THE ROLE OF CULTURAL ARTIFACTS IN THE PRODUCTION OF MEANING OF MATHEMATICAL KNOWLEDGE INVOLVED IN EMBROIDERY ACTIVITIES FROM THE HÑAHÑU CULTURE

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ABSTRACT

We identified mathematical knowledge involved in embroidery activities of the Hñahñu culture from Valle del Mezquital (Mexico) from a cultural-semiotic approach. The motivation that led to this research was to test the viability of a teaching model (Filloy, Rojano & Puig, 2008), which considers the social and cultural aspects of the embroidery activity. We resorted to the *Theory of objectification* (Radford, 2014) as it allows us to study the appropriation process of the mathematical knowledge involved in this activity, taking into account the cultural artifacts and the interaction between embroiderers. Among the results of this study, we can say that embroiderers establish a correspondence between the symmetries and patterns of the geometric motifs and the "symmetries" and patterns of the numerical sequence of stitches. We also found that they develop different strategies for counting and measuring the motifs, stitches and yarn strands.

Introduction

In this research we identified mathematical knowledge involved in the production of embroidery of Hñahñu culture (also called Otomí) in the region of Valle del Mezquital, Hidalgo State, Mexico. The question that guided our research was: What math knowledge is implicit in the production process of hñahñu embroidery?

We were specifically interested in the processes of acquisition and communication of this knowledge. For that purpose, we adopted a semiotic-cultural approach (Radford 2006, 2013, 2014), in which we considered that learning is comprised of active processes of acquisition of the meanings implicit in cultural artifacts and social interaction:

> It deals with making sense of conceptual objects that the student finds in his culture. The acquisition of knowledge is a process of active elaboration of meanings. It is what we will call later a process of objectification. (Radford, 2006, p. 113)[1]

We sought to answer this question considering that the findings could be useful for researchers on mathematics and culture as well as for teachers and designers of curriculum for indigenous education.

1 Previous research

In recent years, several theoretical approaches have been developed for considering the influence of context in mathematical knowledge learning. These approaches highlight

[1] Author's translation

the importance of social, cultural and historical aspects on the construction and dissemination of mathematical knowledge (D 'Ambrosio, 2002; Radford, 2006, 2013, 2014). They also consider that mathematical skills, knowledge, values and beliefs are essential part of any culture (Aroca, 2008; Oliveras & Albanese, 2012; Silva, 2006).

In this research, we made use of results of several studies that work with the mathematical knowledge involved in the construction of the geometric patterns of some embroideries (Gerdes, 2012; Gilsdorf, 2012; Sánchez, 2014; Soustelle, 1993). According to them, in carrying out this activity, embroiderers resort to various forms of the notion of symmetry; they combine aspects equally inspired by decorative aims, spiritual purposes and abstraction.

Some other studies dealing with the development of the geometry in several cultures conclude that mathematics and geometry are developed by all human group (Diaz Escobar & Mosquera, 2009; Fuentes, 2012; Micelli and Crespo, 2011; Scandiuzzi & Regina, 2008; Sufiatti, Dos Santos Bernardi & Duarte, 2013; Urban, 2010). They affirm that geometry arises from the survival needs of the people, and it is unfolded in the ways people organize and build their homes, in the ways they produce their basketry, ceramics and textile, and in all the activities they do for satisfying their needs.

Despite the large body of research dealing with the study of mathematical knowledge involved in the activities of specific social and cultural groups, it is still needed to address the acquisition of meaning of mathematical objects involved in such activities. Our study sought to contribute on this direction.

2 Theoretical approach

We adopted the theoretical approach of objectification (Radford, 2006; 2013; 2014) as it allowed us to study the acquisition of mathematical objects from a sociocultural perspective. According to this theory, the meaning of mathematical objects has two essential aspects: on the one hand, it is a subjective construction, linked to the history and experiences of each person; and, on the other hand, it is a cultural construction, previous to the subjective experience, in which the objects have their own values and cultural content.

Thus, in order to study the acquisition process of the meaning of mathematic objects is important to take into account two sources: the cultural artifacts with which people carry out their activities, and the social interaction. The artifacts are repositories of historical knowledge from cognitive activity of past generations. Radford argues that 'the human being is profoundly affected by the artifact: because of the contact with it, human being restructures its movements and creates new motor and intellectual capacities, such as anticipation, memory and perception' (Radford, 2006, p. 113). Regarding the interaction, he claims that "objects can not make clear the historical intelligence embodied in them. For this it is necessary to use them in activities, and the contact with other people who know 'read' such intelligence and help us acquire it" (Radford, 2006, p. 113).

3 Methodology

The methodological design had two phases. In the first one, we studied the "formal" mathematical knowledge that can be identified in a collection of 271 hñahñu traditional geometric motifs, which formed part of this research. We used the analytical tools proposed by Jaime and Gutiérrez (1995) to identify the *systems of generating transformations of figures* from 271 embroideries (Barquera & Solares, 2016).

In the second phase, we used an ethnographic methodology and developed field observations and interviews for analyzing the mathematical knowledge that women really use for the production of their embroideries. The field observation entailed accompanying the embroiderers during their daily work life, in the places and moments of the day when they usually embroider. We were interested in knowing how they learned to embroider, who taught them, what geometric motifs were the first that they learned to embroider, in what moments of the day and under what conditions they perform their work. We worked with 10 experienced embroiderers, who began with this work activity since childhood. For the interview we chose 4 of them, the more willing for being interviewed. The interview consisted of presenting one embroidery design and asking to the interviewee to reproduce it in a piece of fabric, according to a frequent custom of sharing geometric motifs. This phase is still ongoing.

4 Results

In this presentation we focus on some of the results of the second phase of the study. While research is still ongoing, the results presented show the kind of mathematical knowledge that hñahñus embroiderers put into action to carry out their activities.

4.1 Some results of phase 2: the ethnographic study

Among our results, we found that embroiderers structure their strategies considering both the characteristics of their work tools (size of fabrics, types and colors of yarns, etc.) and the characteristics of the geometric motifs that they construct. The uses of material resources (the work tools) and the conceptual resources (for instance counting, perception of symmetries and hñahñu cultural symbolism) determine strategies for building the geometric patterns of their embroidery.

Here we refer to the interviews of Isabel and Sabina, two older embroiderers, to show some strategies of thread counting resorted to when carrying out their activity. At the beginning of the interviews they were presented with the embroidery shown in Figure 1. This embroidery was chosen because is rich in geometric figures and has a number of symmetries. Isabel and Sabina were asked to reproduce it in a piece of fabric, as they do in the exchanges that commonly occur between the embroiderers.

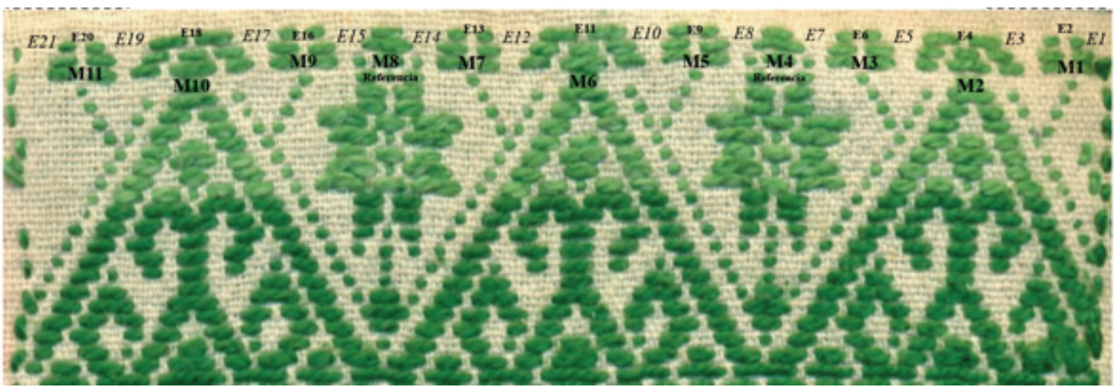

Figure 1. Embroidered motifs

In Figure 1 letter "M" indicates a geometric motif that forms part of the embroidery. In this case, we have 11 motifs, from which M2, M6 and M10 are rhombuses; M1, M3, M5, M7, M9 and M11 are motifs representing "leaves," as the embroiderers call them; M4 and M8 are "stars."

The distances between the motifs are counted as the number of fabric threads between them. In Figure 1, these distances are indicated by letter E. Distances E1 to E21 are measured by considering the first line of the embroidery. There are distances of two types: the first type separates parts of the same motif, for example, into the leaf M1 there is a distance of E2 between the two parts of the leaf; the second type measures the distance from one motif to another. In this case, E2, E4, E6, E9, E11, E13, E16, E18, E20 are distances of the first type and each measures 2 threads. E1, E3, E5, E7, E8, E10, E12, E14, E15, E17, E19 and E21 are of the second type, each measuring 10 threads.

Both embroiderers indicated that the count of the threads of the first line determines the success or failure of the reproduction of the given embroidery. The following episode shows what Sabina said about the importance of well counting in the first line:

> Interviewer: What is the hardest thing in embroidery?
>
> Sabina: The start. Once you have started, it's easier ... The start is more difficult because you have to count. In the second round you can just increase to go forming little leafs...

Isabel and Sabina counted by considering that the fabric is a grid. They proceeded line by line of the grid. Both Isabel and Sabina began embroidering the first line from right to left and, once reached the left side, they changed direction: now from left to right; and so on.

Isabel counted both the threads forming the motifs and the distances that separate them. Whenever she counted the threads of one motif, she verified that the similar motifs measured the same. Figure 2 shows the results of the count carried out by Isabel in the first line of this embroidery.

10 4 2 4106 2 6 10 4 2 4 10 6 10 4 2 4 10 6 2 6 10 4 2 4 10 6 10 4 2 410 6 2 6104 2410

Figure 2. Sequence of numbers obtained by Isabel in counting the threads of the embroidery first line

In this figure, the numbers with a line represent distances between motifs, measured by counting the number of threads of the fabric that are in sight. The distances of the first type (between parts of the same motif) are highlighted in green and the second type (between different motifs) in yellow. The numbers having no lines represent the motifs lengths, measured by counting the number of threads of the fabric under the stitches.

Isabel showed ease in producing the embroidery. She was meticulous in her counts and systematically verified them. In her explanations, she said that located similar motifs to one already counted and gave the corresponding stitches on piece of fabric. So, she did not need to do all the count, thread by thread. Moreover, she established that the "stars" (M4 and M8) were benchmarks, allowing her to look for patterns and symmetries. Figure 3 shows the sequence numbers determined by Isabel and its correspondence with the benchmarks.

10 424 10 626 10 424 10 6 10 424 10 626 10 424 10 6 10 424 10 626 10 42410

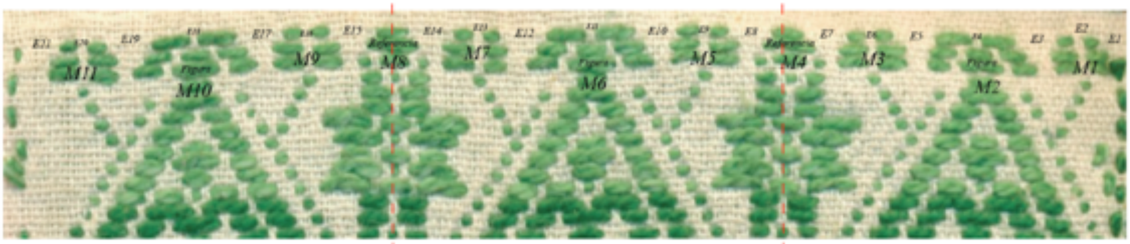

Figure 3. Symmetries and benchmarks in the embroidery

The embroidery symmetries allowed Isabel to reproduce it without counting all the motifs. We can say that Isabel established a correspondence between the geometric symmetries and patterns of the embroidery and the "symmetries" and patterns of the numeric sequence, line by line.

These symmetries also allowed Isabel to correct mistakes. For example, when embroidering the first three lines of the upper part of M2 and M6 motifs (rhombuses), Isabel counted as follows:

6 2 6	6 2 6
4 2 6 2 4	4 3 5 2 4
3 3 10 2 4	4 2 10 2 4
M6	**M2**

The red numbers indicate errors in stitching. To correct her mistakes, Isabel compensated them in the immediate following stitches. In M2, for example, to correct the stitch where she spent his needle beneath 3 threads (instead of 2), in the next stitch Isabel spent his needle above 5 threads (instead of 6). Thus, Isabel controlled the overall size of the motifs (motifs M2 and M6 measure 18 threads in the second line). Isabel considered not only the numeric characteristics of the motifs (number of threads), but also their symmetries and geometric regularities.

Final remarks

The practical knowledge of hñahñu embroiderers are deeply affected by artifacts, both material and conceptual, that they use to carry out their activities. By using tools such as threads, needles and fabrics, embroidery women mobilized and provide with meanings to their knowledge on counting, compensation through addition and subtraction, and numerical patterns and symmetries. They construct (and we have found that also share as a community) strategies that allow them to efficiently perform their activities.

Our research raises the possibility of characterizing a "practical geometry" used by the embroiderers, whose study can provide of teaching alternatives for school geometry. The richness and variety of meanings of mathematical objects found in embroidery activities show the educational potential of the contexts that can be exploited for the formal teaching of geometry in schools of hñañhu communities. We consider important that school recognize the practical geometry developed in practical activities, where geometric transformations and symmetries are used through embroidery tools not through educational tools, such as ruler and compass, or dynamic geometry software.

Finally, it is important to say that to unfold the details of the acquisition process of mathematical knowledge of hñahñu embroiderers is necessary to deepen the analysis of the relations between subjectification and objectification in this activity (Radford, 2014). This analysis is still ongoing (Solares and Barquera, forthcoming).

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