FATHER PADILLA'S *ARITHMETICA PRACTICA* (1732) IN ITS CULTURAL COLONIAL GUATEMALAN CONTEXT

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ABSTRACT

In this article I focus on the oldest extant mathematics book published in the Colonial period in Central America: Padilla's (1732) *Arithmetica practica*. In the first part of the article, I briefly recount my encounter with Padilla's book and the contextual problems that I found in order to offer a facsimile edition of it. In the second part, I define the historiographic approach that I follow. I argue that to inquire about the history of mathematics is to inquire about mathematics as it was imagined, thought, and practised within a certain cultural context. But I also argue that we can go further: I contend that a certain cultural historical context can be understood only if we understand its intellectual life, and more specifically its mathematics. It is in this second line of thought that I wish to see how the book fit and responded ideologically to the constraints generated by the economic, political, and educational colonial structures. To do so, before dwelling on the book and its author, I discuss the cultural context of Padilla's *Arithmetica*.

1 Introduction

In 1732 the book "Noticia Breve de Todas las reglas mas principales de la Arithmetica practica con q ſe puede deſatar, no ſolo las demãdas ordinarias, ſino tãbien muchas difficultoſas, que de otra ſuerte ſolo por la Algebra ſe respondieran" [Short Notice of all main rules of practical arithmetic with which to solve not only the ordinary questions, but also difficult ones that otherwise would be solved by algebra] was published in Santiago of Guatemala. It was published by the printing press of Ignacio Jacobo de Beteta. The author of the book was Juan José de Padilla. Padilla's *Arithmetica* is the oldest extant mathematics book published in the Colonial period in Central America. There is, to my knowledge, only one copy of the book. This copy is kept at the *Museo del Libro Antiguo* in the colonial city of Santiago of Guatemala, known today as Antigua Guatemala—about 50 km from the current capital of the country.

My encounter with the book was rather accidental. One Sunday in the late 1980s, I was visiting Antigua Guatemala and ended up in the *Museo del Libro Antiguo*. While browsing the collection of old books I noticed Padilla's *Arithmetica*. The director of the museum was not there and I was unsuccessful in convincing the personnel of the museum to open the bookshelf door to have a look at the book's contents. I left a note addressed to the director and came back the next week. At the time, I was the director of the mathematics department of the Escuela de Formación de Profesores de Enseñanza Media (EFPEM, a High School Teachers Training Centre) of the Universidad de San Carlos. The historical prestige of the university in the intellectual development of the country was crucial element to how events transpired. When I arrived at the museum, the director greeted me, and unlocked and opened the shelf. I quickly went over the contents. After the visit to the museum, the idea of a facsimile edition of the book started taking place. The museum was part of the Ministry of Anthropology and

History. I drafted a collaboration letter between the Faculty of Humanities (of which EFPEM was a part) and the ministry. Once the letter was signed, I had access to the book, under certain conditions. EFPEM hired a professional photographer with whom I travelled to Antigua Guatemala several times a week. The book was made available to us if the climatic conditions were deemed suitable for having the book exposed. After that, each page was photographed and printed, and the material was prepared to go to the press. That was the second part of the project. Unfortunately, I was not able to secure the money to print the 200 copies that had been planned. At the time, I was moving to Canada for one year, to work at a research centre of the Université du Québec à Montréal. I decided to leave the material to a professor of the Universidad de San Carlos with the hope that he would continue with the project. I kept with me the photo rolls. A few years later, when CDs were invented, the photos were saved on CDs, and some years later, technology permitting, transformed into PDFs. The idea of a digital version of Padilla's book was finally possible. The PDF can now be downloaded (http://luisradford.ca/publications/).

2 Historiography: mathematics and the cultural context

The relationship between a cultural context and the ideas that emerge and evolve in such a context has been a crucial question in disciplines such as anthropology (Geertz, 1983), epistemology (Foucault, 1966, 1969), and psychology (Luria, 1931, 1934). Yet, supported by rationalist epistemologies, mathematical ideas have long been mainly considered as independent of their cultural context. While it is true that within this tradition (which, for obvious reasons, we may term *a*culturist), it is conceded that the environment may have some influence on the evolution of mathematics (e.g., by accelerating or deaccelerating it), the cultural context is, at the same time, considered as something that cannot modify the mathematical content. That is, the context cannot determine the essence of mathematics and its objective nature.

The aforementioned relationship between context and mathematical ideas has been theorized in different ways. For instance, the reconstruction of mathematical ideas has been considered as having its internal law, while the context in which mathematics emerges and develops is assumed to have a developmental law that is external vis-à-vis the internal one. This is the view that Lakatos adopts in *The methodology of scientific research Programmes.* In the famous *Internal and external history* section of this book, Lakatos argues that "Rational reconstruction or internal history is primary, external history only secondary" (Lakatos, 1978, p. 118). Piaget and Garcia (1989) took a similar path. Although it was recognized in their work that society provides mathematical and scientific objects with specific meanings, Piaget and Garcia traced a clear frontier dividing the social and the individual. For them, a distinction must be made between mechanisms to acquire knowledge and the way in which objects are conceived by the subject. In a concise and clear phrase, they said: "Society can modify the latter, but not the former" (1989, p. 267; for a more detailed discussion, see Radford, 2000; Furinghetti and Radford, 2008). The aforementioned distinction between the social and the mathematical finds, I believe, its most tremendous tension in the work of Glas (1993).

Recent historiographical approaches have nonetheless stressed the local nature of mathematics and the manner in which mathematics is conceptualized and practised. In these approaches, context and mathematics cannot be separated (e.g., Høyrup, 2007; Lizcano, 2009; Rowe, 1996). To inquire about the history of mathematics is to inquire about mathematics as it was imagined, thought, and practised within a certain cultural context. But we can also go further and argue the opposite, namely that a certain cultural historical context can be understood only if we understand its intellectual life, and more specifically its mathematics. This is the line of inquiry that I follow in this article. More precisely, I wish to see how Padilla's *Arithmetica* fit and responded ideologically to the constraints generated by the economic, political, and educational colonial structures. To do so, before dwelling on the book and its author, in the next section I discuss the cultural context of Padilla's *Arithmetica*.

3 The cultural context of Padilla's *Arithmetica*

Padilla published his *Arithmetica* in the city of Santiago de Guatemala, which was founded in 1543. Santiago de Guatemala was one of the main political and military centres of the Spanish colonies—the two other major centres were in Mexico and Peru. Santiago de Guatemala hosted the Audiencia de los Confines, the Capitanía General, and other institutions whose goal was to regulate and control life in the colony.

By the early 18th century, when Padilla published his book, the Spanish colonizing apparatus had reached an extreme degree of sophistication. In its beginning, such an apparatus was organized around the concept of "encomienda." The encomienda was a political concept of government. Its theoretical foundation rested on the alleged social and natural inferiority of the Native people. Its real practical basis was to repay the conqueror by making him the guardian of the conquered land (Barbosa-Ramírez, 1971, p. 43). Conquerors received a certain number of Native people to be placed under their tutelage. As a result, they became "encomenderos," or trustees of those assigned to them. An encomendero had to oversee the Christianization of "his" Native people and organize their work on the lands. In exchange, the Native people entrusted to an encomendero had to pay him an annual tribute in cash, fruits, products of the land, and personal work (Contreras, 2007). Guzman-Bockler and Hebert summarize the encomendero as follows: "grim character, who to save the natives' soul, Christianized Indians, then, against all Christian rule, treated them as if they were beasts" (1975, p. 43). The encomienda evolved later into a concept of "repartition of natives." Organized by the mayor or corregidor, the repartition of Native people ensured the supply of labour for the Spaniard entrepreneurs. The encomienda and its evolved form —the repartition of Native people—were at the centre of the relations of production, which appeared as a complex system of production and extraction of goods, consumption, distribution, importation, and exportation. They were part of a political mechanism that sought to legitimize the subjugation of the Native people and the appropriation and distribution of production.

Historians of the Spanish colonisation have argued that the colonisation was carried out within the Spanish medieval epic mindset of the simultaneous religious and military war against Muslims. Guzman-Bockler and Hebert note that in the American continent "the

Christian Spanish caste repeated this holy war, but this time to Christianize. The American conquest is the culmination of the [medieval] Spanish epic effort" (1975, p. 42). But at the end of the 15th century, with the "Reconquista"—that is, the end of Muslim rule in Iberia—a new relationship to the land emerged. The concept of property changed and land appeared no longer as a medieval lordship space but "as an instrument of production where the essential factor is what the land can produce" (Barbosa-Ramírez, 1971, p. 30). It is not surprising, then, that with the arrival of the Spaniards in the American continent, the land became a new mode of exploitation with new characteristics alien to its Native people. This concept of land as something to be owned and as an object of production in the emerging modern sense of the term, was indeed at odds with the Native people's concept of land. In their communities, the land was an object of "biological work" (Barbosa-Ramírez, 1971, p. 59); that is, something through which they responded to the needs of their group. In the pre-Columbian communities, "there [was] an intimate cohesion between the individuals and the land of each community. . . The Indians fought throughout the colonial period to safeguard this coherence against the attacks of all adverse factors, including the Spanish mentality of possession" (1971, pp. 59-60).

When Padilla's *Arithmetica* was published, Santiago de Guatemala had about 38,000 inhabitants—about 25,000 "gente ordinaria" (ordinary people), 5,500 Spaniards, about 1,000 clergy, and the rest were Native people. Around 1549, Santiago "consisted of a central core of Spanish households and church and Crown institutions ringed by barrios populated by newly emancipated Indians" (Lutz, 1997, p. 155). In the early 17th century, "Spaniards in the urban core were most often merchants, owners of rural agricultural estates, encomenderos, master craftsmen, and government officials" (p. 159). When Padilla's *Arithmetica* was published, the city was heading towards a "gradual disintegration of the barrios and associated institutions… [The] urban core and periphery [were becoming] more similar in socioracial composition" (Lutz, 1997, p. 156). In the surrounding barrios

> castas (including ladinos), free blacks, urban Indians, and poor Spaniards constituted a multiracial urban *plebe común* (commoners) or "laboring poor," living together as neighbors, spouses, in-laws, employees, employers, attending the same churches, even sending their children to some of the same schools. They drank, celebrated, and mourned together, endured low wages and high food prices together. (Lutz, 1997, p. 160)

At that time, the city had well-established political and economic structures. The political structure included a General Captain, the Real Audiencia (which was a justice court comprised of a president, judges, and other public servants), a cabildo (a city council), and "serenos" whose role was to patrol the city. While the Real Audiencia was the direct representative of the Crown's interests, the cabildo was the political body of the most important Spanish local group. The Real Audiencia and the cabildo were generally opposed to each other in a contradictory relationship that always had as its basis the appropriation and distribution of the colonial production surplus. The city council designed and enforced a power apparatus to control the Native people, the repartition of the conquered land, and the economic organization of the colony.

The economic structure included, at the local level, some markets, shops, artisanal workshops, etc. A complex system of food supply was in place to bring to the city the goods it required on a daily basis. "[H]igher-priced goods included maize, wheat, and meat, as well as sugar, tobacco, eggs, poultry, meat byproducts, cotton thread, cloth dyestuffs, and clothing" (Lutz, 1997, p. 143). These and other goods were distributed through regular and black market circuits. At the international level, the economic structure included an importation and exportation system. It is important to bear in mind that the international commercial activity was shaped by an important factor: the Spanish Crown banned its colonies to participate in international trade. Thus, commercial transactions in colonial Guatemala were made primarily with Mexico and Spain, and included cacao, indigo, and other highly regarded goods in the international market. The craft industry also occupied an important place, with exports of silverware, paintings, ceramics, textiles, and leather products. Imported goods included wine, iron, clothing, ink, olive oil, sweets, weapons, and religious objects (Polo, 1988).

Along with the political and economic structures was an educational structure. These three structures were, of course, deeply intertwined. A predominant role in the educational structure was played by the church. An impressive array of religious orders travelled to Santiago with the aim of evangelizing the Native people. To do so, the religious orders quickly acquired lands that were sown and harvested by indigenous labour. In this manner the religious orders became active agents of the new political and economic apparatus of the colonial system. Referring to the Dominicans, Pinto Soria (1969, p. 57) writes: "the Dominicans became another oppressive group [hiding] a disguised form of domination."

The lands that the religious orders acquired were not only sites of agricultural production. On these lands the religious orders also erected churches to which the so-called escuelas de indios (Native people's schools) were often appended. In these schools the clerics started teaching the conquerors' language and some basic techniques of agricultural production. These schools were part of the dissemination of an ideology pulled by the internal contradictions of the spiritual worldview of medieval Christianism and nobility on the one hand, and the ambition of becoming rich through the possession of the land and the exploitation of the mines, on the other hand. As we can see, to reduce the role of the church to an evangelizing mission would miss the most important point. Pinto Soria (1969) notes that the church was instrumental in breaking the Native people's insurrectional spirit and in the expropriation of their lands. The church was also instrumental in stripping away their cultural traits and replacing them with a foreign worldview and values. As Pinto Soria (1969, p. 62) argues, "The Spanish Crown had in the Catholic Church a great ally; without the Catholic Church's presence the imposition and maintenance of the colonial domination is almost unthinkable."

The religious orders also created the "schools of first letters," where children learned to read, write and count, and the Christian doctrine. Counting does not mean knowing the "core operations" of arithmetic only, but also the resolution problems through the Rule of Three, and applying this rule, to various kinds of problems—e.g., revenue sharing among members of a mercantile corporation.

In addition to the escuelas de indios and the schools of first letters, the religious orders also created houses for orphans and maidens (doncellas), and "colegios mayores" (i.e., advanced schools frequented by the Spaniards' children, generally intended to produce clerics). In these advanced schools, the focus was on the teaching of grammar, canons and theology. Mathematics was not a part of such curriculum. The assumption was that the education provided in elementary schools (the schools of first letters), should allow the children of merchants to continue studying at home (with the help of tutors) the most advanced commercial applications (cf. Gonzalez Orellana (1970), pp. 95-96).

The first of these advanced schools was the school of Santo Tomás created in 1529 as part of the Dominican Convent to provide instruction to the children of poor Spaniards. The historian Contreras writes: "The most important [of the colegios mayores] were the school of Santo Tomás and the school of San Francisco de Borja. They conferred titles of bachelor and masters (maestros and licenciados) to those without religious affiliation" (Contreras, 2007, p. 48).

When Padilla published his *Arithmetica*, Santiago was not only an accomplished political and military centre; it also had an intellectual and cultural life for its elite. It already had a university—the Universidad de San Carlos de Borromeo (Rodriguez Cabal, 1976), created in 1676—with studies in theology, law, and medicine. Santiago's cultural milieu at the time of Padilla included the printing press, which arrived in 1660, and the first newspaper, called *Gaceta de Goathemala*, which started circulating in 1729—only three years before the publication of the *Arithmetica*.

4 Who was Padilla?

Juan José de Padilla was born in the city of Santiago, studied theology, and served as Master of Ceremonies of the Cathedral. The Guatemalan historian Domingo Juarros (1808) notes that Padilla taught himself mathematics, a discipline in which he made great progress "with a few books." Padilla died on July 17, 1749, when he was over 65 years old. Juarros also tells us that Father Padilla was an excellent watchmaker. We know that he built the clock of one of the towers overlooking the College of Christ—a clock that marked the flowing of hours by sounds (Gavarrete, 1980, 268 p.). The *Gaceta de Goathemala* for the month of February 1730 highlights the merits of Padilla and refers to him as "famous in the art of making clocks of all sizes."[1]

We do not know what books Padilla read to learn mathematics, nor do we know exactly what books inspired his *Arithmetica*. Probably one of those "few books" to which Juarros refers is Joseph Zaragoza's *Trigonometria hispana resolution triangulorum plani, & sphaerici* (1673), which is mentioned on page 32 of the *Arithmetica as Trigonometria* only. As a reviewer of this paper noted, tt is not clear, if Padilla refers to the Latin version or to the previous Spanish version of the book, *Trigonometria española: resolucion de los triangulos*

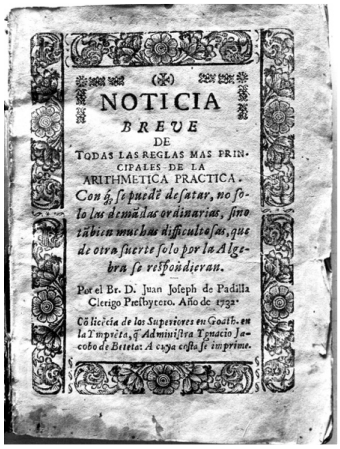
[1] As mentioned before, the *Gaceta de Goathemala* began to circulate in November 1729. It was the first newspaper published in Guatemala and was the second journal in the American continent, preceded only by a few years by the *Gaceta de Mexico*, founded in 1722 (González Orellana, 1970, p. 165). The *Gaceta de Goathemala* played an important role in the dissemination of new scientific ideas (Cf. Tate, 1978, p. 261 ff.)

planos y esfericos, fabrica y uso de los senos y los logaritmos (Mallorca: Francisco Oliver, 1672). To quote the reviewer: "I think it is more plausible that the Spanish version travelled to America than the Latin one."

Be it as it may, in Chapter V of the *Arithmetica*, Padilla mentions Simon Stevin's *Disme*. He also mentions the Jesuit Andrés Tacquet without mentioning, however, the title of the work. As we shall see in Section 6, it is reasonable to assume the (direct or indirect) influence of the *Arithmetica demostrada teorico-practica para lo mathematico y mercantil* of Juan Bautista Corachan (1699/1719) and the *Tratado de mathematicas en que se contienen cosas de arithmetica, geometría, cosmographia, y filosophia natural* of Pérez de Moya (1573).

5 The *Arithmetica*

Written in Spanish, Padilla's *Arithmetica* contains 237 pages (see Figure 1). The size of the pages is 16 cm x 12 cm. Before Chapter I, there are three pages containing a dedication to Saint Gertrudes; a tribute by the author to Jesus, Mary, and Joseph; and a definition of arithmetic. Page 237 of the book, which closes the last chapter of arithmetic, is followed by an index of six pages and two pages of errata.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Por el Br. D. Juan Joseph de Padilla
Clerigo Presbytero. Año de 1732.
Cõ licẽcia de los Superiores en Goath. en
la Imprẽta, q̃ Administra Ignacio Ja-
cobo de Beteta. A cuya costa se imprime.

Figure 1. Photo taken from the cover page of the copy of the *Arithmetica* in the Museo del Libro Antiguo, in Antigua Guatemala.

The content of the book is as follows:

Definition of arithmetic.

Chapter I. Of the letters or characters of arithmetic and mode of numbering [modo de numerar].

Chapter II. Of the four rules of arithmetic.

Chapter III. Broken Numbers [i.e., fractional numbers]

Chapter IV. Of the four general rules with broken numbers.

Chapter V. Of decimal calculations.

Chapter VI. Of powers of numbers and their roots.

Chapter VII. Of proportions.

Chapter VIII. Of progressions.

Chapter IX. Of the Rule of Three.

Chapter X. Of the rules to find the measure of plane and solid [objects]

Chapter XI. Of the rules of combinations and permutations.

Chapter XII. Of other counts and things of Arithmetic.

Padilla's *Arithmetica* was intended for students attending the schools of first letters, while allowing the children of the Spaniards devoted to trade to continue the study of arithmetical methods. Those students probably took private lessons on the subjects addressed in the *Arithmetica*. It is likely that Padilla gave private lessons and that his book is the result of these lessons. What we do know for sure is that, in the early 19th century, the Bishop of Guatemala, Cayetano Francos y Monroy, recommended that the *Arithmetica* be used in his schools of first letters.

The *Arithmetica* is considered the first *pedagogical* treatise of Colonial Guatemala. The content of the books that were published in Santiago before the *Arithmetica* and even after were indeed of a religious or historical nature.[2] Thus, the first book published in Santiago was *Explicatio Apologetica,* written by Fray Payo Enríquez and published in 1663, followed by Roque Núñez' (1673) *Solemne Novenario*, both from the same publishing house that published the *Arithmetica*.

In the *Arithmetica*, Padilla deals with different types of problems of a commercial nature that were relevant in the economy of the colonial period. Let me mention three here.

A first type of commercial problem is the so-called "society problems." A number of people invest different amounts in a business and the problem is to determine how the profit should be distributed. This type of problem was solved using the Rule of Three.

A second type of commercial problem revolves around the mixing of products; that is, how to calculate the price of a product from the price of its components. In Section 12 of Chapter 9, a section entitled "From the rule of three to compound and mix prices and various other things" we read:" This rule teaches first to calculate an average price or value of a mixture: as if several portions of indigo ink of several prices are mixed, we have to find the price of the resulting mix"(Padilla, 1732/2013, p. 146).

A third type of problem deals with the calculation of the amount of ingredients of known prices to be mixed to obtain a mixture at a given price. This type of problem has a wide variety of applications: for example, mixing different qualities of wines, such that the price of the resulting mixture is attractive for sale. Another example is the mixture of precious metals in the craft industry or in minted coins.

[2] See https://issuu.com/informaticapatrimonio/docs/catalogo_museo_del_libro_antiguo.

Let us see in more detail how Padilla tackles the mixing problems (the aforementioned third type of commercial problems). Padilla observes first that the final price of the mixture must be chosen between the highest price and the lowest price of the ingredients of the mixture. The solution is based on proportional reasoning, which takes into account the difference between the price of the final mixture and the price of the ingredients. The numbers are placed around a cross, allowing for convenient and easy organization of data in order to apply multiple rules of three. Padilla states the rule to solve those problems as follows:

> Elegido el precio medio entre el menor, y mayor de todos los diver*ſ*os, que *ſ*e han de me*ſ*clar, *ſ*e *ſ*acará la diferencia, que hai del medio elegido â cada vno de los otros: y e*ſ*tas diferencias en derecho de los precios; pero cada vna en derecho del precio opue*ſ*to: e*ſ*to es que las diferencias, que *ſ*e *ſ*acaren del precio medio â los infimos, *ſ*e pongan con los *ſ*upremos; y las que *ſ*e *ſ*acaran del medio â los *ſ*uperiores, *ſ*e pongan con los inferiores. Y quando los *ſ*uperiores *ſ*on mas, que los inferiores, *ſ*e repite el mas inferior; y *ſ*i al contrario los inferiores *ſ*on mas, que los *ſ*uperiores, *ſ*e repite el mas *ſ*uperior. (Padilla, pp. 148-149)
>
> [Having chosen a price between the lowest and highest [price] of the diverse [components] to be mixed, calculate the difference between the chosen price and each one of the other prices. Put the difference to the right of the prices; but each to the right of the opposite price: that is, the differences between the chosen price and the prices lower [than the chosen price] have to go with the prices that are higher [than the chosen price]; and the differences between the chosen price and the higher prices have to go with the lower prices. And when the higher prices are more than the lower prices, repeat the lowest price [as much as required]; and if by contrast the lower prices are more than the higher prices, repeat the highest price [as much as required. (Padilla, p. 146)]

Padilla gives the following example:

> Como *ſ*i *ſ*e han de me*ſ*clar dos porciones vna del precio de â 2, y otra de â 7, y *ſ*e quiere que la me*ſ*cla *ſ*alga â 5: *ſ*aque*ſ*e la diferencia de 5 â 2, y ponga*ſ*e en derecho del precio 7: y la diferencia de 5 á 7 ponga*ſ*e con el precio 2. (p. 149)
>
> [As if you have to mix two parts, one of a price of 2, and the other of a price of 7, and you want the mixture to be of a price of 5: calculate the difference between 5 and 2, and put it to the right of price 7: and put the difference between 5 and 7 to the right of the price 2. (Padilla, p. 149)]

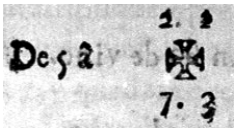

Figure 2. Shows the organization of data around the cross.

The distribution of numbers around the cross is the first step in solving the problem. Padilla spends some time explaining this step through two more examples. It is only when he has sufficiently explained the first step that he ventures into explaining the second step, which

consists of a series of calculations with the numbers around the cross. He considers the following problem: to find the amount of water and wine to be mixed in order to produce 100 quartillos of wine to be sold at 3 reales/quartillo, knowing that the wine to be mixed costs 5 reales/quartillo. The calculations are expressed as follows:

> calculate the difference between 5 and 3, and the difference between 3 and 0 [the cost of water]. And put each one with the opposite price, and add, which gives 5. This will be the first term of the rule. The second term will be 100, the third 3 and 2. And to calculate each fourth term it will be better to divide 100 by 5 and then to multiply 20 [the result] by 3, and 20 by 2, and you get 60 quartillos of wine and 40 [quartillos] of water (p. 150)

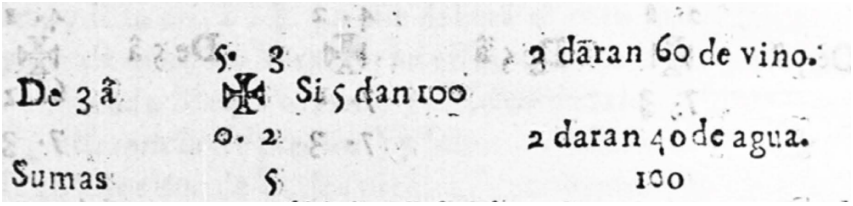

Figure 3. Padilla's distribution of numbers and the two rules of three that solve the problem.

The known prices are placed on the first column (left of the cross). On the second column, the differences between the prices and the chosen price (3 reales in this example) are placed, but with the opposite prices. Therefore, the rule is: If 5 [the sum of differences] give 100, the first difference (the second difference, respectively) gives the amount of wine (the amount of water, respectively) to be mixed.

Padilla gives more examples, one with three ingredients to be mixed. Translated into modern symbolism and its concomitant ideas, the problem becomes an indeterminate linear system. By repeating the highest or lowest price [as much as required], the indetermination is removed and one possible solution is found (see Radford in Padilla 1732/2013).

6 Summary and concluding remarks: The ideology of the *Arithmetica*

I started this article with a short account of my encounter with Padilla's *Arithmetica* and the contextual problems that I found in order to offer a facsimile edition of it. My hope was that making available the oldest known colonial book of mathematics of what is today called Central America would provide us with an interesting window through which to better understand the history of mathematics. From this line of thought, Padilla's *Arithmetica* appears as a cultural artifact that refracts the mathematics that was practised in the colony. This is the argument that I submitted in Section 2, where I argued that to inquire about the history of mathematics is to inquire about mathematics as it was imagined, thought, and practised within a certain cultural context. I also argued the value of considering the opposite proposition, namely that the understanding of a certain cultural historical context can only be achieved if we understand its intellectual life, and more specifically, its mathematics. While in a previous article devoted to the *Arithmetica* (Radford, 2007; reproduced in Padilla (1732/2013)) I engaged with the artefactual view of the book, in this paper I moved to the second (opposite and dialectically complementary) view. I wished to see how the book fit and

responded ideologically to the constraints generated by the economic, political, and educational colonial structures. I think that this line of investigation has not yet been explored in past or contemporary historiographical approaches to the history of mathematics (with perhaps the exception of Høyrup (2007), Restivo (1992, 1993) and a few other scholars). In the second view, mathematics and mathematicians are investigated as elements of an ideological apparatus. By ideological I do not mean something as a false consciousness or as a deception. I rather mean a system of cultural ideas in which mathematics and mathematicians unavoidably live, breathe, think, and act. Padilla's *Arithmetica* appears in this view as a book that conveys a worldview—the mercantilist view that started being shaped at the end of the Middle Ages and the dawn of the Renaissance in Europe and of which the *Tratado* of Perez de Moya (1573) and the *Arithmetica demostrada* of Corachan (1699) are two extraordinary examples. These books, which may have influenced Padilla, deal indeed with mixing problems in a manner that is similar to the one we find in Padilla's *Arithmetica* (see Perez de Moya, 1573, p. 290; Chorachan, 1699, p. 295). More research is required in order to ascertain the differences between Perez de Moya and Padilla's methods. But the point, along the lines of the second view, is that Padilla's *Arithmetica* comes to be part of and support an oppressive economic and political apparatus—one that distinguishes, for example, the education of Spaniards from that of the Native people; one that reaffirms the Spaniards and their children as the masters and the Native people as their slaves—even if "theoretically" by Crown law they are not slaves. By appearing as it does, the book naturalizes the oppression of the system. It helps to offer the practical knowledge required to maintain specific forms of the production of life and existence in the colony—both in its material, intellectual, and spiritual dimensions.

Does it mean that we should have expected Padilla to embrace the Native people's cause and fight for them, as the Dominican Bartolome de Las Casas did in the 16th century? While de Las Casas' countrymen saw in the Native people a formidable means to enrich themselves by occupying their lands and exploiting them as free labour, the Dominican priest made, through the presence of the Native people, the extraordinary cultural experience of *alterity*; that is, the encounter of the Other (de Las Casas, 1552/1994). Referring to his countrymen's actions vis-à-vis the Native people, de Las Casas notes: "I do not say that they [the Spaniards] want to kill them [the Indians] directly, from the hate they bear them; they kill them because they want to be rich and have much gold, which is their whole aim, through the toil and sweat of the afflicted and unhappy" (Cited in Todorov, 1984, p. 142). De Las Casas was confronted in an extraordinary new way by the problem of the Other, anticipating the current problems of social justice and equity with which contemporary societies are faced today and which are, to a large extent, sequels of colonialism.

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