MACHINES DESIGNED TO PLAY NIM GAMES

Teaching supports for mathematics, algorithmics and computer science (1940-1970)

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ABSTRACT

This article deals with Nim games and machines built to play against a human being between the 1940s and the 1970s. They were designed not only to entertain, but also to explain concepts in mathematics, algorithmics and computer science to a general public. Moreover, they were exhibited during fairs or science shows and manufactured later for personal use. Nim games are take-away games without chance whose winning strategy relies on the binary system, easily characterized by bistable circuits called flip-flops. The first electromechanical Nim player machine, called The Nimatron, was invented in 1940 and exhibited during the New York World's Fair. Its success led to the construction of another electromechanical machine (1951). Then electric or purely mechanical inventions were patented for their cheaper production cost and their pedagogical aspects to understand elementary level instructions in computers as well as the rules of the binary system and notions of Boolean algebra. This article provides examples of such machines and shows their pedagogical interest.

1 Introduction: what is a Nim game?

Nim game falls within the class of combinatorial games. In a combinatorial game, there are only two players, playing alternately. Usually, there are a finite number of positions and the information is complete – which means both players know what is going on at any moment of the game. There are no chance moves such as rolling dice or shuffling card and in the *normal* play convention the player who finds himself unable to play loses.¹

Nim game is considered as a take away game: usually three (or more) piles of counters are set on a table. Each pile contains a different number of counters. Alternatively, both players select one of the piles and remove as many counters as they want: one, two... or the whole pile. The first player who takes the last counter(s) wins the game.

Nim game was first mentioned under this name in a 1901 article published in the *Annals* of *Mathematics* and written by a mathematician from Harvard: Charles Leonard Bouton (1869-1922). In this article, Bouton gives the complete mathematical solution to Nim (Bouton 1901), i.e. a strategy to win every game; first, each pile of counters must be written in the binary scale. Then these binary numbers are placed in three horizontal lines so that the units are in the same vertical column. The sum of each column is calculated and if all of them are congruent to 0 mod. 2, the position left on the table is called a *safe combination*. Such positions should be reached at each move in order to win the game, and as soon as we reach one, it is possible to obtain another one on our next move, but not for our opponent. Indeed,

¹ In the *misère* play convention the player who finds himself unable to play wins.

safe combinations verify the following properties: "I. if A leaves a safe combination on the table, B cannot leave a safe combination on the table at his next move. II. If A leaves a safe combination on the table and B diminishes one of the piles, A can always leave a safe combination" (Bouton 1901, p. 36). This use of the binary system is the basis of the functioning of the future machines designed to play Nim. Thus the game of Nim is particularly suited to the use of the usual binary computing elements, such as bistable circuits (flip flops). Later, *Geniacs*, simple electronic brain machines, was to be designed for educational purposes to translate from decimal notation to binary notation and vice versa, but also to add, multiply and compare numbers in the binary system (Geniacs 1955a).



Figure 1. A young lady playing against Nimatron (Condon, 1942, p. 330)

Bouton's article is considered as the starting point of the relatively recent (twentieth century) mathematical theory of combinatorial games. The theory was developed among the mathematical field and became a beautiful abstraction with John Conway's surreal numbers in 1976. Conway's construction admirably generalizes Dedekind cuts and his theory distances itself from its subject of study, namely games. Meanwhile, Nim game was introduced to the general public through machines designed to play against human players – and always win – but the initial ambition was to explain to people how machines operated, and which mathematics was behind. We will see that the first machines were so big that they were exhibited during fairs or science shows. Then, personal machines, totally mechanical or slightly electric, were designed, especially to explain basis of computer science (and also to reduce the production costs). Nowadays, these machines are of highly interest since the new French educational program in mathematics high school classes, which will come into effect in September 2016, recommends to develop the new theme "algorithmic and programing" through examples of games to help pupils to break down a problem, recognize ways of thinking, translate logical thinking to conditional instructions and create an algorithm that solves the problem.

2 Machines exhibited during fairs and science shows

In the spring of 1940, an electromechanical Nim player machine weighing a ton, called *The Nimatron* (see Figure 1.), was exhibited at the Westinghouse Building of the New York World's Fair and played more than 100 000 games (and won 90 000 of them). Two members of the Westinghouse Electric Company staff invented it during their lunch break. Condon, the signatory of the US Patent (Condon et al. 1942), underlined the entertaining purpose of the

Nimatron, but also specified that it could illustrate "how a set of electrical relays can be made to make a decision in accordance with a fairly simple mathematical procedure" (Condon 1940, p. 330).

The Nimatron was set to play Nim with 4 piles containing up to 7 counters. The human player began the game, and only 9 initial configurations were possible (due to space limitations), each of them was unsafe combination, so that the human player had a chance to win. The lamps (which can be seen on Figure 1) a1 to a7 (7 lamps for the 7 counters of the column a), b1 to b7, c1 to c2 and d1 to d7 were connected in circuits, which were controlled by relays A1 to A7, B1 to B7, C1 to C7 and D1 to D7 respectively, each of them controlled by a master relay A, B, C and D (one for each column). Other relays, AZ, BZ, CZ and DZ were actuated when the number of energized lamps in the corresponding columns a, b, c and d, respectively, contained a zero power of 2; the relays AF, BF, CF and DF were actuated when the number of energized lamps in the corresponding columns a, b, c and d, respectively, contained a first power of 2, and the relays AS, BS, CS and DS were actuated when the number of energized lamps in the corresponding columns a, b, c and d, respectively, contained a second power of 2 (the maximal number of lamps being 7, 111 in binary). To properly play the game, the machine had to determine whether any power of 2 was contained in the number of energized lamps in an even or an odd number of columns. If the three powers of 2 appeared in an even number - which meant that the human player left a safe combination – the machine played randomly, otherwise it analyzed in which column a change should be made to obtain an even number of the three powers of 2.

In 1942, the Nimatron was exhibited for the last time at the convention of the Allied Social Sciences associations in New York City under the sponsorship of the American Statistical Association and the Institute of Mathematical Statistics. Then the machine belonged to the scientific collections of the Buhl Planetarium in Pittsburgh (Condon 1940, pp. 330-331).

A few years later, Ferranti, the electrical engineering and defense electronics equipment firm, designed the first digital computer dedicated to play Nim, *The Nimrod*. It was exhibited at the Festival of Britain (Exhibition of Science) in May 1951 and afterwards at the Berlin Trade Fair (Industrial Show) in October of the same year. These exhibitions were a great success and many witnesses related that the most impressive thing about the Nimrod was not to play against the machine but to look at all the flashing lights which were supposed to reflect its thinking activity. It had even been necessary to call out special police to control the crowds (Gardner 1959, p. 156). This particular display was built for the purpose of illustrating the algorithm and the programming principles involved. The instructions followed by the Nimrod were written on the left side, as shown in Figure 2. More over, a booklet was available for visitors,² for the price of one shilling and six pence, which contained a lot of information about automatic digital computers in general. The introduction states: "the

² The booklet, released in 1951, *The Ferranti Nimrod Digital Computer*, is available at the following address: <u>http://goodeveca.net/nimrod/NIMROD_Guide.html</u>

machine has been specially designed to demonstrate the principles of automatic digital computers [...] the booklet has been prepared for those persons who desire to learn a little more about computers in general and of Nimrod in particular." Explanations of main notions, such as "electronic brains", "calculating machines", "automatic computers", 'automatic sequence control" and characteristics of automatic computers (calculation, memorization and making decisions) are given in the first part of the booklet. The second part is devoted to Nimrod functioning ("some details of the machine", "the way in which Nimrod plays Nim"...). "Nimrod has been designed so that it can play the game of Nim with an opponent from the general public or it can be given a "split personality" so that for demonstration purposes it will play a game without an opponent" (Nimrod 1951).

For instance, the instructions run by the program while Nimrod plays Nim are the following:

- 1. Examine the column specified by the column counter. If this is even transfer control to 7.
- 2. Examine the digit specified by the column and heap counters. If this is 1 transfer control to 4.
- 3. Add 1 to heap counter. Transfer control to 2.
- 4. Examine heap selected memory. If a heap has previously been selected transfer control to 6.
- 5. Operate heap selected memory.
- 6. Substitute modifying number in heap specified by heap counter.
- 7. Deduct 1 from column counter. If this does not complete examination of columns transfer control to 1.
- 8. STOP.

This terminology is specific to algorithmic and programming: a set of instructions is provided, ordered in a logic way and described with conditional statements proper to Boolean data type. In a common language that everyone can understand, the procedure executed by the machine is thus explained (the earliest high-level programming languages with strong abstraction were written in the 1950s).

Nimrod booklet also contains a glossary with definitions of the terms used. The authors highlight the quick evolution of automatic digital computers during the 1950s and want to clarify the new terminology that arose for describing the machines. Once again, there was a real will to use the simplest possible explanations in order to embrace the widest field. We do not know for sure if the Nimatron or the Nimrod had a pedagogical impact in mathematics education. But few years later, smaller machines, cheaper to produce, appeared for

pedagogical purposes, and were intended for a wide audience.³ They are the subjects of the next section.

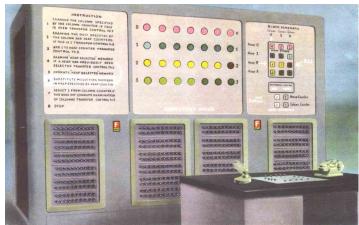


Figure 2. Drawing of the Nimrod, with instructions on the left side. "The display in front of the machine is used to demonstrate the process involved when Nimrod carried out a move" (Nimrod, 1951)



Figure 3. *Popular Electronics* cover, January 1958

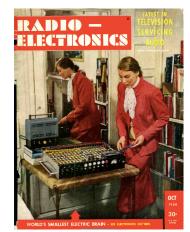


Figure 4. *Radio Electronics* cover magazine, October 1950

³ Other machines designed to play Nim were created between 1941 and 1958, but in a more mathematical sphere. In 1941, an assistant professor of mathematics at the University of California at Los Angeles (Gardner 1959, p. 156), Raymond Moos Redheffer, improved considerably the Nim-playing machine. (Redheffer 1948) To our knowledge, Redheffer's machines were not exhibited to a broad public, consequently they were less known. In 1952, engineers from W. L. Corporation, Hubert Koppel, Eugene Grant and Howard Bailer, developed a lighter machine than Nimatron or Nimrod, as it weighed less than 25 kg and cost \$2000 to build. We would like to note that machines mentioned in this note had no clearly expressed pedagogical or educational aspirations and were probably not much widespread. Nevertheless, Pollack's DEBICON (1958) can be found on *Popular Electronics* magazine cover, which soon became the "World's Largest-Selling Electronics Magazine". (see Figure 3)

3 Machines designed for personal use

Since 1945 there has been interest in helping people understand how automatic machines reason, calculate, and behave.

And we know that equipment that you can take into your hands, play with, and do exciting things with, will often teach you more, and give you more fun besides, than any quantity of words and pictures. (Geniacs, 1955a, p. 2)

This justifies the educational toy Geniac designed and marketed by Edmund Berkeley and Oliver Garfield between 1955 and 1958. Edmund C. Berkeley (1909-1988) was a mathematician, insurance actuary, inventor, publisher, and one of the founders of the Association for Computing Machinery (ACM) (Longo, 2015). In 1949, he published a book titled Giant Brains or Machines That Think, which was the first explanation of computers intended for a general readership.⁴ In the 1950s, Berkeley developed mail-order kits for small, personal computers such as Simple Simon and the Brainiac. At a time when computer development was on a scale barely affordable by universities or government agencies, Berkeley took a different approach and sold simple computer kits for average Americans. He believed that digital computers, using mechanized reasoning based on symbolic logic, could help people make more rational decisions. These considerations show that the idea of handling objects to help the teaching of mathematical – here logical – concepts is not new. It has been shown (Brougère 1995) that until the 20th century, games were not considered as direct educational tools. "Recreation is essential but game has no status beyond it." (Brougère, 1995, p.135) This position has been evolving and now it is acknowledged that to construct a mathematical concept, a first phase of action is essential to build a mental representation (even if this handling phase alone cannot be enough to learn, and a mediation with the teacher - or someone who knows - is necessary). Construction kits such as Simple Simon and Geniac provide the necessary material to grasp mathematical and easy computer science notions and could be revisited nowadays in mathematic classes to illustrate the procedure of an algorithm.

When it was released in 1950, *Simple Simon* was the "World's Smallest Electric Brain" (see Figure 4). It weighed 39 pounds and showed how a machine could do long sequences of reasoning operations. "The machine itself has been demonstrated in more than eight cities of the United States."⁵ (Geniacs 1955a, p. 1) "He will be useful in lecturing, educating, training and entertaining." (Berkeley & Jensen 1950, p. 29) By 1959, over 350 sets of Simon plans had been sold, but it cost over \$300 for materials alone, and Berkeley and Garfield admitted: "it is therefore too expensive for many situations in playing and teaching." That is the reason why they worked four years long to develop a really inexpensive electric brain: *Geniac*, a construction kit costing less than \$20.

⁴ His journal *Computers and Automation* (1951-1973) was the first journal for computer professionals.

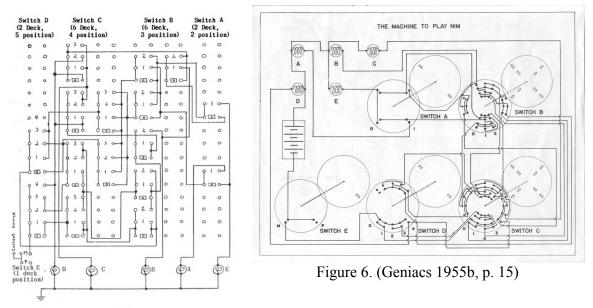
⁵ Simple Simon was exhibited in New York, Seattle, Philadelphia, Boston, Washington, Detroit, Minneapolis, Pittsburgh, and other smaller cities. The fact that Berkeley could take Simon from place to place meant that students and other non-experts could have first hand contact with automatic computing equipment "for real".

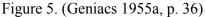
3.1 Geniacs

The name Geniac stood for "Genius almost-Automatic Computer" (Geniacs 1955a, p. 2). The construction kit consisted in 30 small electric brain machines – each one being a Geniac – which could be made with very simple electrical equipment. The guide supplied with the kit first gave a general description of the material and the way the different components worked.

One of the proposed problems was to design a Geniac that could play Nim in normal convention with four piles of matches, containing respectively 4, 3, 2 and 1 matches. The solution of the wiring is shown Figure 5 and Figure 6.

Besides the construction of machines to play games such as Nim, Tit-Tat-Toe or to answer recreational mathematical riddles such as the Two Jealous Wives, the kit provided other Geniacs to illustrate more purely mathematical problems such as the adding machine, the multiplying and the dividing machines, or the machine for arithmetical carrying (Geniacs 1955a, p. 4). A 1958 advertisement explained all the interesting aspects of Geniac and highlighted its popularity, its pedagogical interest and its low price (see Figure 7).





For only \$19,95, Geniac offered a complete course in computer fundamentals used by thousands of colleges, schools and private individuals. It seems that in October 1958, more than 30,000 Geniacs kit were in use by satisfied customers. The advertisement clearly underlined the pedagogical purposes of Geniac for understanding notions of mathematics and computer engineering:

Geniac is a genuine electric brain machine, not a toy. The only logic and reasoning machine kit in the world that not only adds and subtracts but presents basic ideas of cybernetics, Boolean algebra, symbolic logic, automation, etc. So simple to construct that a twelve-year-old can construct what will fascinate a Ph.D. (Geniac 1958, p. 29)



Figure 7. Advertisement for Geniac, (Geniac 1958, p. 29)

Berkeley designed Geniac to be a tool for educators and it seemed it had some success in this area (Longo 2015). In 1958, the *Mathematical Gazette* published an article of a mathematics teacher, Martyn H. Cundy, who developed plans for a binary adding machine for classroom use. The machine could add two binary numbers of two digits or of three digits (with or without carry). Cundy credited his work to Geniac that taught him the fundamentals for building his own machine and justified that some knowledge of binary arithmetic should be part of the mathematical equipment of the normal grammar-school pupil (Cundy 1958, p. 272). In 1956, in the magazine about education *Phi Delta Kappan*, Daniel Davies⁶ covered "breakthroughs" in educational administration, in which he detailed areas where new developments were having impacts on education. These included mathematics, for example game theory or binary numbers systems, and Davies claimed: "Boolean algebra is already at work in problem solving. One firm is advertising a kit for setting up an ingenious device known as Geniac which can quickly solve a wide range of problems involving multiple choices." (Davies 1956, p. 276) Moreover, as the 1958 advertisement stressed it: "In addition

⁶ Daniel R. Davies was executive director between 1954 and 1959 of the UCEA (University Council for Educational Administration), an organization aimed to improve the professional preparation of educational administrators.

to its value as a source of amusement and education the kit exhibits certain technological features that may have widespread implications in other areas." (Geniac 1958, p. 28)

The use of Berkeley small electrical brain machines in classroom and how they impacted the teaching of mathematics is difficult to ascertain and this part of the work is still in progress. However, popularity of Geniac during the 1950s was an acknowledged fact, as a lot of electronics magazines or science journals can prove it.⁷

3.2 Machines or computer type devices patented

We have already mentioned that the main problem of electromechanical machines such as *Nimrod* and *Nimatron* was, first of all, their size, and also their expensive production cost: "Standard electronic computers [...] have been both bulky and expensive." (Du Bosque 1962) That is why during the 1960s electric or purely mechanical inventions, in the same vein as Geniac,⁸ were patented for their "durability and reliability in use." (Weisbecker 1968) For example, Joseph Weisbecker's invention related to a unique mechanism in the nature of a computer for use as a toy, game, puzzle or educational device, "say to illustrate computer operation and logical techniques [...]" (Weisbecker 1967).

Moreover, these inventions permitted "the achievement of elementary level instructions in computers." (Godfrey 1968) "[...] the invention is an educational device for indicating the best play to be made [...]" (Du Bosque 1962). "It is a principal object of the present invention to provide improved educational game apparatus which permits the learning of strategy techniques, logic methods, and mathematical systems." (Morris 1971) Authors of such patents also justified the interest of their machines by filling the gap left "in the ability of the student to understand and comprehend what a computer is all about" since the venue of high-speed electronic digital computers (Godfrey 1968). They emphasized the importance of presenting an invention that would provide a game and a teaching aid "so as to attract persons of higher intellectual level, while maintaining relative simplicity for attention arresting use as a toy by relatively young children." (Weisbecker 1968)

3.3 A marketed patent: Dr. Nim

Some of these inventions were marketed and sold as family parlor games. *Dr. Nim* is an example: it was manufactured by E.S.R. Inc., a company specialized in education toys during the 1960s, and was equivalent to a single pile Nim game where 1, 2 or 3 counters could be removed at each move. Dr. Nim was played by one player – against the machine – and offered several starting positions: the game could be played in normal or in misère convention (the last player to move loses) and the initial number of marbles could vary between 9 and 20 (see Figure 8).

⁷ For example *Popular Electronics* or *Galaxy Science Fiction* magazines. A Ngram research in the English Google books corpus shows a net increase of the use of the term "Geniac" between 1955 and 1960: https://books.google.com/ngrams/graph?content=Geniac&year_start=1940&year_end=2000&corpus=15&smoot hing=3&share=&direct_url=t1%3B%2CGeniac%3B%2Cc0

⁸ By the way, during the early 1960s, Berkeley created other electric brain machines with Brainiacs and Tyniacs kits.

Dr. Nim device included a plurality of flip-flops – bistable circuits (see Figure 9) – being moved by marbles when they fell down, "so as to allow mathematical computations to be effected upon binary numbers to which the flip-flops are set." (Godfrey 1968)

Inclusive among the concepts which may be explained and understood by this computer invention are the following: the binary number system; the simplicity for machine design using binary arithmetic; [...] the rule for binary counting and addition; modular arithmetic; the use of two's complement arithmetic to achieve subtraction with only its add capability; [...] binary multiplication. (Godfrey 1968)

Like the Nimrod, Dr. Nim provided a manual of fully detailed instructions (23 pages, A4 paper sized) with the rules of the game, its variations, how it was programmed... and also deeper considerations such as "can machines really think?" Few pages were devoted to the explanation of Boolean algebra in use behind flip-flops mechanisms. A capital letter was assigned to every flip-flop (A, B, C, D and E) and a bar was put over the letter when the flip-flop was open, for instance \overline{A} . Then, every number of marbles left in the top row of the machine was written in the form of equation, for example: when 13 or 9 or 5 or 1 marbles are left in the top row, the corresponding flip-flop configuration is $\overline{A} \ B \ C \ D \ E$. Instructions given to the machine could be expressed with Boolean algebra operations *and* and *or*.



Figure 8. Dr. Nim red plastic board with white flip flops, owned by the author

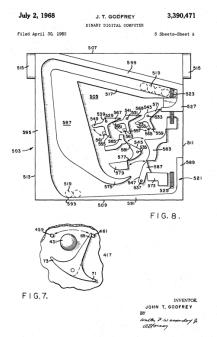


Figure 9. Patent of a binary digital computer (Godfrey 1968)

At the beginning of the 1970s, other inventions were patented and "relate to educational game apparatus", for example a "computer-controlled apparatus for playing the game of NIM" (Morris 1971), but with the advent of electronic toys, the number of purely mechanical inventions declined. Moreover, the increase of personal computers favored the development of programs, and by the middle of the 1970s one could find game programming books.

4 Conclusion

In 1973 one of the first compilations of computer games in BASIC programming language was published: *101 BASIC Computer Games*, in which the Nim game was presented. The author, David Ahl, explained this interest in computer games by the expansion of minicomputers and timesharing networks that enlarged an emerging group of "computer hackers and of people who were furtively writing and playing game at lunchtime, before and after works on their employers' computer." (Ahl 1978, p. x) The study of first programs created to play Nim would go beyond the period covered in this article; however, it should be noted that the PCC (People's Computer Company),⁹ created in the early 1970s, was one of the first organizations to recognize and advocate playing as a legitimate way of learning. PCC recognized the potential of BASIC and helped installing computers for children in libraries or schools to encourage hands-on learning approach.

In the 1980s, construction of mechanical Nim playing machines was still of interest, as "there are no commercial teaching materials that provide concrete modeling of Boolean algebra" (Cohen 1980) and that such mechanical models could render this algebra intelligible. Nowadays, Nim game is set for a comeback in French educational program in September 2016 as a recreational application to tackle algorithmic and programing in mathematic high school classes, and the creation of machines such as Geniacs or a simpler Dr. Nim could help pupils to acquire methods that build their algorithmic knowledge and develop skills in problem solving.

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⁹ In honor of Janis Joplin's rock group Big Brother and the Holding Company (Levy 1984, p. 136).

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