20).

The changes of beliefs about the connections between mathematics and real world, and between mathematics and beauty can be confirmed from their reflexions. For instance, a Ms H (first-year medicine major) was surprised when she learned that the form of the *United States Declaration of Independence*, a real-world political document of great importance, is related to that of the *Elements*. Students did pay much attention to the utility of mathematics, which can be seen from the high scores of q.2 and q.6 in the pre-test, and from the changes of the two questions in the post-test.[2] Moreover, several students, including a Ms X (second-year dentistry major), used mathematical concepts such as parallel lines or circles as metaphors to write proses in their reflexions. This could very well be the reason that they saw the connections between mathematics and beauty.

Results for the dimension of the values of mathematics are similar to those reported in the authors' previous study, showing that liberal-arts mathematics courses with contents in the history and culture of mathematics can usually strengthen students' beliefs that mathematics is useful in the real world, but it also lets them be more prone to believe that mathematics is linked to creativity and sensibility to beauty (Ying, Huang & Su, 2015, pp.19-20).

For the nature of mathematics, statistics show that answers of only two of the questions have significant changes. After taking the course, students were more prone to believe that

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[2] It seems that the students believed mathematics could solve real-world problems, although they did not have so much related experience in their professional training. Beside the examples they saw in class that could slightly influence their beliefs, our guess for the explanation to this phenomenon is that it could be due to certain 'indoctrinations' they received from their primary and secondary education, that mathematics is somehow useful, even though they did not have so much related experience. This could be the topic of future studies.

results calculated with mathematics could accurately explain many phenomena (q.11), and that proof was the only method that justifies mathematical knowledge (q.15).

The result about question 15 was naturally expected, since students had read a classic that was organised according to a strict logical sequence. That result is also very interesting compared to the authors' previous study. We found that although a liberal-arts course in mathematics with an emphasis on history and culture had many advantages, it might also confuse students about the context of discovery and that of justification, because they were also more inclined to believe that both proof and exemplification could justify mathematical knowledge stated in questions 15 and 17 (Ying, Huang & Su, 2015, pp.18-19). In the present case, however, students' beliefs had significant changes for question 15 but not for question 17, showing that they were not more inclined to believe that exemplification could justify mathematical knowledge. In other words, they were not as confused as those in the previous study.

This kind of beliefs about logic and justification can also be confirmed from students' reflexions in the present study. For instance, a Mr S (second-year pharmacy major) wrote:[3]

> I felt great when the professor used some kind of 'map' to show the context of Book I and the relations among the theorems. […] The *Elements* is a book linked by logic and has its own thinking contexts. A small change can affect everything.

Another student, a Mr T (first-year medical technology major) also wrote:

> I have gradually accepted the rigorous, un-intuitional mathematical method of the *Elements*. I was originally reluctant to try to prove propositions that could be 'taken for granted.' I felt that was only constantly circling some known facts; there would not be any new discovery so mathematics would stop improving. Later I have gradually felt that those things about foundations have their necessities to exist […].

Therefore, it can be seen that reading the *Elements* did help students see the necessity and advantages of logic.

If we try to consider Taiwanese students' typical mathematics-learning experiences, we might also understand why reading *Elements* can bring changes in their beliefs. In the culture of problem-solving and examinations, students rarely have the opportunities to appreciate the logical structure of certain mathematical theories. Learning to use problem-solving techniques and earing high scores in tests are the most important, if not the only, goals for mathematics learning. Besides, geometrical topics in Taiwanese high school textbooks are mainly treated with analytical approaches, so students are still using algebra to solve problems. Reading Euclid's *Elements* gives students a chance to slowly understand and appreciate the structure and beauty of an axiomatic-deductive system, and thus could bring the changes reported in this article.

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[3] The students in this course usually wrote in Chinese, and the English translations of their words were done by the authors.

To conclude, we believe that in a liberal-arts course about mathematical classics reading, with careful choices of reading material and related cultural backgrounds for teaching, and providing students with opportunities for discussions and reflexions, it may not only be easier for university students to see the connections between mathematics and the real world, and between mathematics and aesthetics, it may also help students see the differences between the context of discovery and that of justification in mathematics, which is especially meaningful in a society where problem-solving, not logic, is believed by many to be the most useful aspect of mathematics. We are hoping that this kind of courses can bring medical university students in Taiwan more diversified views about mathematics, and then they may, in turn, influence the general public in Taiwan.

REFERENCES

Cifarelli, V., & Goodson-Espy, T. (2001). The role of mathematical beliefs in the problem solving actions of college algebra students. In *Proceedings of the 25th Conference on The International Group for The Psychology of Mathematics Education*, *2*, 265.

Furinghetti, F., & Paola, D. (2003). History as a crossroads of mathematical culture and educational needs in the classroom. *Mathematics in School, 32*(1), 37-41.

Goldin, G. A. (2002). Affect, meta-affect, and mathematical beliefs structures. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 59–72). Dordrecht, the Netherlands: Kluwer.

Hong, H. C. (2009). *A study about gender-stereotypes threat and mathematic-related professional development for female students major in engineer at universities of Science and Technology in Greater Kaohsiung Pintung Area*. Unpublished master's thesis. Kaohsiung: National Kaohsiung Normal University. (In Chinese).

Horng, W.-S. (2000). Euclid versus Liu Hui: A pedagogical reflection. In V. J. Katz (Ed.), *Using history of mathematics in teaching mathematics* (pp. 37-48). Washington, D.C.: MAA.

Jankvist, U. T. (2011). Students' beliefs about the evolution and development of mathematics. In V. Katz, & C. Tzanakis (Eds.), *Recent developments on introducing a historical dimension in mathematics education* (pp. 123-131). Washington, DC: MAA.

Katz, V., & Tzanakis, C. (Eds.) (2011). *Recent developments on introducing a historical dimension in mathematics education*. Washington, DC: MAA.

Kjeldsen, T. H. (2011). History in a competence based mathematics education: A means for the learning of differential equations. In V. Katz, & C. Tzanakis (Eds.), *Recent developments on introducing a historical dimension in mathematics education* (pp. 165-173). Washington, DC: MAA.

Kloosterman, P., & Stage, F. K. (1991). Relationships between ability, belief and achievement in remedial college mathematics classrooms. *Research and Teaching in Developmental Education, 8*(1), 27-36.

Liu, P. H. (2003). Do teachers need to incorporate the history of mathematics in their teaching? *Mathematics Teacher*, *96*(6), 416-421.

Op 't Eynde, P., De Corte, E., & Verschaffel, L. (2002). Framing students' mathematics related beliefs: A quest for conceptual clarity and a comprehensive categorization. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 13-37). Dordrecht, The Netherlands: Kluwer.

Schoenfeld, A. H. (1985). A framework for the analysis of mathematical behavior. In A. H. Schoenfeld (Ed.), *Mathematical problem solving* (pp. 11-45). New York: Academy Press.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

Schoenfeld, A. H. (1994). Reflections on doing and teaching mathematics. In A. H. Schoenfeld (Ed.), *Mathematical Thinking and Problem Solving* (pp. 53-75). Hillsdale, NJ: Lawrence Erlbaum Associates.

Tzanakis, C., Arcavi, A. *et al*. (2000). Integrating history of mathematics in the classroom: An analytic survey. In J. Fauvel, & J. van Maanen (Eds.), *History in mathematics education* (pp. 201-240). Dordrecht: Kluwer Academic Publishers.

Ying, J. M., Huang, J. W., & Su, Y. W. (2015). An exploratory study on influences of a mathematical culture course on university students' mathematics beliefs – The case in a medical university. *Taiwan Journal of Mathematics Education*, *2*(2), 1-24.

Zheng, L. L. (2002). *The establishment and development of imperial universities in colonies – A perspective centred in Taipei Imperial University*. Taipei: Ph.D. Thesis in National Taiwan Normal University. (In Chinese).