ENGLISH MATHEMATICAL PRACTITIONERS AND THE NEW MECHANICAL PHILOSOPHY

Emerging epistemologies in the development of English Gunnery in the 16th and 17th Centuries: social contexts, technologies, implications for learning

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ABSTRACT

Tartaglia's *Nova Scientia* published in 1537 heralded a new approach to the problems of military technology, by adopting neo-platonistic principles to physical phenomena. However, his enquiry was limited by his Aristotelian viewpoint, and his final edition of 1558 offered the idea that nature could be explained by knowledge applied through Platonic philosophy. Tartaglia had accepted that the flight of the missile, hitherto thought to comprise distinct violent and natural phases, had to be of a mixed nature. While application of mathematics in a mechanical paradigm offered a practical validity, readers needed convincing by a formal proof. The problem of the Gunner was finding a reliable way of firing his shot with reasonable accuracy. Neo-platonist philosophy needed a convincing solution to this problem. In his Stratioticos (1579) Thomas Digges' chapter on gunnery proposed an extensive list of parameters for investigation, thus offering an experimental programme for the English Gunners to follow over the next century. Some thoughts on learning in communities and epistemological contexts are considered.

1 Introduction

Tartaglia's Nova Scientia published in 1537 considered as a Mechanical text heralded a new approach to the problems of military technology, by adopting neo-platonist principles of mathematical reasoning to physical phenomena. In the sixteenth century mechanics was seen as the study of phenomena that happen 'against nature', and inspired by the traditional anecdotes about Archimedes' achievements people believed that one could gain knowledge to achieve power over nature, and with this knowledge gained some 'epistemic authority'. By the use of mechanics, man would be able to master natural phenomena (Cuomo 1997).

After consulting with artillerymen, Tartaglia made some important modifications in his *Quesiti et invenzioni diverse* (1546). He stated that a body could possess violent and natural motion at the same time, so that unless the cannon was fired straight upward the projectile was bound to have a curved path. He proposed that this would obtain a maximum range at an angle of 45^0 elevation of the gun, but some insisted that the impetus given to a shot guaranteed that it would move in a straight line for part of its flight.

Tartaglia's enquiry was limited by his Aristotelian viewpoint and his final edition of the *Nova Scientia* in 1558 offered the idea that nature could be explained by a Euclidean pathway to knowledge giving access to mathematical discipline and true Platonic philosophy, accepting the gunners' experience that the flight of the missile, hitherto thought to comprise distinct violent and natural phases, had to be of a mixed nature. While the potential of the certainty of mathematics applied in a mechanical paradigm offered the belief in an epistemic potential that had practical validity, readers needed convincing that validity had to be proved formally, so while practitioners both praised and criticised the book, it was apparent that the lack of internal coherence was problematic.

The problem of the Gunner was finding a reliable way of aiming, ranging and firing his shot with reasonable accuracy. The neo-platonist philosopher had to find tools to provide a convincing solution to this problem. The sixteenth century English practitioners, particularly Thomas Digges, were pursuing similar aims first seen in his *Pantometria*[1] (1571) and in his chapter on gunnery in *Stratioticos* (1579) Digges prepared an experimental programme, listing a considerable number of parameters for investigation; the dimensions of the canon, its length, bore, and manufacture, the qualities of the powder, shape of the shot, air resistance, and the geometrical path of the missile, thus proposing an experimental programme for the English Gunners to follow.

This paper continues the theme of the community of practitioners in the new Common Wealth (Rogers 2012) and follows the development of the Art of Gunnery to the end of the seventeenth century; through improvement of metals technology, the uses in war, and gradual standardisation of the canon and the epistemological value of the development of instruments like the gunners' quadrant, gunsights, and other devices, described in the texts of William Bourne (1535-1582), Robert Norton (1575-1634), Nathaniel Nye (fl. 1647), Samuel Sturmy (1633-1669), an unknown author in 1672, and Robert Anderson (fl. 1668-1696).

1.1 The English gunners in the seventeenth century

There were a large number of manuals for practitioners of every kind published during the 17th century as a consequence of the work of the English mathematicians, astronomers and practitioners like Robert Recorde (1510-1558), John Dee (1527-1609), Leonard (1515-1559) and Thomas Digges (1546-1595) and many others who were regarded both as mathematicians and people who wrote about their practical work. Dee's *Mathematical Preface*[2] had claimed a large umber of pseudo-physical activities as minor branches of mathematics, thus creating an agenda for mechanical investigation (Culee, 1988; Rampling, 2011). The Preface had a major influence on the development of practical mathematics in England, stressing its many applications in navigation, architecture, geography, and even stagecraft. Dee also espoused the virtues of scientific method and vernacular language expressed in Francis Bacon's *Novum Organum* (1620)[3] where the intellect could pass beyond ancient arts and produce a radical revision of methods of gaining knowledge. By the dawn of the 17th century, London was a centre for instrument makers and practitioners of all kinds, and among this new social mix we find those who were writing manuals where gunnery was part of their profession. The men I

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[1] Digges, Thomas (1571). *Pantometria: Longimetria,* First Book Ch. 30 (folios *Jiy* r – *Jv* v) discusses the problems of the table of Randons, the length of the piece, weight of bullet and force of the powder

[2] Dee, John. (1570) *The Mathematicall Praeface to The Elements of Geometry of Euclid of Megara*. Translated by Henry Billingsley.

[3] Francis Bacon (1520). *Novum Organum Scientiarum*, see https://en.wikipedia.org/wiki/Novum_Organum (retrieved June 20 2016)

mention here are some of the most well-known in the field at the time, often writing on practical skills as typical representatives of their craft and I focus on the parts of the works that are devoted to the development of the practice and theory of gunnery.

Thomas Digges clearly indicated problems with gunnery in his *Pantometria* of 1571, demonstrating that to achieve consistent results requires both the *experience of experiment and sound mathematical knowledge*. He then presented a list of problems in his *Stratioticos* (1579) where he devoted the final section[4] to artillery, and raised a number of questions about the efficacy of the canon, proposing to write on the subject, but it was never completed.

He claimed that the Primary problems to be investigated are (p. 181): The Powder mixture, its quality and quantity; the construction of the canon by the foundry, the length and dimensions of the cylinder; the Bullet, its material, (iron or stone or a mixture), weight and dimensions; and the *Randon* (later referred to as Random) which covers a number of effects like the angle of elevation and the effect of these on the range of the gun. Secondary causes he suggested are: the direction of the wind, density of the air; the gun mounting, the boring of the barrel, charging the gun; fitting of the bullet, and the temperature of the gun. Furthermore, he questioned whether the trajectory of the bullet be an ellipse, parabola or hyperbola and whether the trajectory varies continually with the range, and if the angle between the original elevation of the gun and the path of the shot was continually changing (pp.187/188). Accepting the trajectory of the shot comprised violent and natural motion, he insisted that those without practical experience should not make authoritarian statements about the flight of the bullet. Here, Digges had set out a 'research agenda' and his work was often referred to by gunners in the following century where some of these questions began to be approached, but any answer to the question of the range of shot continued to be a serious problem.

William Bourne (c. 1535–1582) was a gunner at Gravesend bulwark, defending the approach to London. In 1574, he produced a popular version of Martin Cortes de Albacar's (1510-1682) *Arte de Navegar*,[5] entitled *A Regiment for the Sea.* Bourne was critical of the original and produced a manual of more practical use to the seaman. He described how to make observations of the sun and stars using a cross-staff and how to plot coastal features from the ship by taking bearings using triangulation. He also published *The Art of Shooting in Great Ordnance* in 1578. He aims to rectify the faults of England's gunners, given their inability to determine relative ground heights, elevations of their pieces, and distances to their target (1578 folios Aiii r -Avi v). Bourne provides ten "Considerations" regarding great ordnance, and while important, they are simple maxims, providing qualitative explanations for missing the target or just an 'aide memoire' for the gunner (1578 pp. 1- 4). He covers topics in four categories: physical characteristics of ordnance (including gunpowder), numerical calculations, use of the quadrant and the foot rule, and the process of 'laying' a shot (estimating the range) and providing tables measuring elevation. His description of the flight

[4] Digges, Thomas (1579). *Stratioticos* Chapter 18 pp. 181 -189; revised and reorganised in 1590 pp. 361- 368.

[5] The *Arte de Navegar* translated in 1561. Bourne was critical of this book and published a more practical *Regiment for the Sea* in 1574. Translations of various European works continued throughout the century.

of the shot is in three parts: a straight line as long as the shot goes violently A; the second part circularly B; and the third part is at the highest distance above the earth C; and the fourth part downwards circularly towards the earth D. (Fig 1). He states that the maximum range is found at 45^{o} and he shows in a diagram that the flight is the same shape for 45, 30 and 15 degrees of elevation. Bourne, as an experienced gunner, had obviously accepted the fact that the flight of the projectile was some kind of curve, and abandoned the Aristotelian theory to use some kind of 'mixed' motion once the initial impetus was lost.

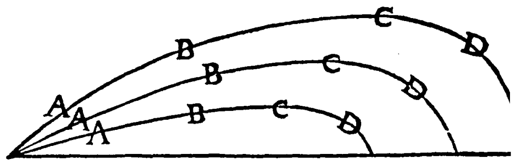

Figure 1. Bourne 1578 Chapter 9 p. 38

Bourne discusses the astronomical quadrant, introducing the measure of degrees as more profitable than the 'gunners quadrant' that had comparatively fewer makings.

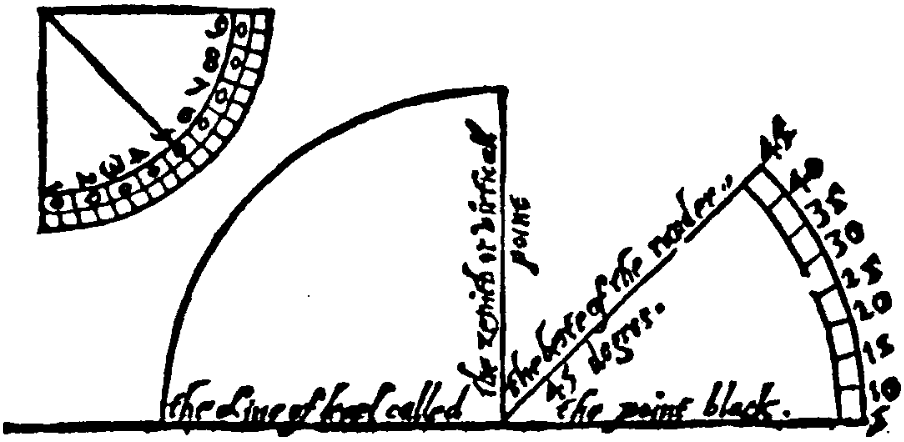

Figure 2. Bourne 1578 page 22 use of the quadrant

The diagram (Fig. 2) shows the vertical, the 'point blank' positions, and the 'best of the random' and he introduced a table comparing the fractions of an inch of a gunners level with the degrees on a quadrant. It is in Bourne's work that we find lists comparing different properties and quantities: diameters of shot to their weight; powder types for different shot, calibre of canon and material (stone, iron, etc.) and weight of shot; and many other comparisons. The lists are difficult to read as text, but he summarises some sections by introducing tables, an innovation which is taken up by succeeding practitioners. Bourne's was the first work in English which defined gunnery as a separate and independent branch of the Art of War by military writers.

Robert Norton (d. 1635) was a military engineer and gunner who studied under John Reynolds, master-gunner of England, and became a gunner in the Royal service. In 1627 he was sent to Plymouth as an engineer, to await the arrival of the Earl of Holland and to accompany him to the Isle of Rhé.[6] In the same year he gained the post of engineer of the Tower of London. He published *Of the Art of Great Artillery,* London 1624. Norton acknowledged Digges' work with expositions and answers of his own, and followed it with *The Gunner, showing the whole Practice of Artillery*, London 1628. Here, Norton supplied tables of various measurements and instructions in decimal arithmetic from Recorde's *Ground of Arts*. Norton had also published *Disme, the Art of Tenths or Decimal Arithmetic* London 1608[7], claiming that decimal fractions make calculation much easier. *The Gunners Dialogue.* London 1643, described the types of artillery[8] used at the start of the Civil War in England. This was also published in an edition together with his *Art of Shooting in Great Ordnance*. Here were more tables of shot sizes and weights, quality and mixtures of gunpowder and use of the gunners' level. The information is cumulative but provides no specific technical advance on earlier publications. Here we are beginning to see some improvements in presentation of data and the introduction new methods of calculation.

Nathaniel Nye (1624-1647?) living in Birmingham developed an interest in canon, the city's principal trade during the English Civil War (1642-1651) and tested cannon there in 1643. From 1645 he was master gunner to the Parliamentarian garrison, and in 1646 directed the artillery during the Seige of Worcester[9], recounting his experiences in his 1647 book *The Art of Gunnery*. For Nye, the 'Art of War' was also a science, and his other work focused on triangulation and cartography, fortification, and mechanics, as well as finding the ideal specification for gunpowder. Nye described the rules and directions in this book both with and without the help of arithmetic. He has the usual contents: description and maintenance of canon, composition of gunpowder and other fire-works, estimating heights and distances, plotting positions of targets and drawing maps of fortified places. He covers ranging and provides tables according to results of experiments with various canon, recounting an experiment (chapter 52) where (under the same conditions) he fired seven shots at a target in 50 minutes, and attempts some explanations of the difference in the distances of shot from the gun: the heat of the barrel, the dryness of the powder, and that the first shot parted the air allowing later shots to fly further. Here the use of tables include comparing a diameter of a canon's bore to the weight of powder, weights of various shot for different diameters, a cube root table to help with the required calculation, and another to compare quadrant degrees to the scale on the Gunners rule. In these tables we are beginning to see gunners focussing on recording and comparing simple experiments, but it did not progress the trajectory problem.

Samuel Sturmy (1633-1669) first published *The Mariners Magazine*, in 1669. It became the most extensive compendium of its kind in the latter 17th century. Subtitled the *'mathematical and practical arts'*, Sturmy had collected a considerable variety of information using instruments on Navigation, Surveying, Gunnery, Fire-Works, Fortification, Sundials,

[6] Norton took part in the expedition to support the Hugenots in the 1627 Anglo-French War.

[7] A translation of Stevin's *De Thiende* (1585).

[8] Civil War Artillery had adapted from heavy siege guns to mounted canon about 9-10 feet (2.7-3.0 metres) in length with bore 3-4 inches (8-10 cm) diameter firing a shot of 5-12 pounds (2.3-5.4 kg) in weight.

[9] The final Battle of Worcester in September 1651 was the last in the English Civil War.

and Spherical Astronomy. Subtitled the *‘mathematical and practical arts’*, Sturmy had collected a considerable variety of information describing the use of instruments on Navigation, Surveying, Gunnery, Fire-Works, Fortification, Sundials, and Spherical Astronomy. Many topics had been addressed by others, but he shows he has command of the mathematics, with sections on geometry and the *‘doctrine of triangles both plain and spherical’*. He describes the use of the quadrant, nocturnals, and other instruments, with problems resolved geometrically, and instrumentally.

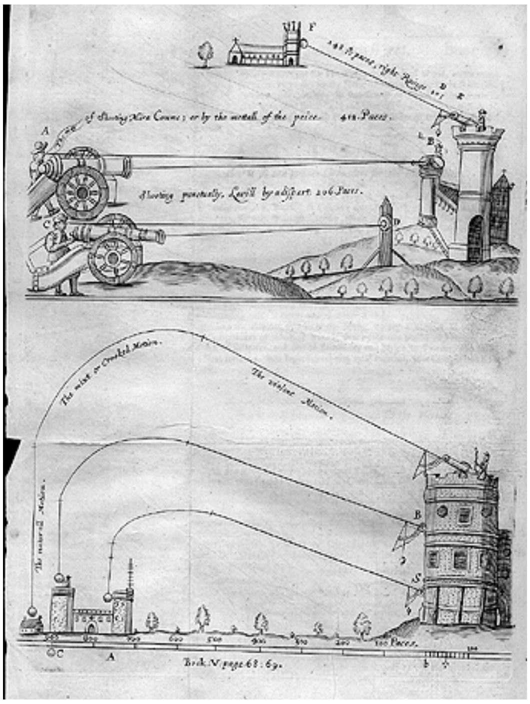

Figure 3. Source, MHS Oxford, Sturmy 1684 edition

Figure 3 shows the problem of the trajectory still unsolved. The upper part of the figure shows firing *Point Blank* where the path of the shot and the direction of the canon are parallel with different calibre canon and firing with different *Randons* below. While the ranges are different, the projectile paths are comparable consisting of violent straight motion, ‘mixt or crooked motion’ and natural straight motion. Sturmy’s 1679 edition was *‘diligently revised and carefully corrected’* by John Colson[10]. Included were tables of Sines and Tangents to every degree and minute of the quadrant, and Logarithms, showing their use in calculations. Sturmy's work also contains what may be one of the earliest complete explanations of the construction of a polar gnomonic chart, presenting a detailed example of a great circle route from the Lizard (SW England) to the Bermudas. His section on gunnery repeats the usual exhortation to gunners to make firing tables and by example, to continue other types of comparative tables.

[10] John Colson was Lucasian Professor of Mathematics at Cambridge, and a Fellow of the Royal Society.

An Anonymous 'W.T.' signed the preface to The Compleat Gunner in Three Parts. London 1672, claiming this is a collection of translations from various sources. The title page refers to material from "Casimir, Diego Uffano, Hexam11 and other Authors" the author claims that he has translated material from French, German, Italian, and Dutch sources. Here we find many familiar topics, but there are some notable inclusions: Founding12 and Casting with discussion of the composition of Metal and Examining Ordnance for flaws. There are Tables of diameter and weight for all ordnance used in England and tables for mixing stone and lead to equal iron canon balls. There is a section on preparing and clarifying saltpetre from Nitrous Earth13 as well as detail on Proving14, and refining the composition of gunpowders. There are descriptions of instruments, and the variety of instructions for training a gunner are quite significant: measuring heights and distances by using a quadrant or without instruments, showing many woodcuts borrowed from other authors15. The Circumferenter (page 68) is a surveying instrument for measuring horizontal angles.16 There is a description of an improved Gunners Scale with multiple uses with a two-sided stepped scale to enable the gunner to measure the diameter of the bore, and know which type of canon he is dealing with (between pages 70 and 71). There are instructions on shooting to make a table of Randoms including measuring distances in Paces. Violent, crooked, and natural motion from discharge to target are discussed with reference to Tartaglia's Nova Scientia and there is an incomplete picture of shooting at Random from Sturmy 1684 (between pages 72 and 73). Much of this was copied form Hexham (1640). In the last part of the book an appendix, has two sections, the first being a description of "The Doctrine of Projects applied to Gunnery by those late famous authors Gallileus and Torricello, now rendered into English." "Together with Excellent Observations out of Mersennus and other famous Authors." It contains some 25 pages of mixed translations of selected parts of Galileo on motion with tables of sines and tangents of the angles of elevation taken in a semi-parabola (p. 78).[17] The proportional calculation is accompanied by the comparison of the angle in a semicircle with a cap of a parabola (Fig. 4).

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[11] These people are: Diego Uffano (or d'Uffano), d. 1613 a Spanish military engineer; and Casimir or Kazimierz Siemienowicz (1600? – 1651) author of *Artis Magnae Artilleriae pars prima* (1650) which was used in Europe as a basic artillery manual, and Henry Hexham, author of *The Third Part of the Principles of the Art Millitarie* 1640 from which the 'Doctrine of Projects' was badly copied and misunderstood.

[12] The Blast Furnace and canon Foundries were well established in England from the late 15th Century.

[13] Nitrous Earth: Manufacture of saltpeter from nitrate deposits and from animal excrement, rotting bodies, and urine by extracting the chemicals ammonia, aluminium sulphate, and acid salts.

[14] 'Proving' is the verb describing the testing a canon for accuracy and the powder for its explosive force.

[15] For example, we can identify here the woodcuts from Digges *Pantometria*1571.

[16] An early Theodolite. See Turner 1973 (page 79 item 16). Leonard Digges *Prognostication Everlasting* 1564 showed an instrument called 'Theodoliticus' that became a standard instrument of surveyors and architects.

[17] Imagine an inverted parabola f(x,y) from $x_0 = 0$ to $x = x_1$ with a chord from x_0 to $y = (x_1/2)^2$; the angle of elevation is contained between this chord and the x – axis. Fig. 4.

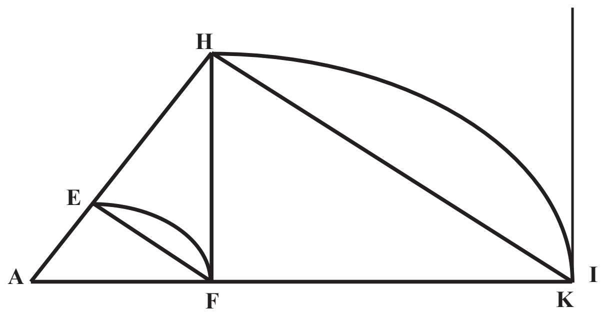

Figure 4. Anon 1672 Chapter XXI, p. 78

"AEF is like and proportional in like and crooked Ranges to HI and their distances or dead Ranges are AF unto AI." (the like Ranges are at Point Blank) The Author has clearly misunderstood the problem and is struggling to fit the results to a 'Euclidean' diagram.

Robert Anderson was helped by John Collins[18] with a loan of books and scientific equipment, and in *The Genuine Use and Effects of the Gunne, as well experimentally as mathematically demonstrated*, 1674 are fifty propositions on the use of all kinds of canon and mortar showing how to calculate ranges for different shot and powder based on his experiments, referring to a set of geometrical diagrams facing page 36. By these experiments he shows that the trajectory is a geometrically constructed curve, and that the ranges for equal elevations above and below 45^0 are equal. He justified this further by analogy, experimenting with water spurting out of two holes in a tube equidistant from the centre of the tube. The distance traversed by water jets is proportional to their horizontal speed and time of fall. Speed is proportional to the square root height of the water above the hole, whereas the time of fall is proportional to the square root of the height of the water below the hole, therefore both jets would hit the ground at the same time. (Anderson 1674: 26-27 and 30-31). The rest of the book contains many pages of tables showing his experimental results.

By this time, the most significant aspects of these publications is the emphasis they put on *accumulating data on all sorts of aspects of the canon and its' firing*. In parallel with this we have the *increasing use of instruments of various kinds, now being fashioned by the practitioner community, and just as important were the technological advances enabling the gradual standardisation of the canon*.

1.2 Gunnery: the general problem

Artillery gun-sights and levels were introduced in the 16th century accompanying the new cast bronze canons that came into use from the late 15th century. Ideally, the canons could be set at various elevations and by traversing around on its mounting, the gun could be aimed in any direction. Gunner's sights and levels were either separate or combined together. The quadrant was placed in the mouth of the gun, so that gunners could elevate the gun to the correct angle for the estimated range, often exposing the gunner to enemy fire, or the level was set up more safely at the touch-hole end. This instrument came in a variety of forms, with

[18] Anderson's book was sent to Newton by John Collins, apparently to provide him with more technical data.

a degree or a tangent scale graduated either in an arc or on a straight rule, usually with a plumb bob on a cord. The levels were set up along the longitudinal axis of the gun. There is often a confusion in the accounts of this instrument; the simple version used during the 16th century was the well-known proportional measure for sighting heights and distances[19]; the instrument used in the 17th century was the *adaption of the astronomer's quadrant marked in degrees of arc* (Fig.2). The early manuals often advised the gunner to make their own quadrant to suit an individual gun, but Bourne (1578) instructed his reader to use a quadrant marked in degrees. In the 17th century mounting the canon on wheels for field use meant that recoil, traversing and resetting the gun could be managed more easily. For maximum destructive force most cannon were fired at 'point blank' range, for which the gun, and hopefully the trajectory of the shot remained horizontal. But after Tartaglia's *La nova scientia* (1537) there was much discussion of the ranges that could be obtained by varying the elevation of the gun. Tartaglia insisted that the maximum range of the gun could be achieved at an elevation of 45^0, and Galileo showed later that a parabolic trajectory was theoretically the case,[20]but Aristotelian philosophers, and to some extent the gunners, relying on 'line of sight' observation[21] were in dispute.

2 Sources of innovation

2.1 The art of the gunner and the role of metallurgy

The emergent science of ballistics was the natural theoretical development following the spread of firearms from the fifteenth century onwards, and their deployment on the field and during sieges became an undeniable reality in the fifteenth century, but considering the level of technological development at the time ordinary gunners felt no need for a science of ballistics such as that formulated by Tartaglia.

Over the course of the fifteenth century, heavy artillery was used during sieges on fortresses and fortified towns. From the beginning, two categories of artillery were produced: one which was able to fire relatively light cannon balls (between 3 and 12 kilograms), and the other intended to destroy fortifications, and therefore capable of firing cannon balls weighing up to several hundred kilograms. Technological developments in the earlier part of the sixteenth century had concentrated primarily on the heaviest artillery, but by the middle of that century the balance became restored, when architecture was able to provide a response to the development of metallurgy, and the construction of the bastion succeeded in putting attack and defence onto a more equal footing, at least as far as sieges on fortresses were concerned.

Durer (1527)[22] had foretold the end of old fortresses, even if they had been readapted.

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[19] Many examples of this instrument can be seen in Digges *Pantometria* 1571.

[20] Galileo (1638) *Two New Sciences*. Fourth Day; Theorem II Proposition II.

[21] From the gunners point of view, observing the shot go straight up and then fall (hopefully) on to the target does not provide clear evidence of any particular trajectory.

[22] In 1527 Albrecht Dürer (1471-1528) published a treatise on fortifications, Etliche underricht zu Befestigung der Stett, Schloss und Flecken. (Several instructions for fortifying towns, castles and small cities), which was the first printed work on the subject of permanent artillery fortification.

Now there was a need for new fortresses to be built, following a geometrical construction on the basis of the strategies of attack and defence made possible by the new firearms. In the sixteenth century, the attack developed during sieges focused on destroying one of the bastions to gain access to an area along the curtain wall where defence became weaker, and where it became possible to move in to attack. At this time, the technology had not yet been developed to avoid serious damage from the dangerous effects of the cannon's recoil. Furthermore, the process of loading the barrel of the cannon, the quality of the gunpowder and the quality of the cannon itself from the metallurgical perspective, made it impossible to fire shots that would follow rectilinear trajectories. Formerly, strategies had focused on destroying as much as possible of the inside of the fortress. A precise shot was not fundamentally important, and the experience of the gunner in battering the walls down was sufficient to achieve the required objectives. Due to developments in metallurgy and to innovations that increased the efficiency of hydraulic apparatus applied to run ventilation systems at the end of the fifteenth century, furnaces became capable of reaching much higher temperatures than before (Simmons 1992).

During the 16th century, metal technology improved considerably. The casting of barrels and smoothing of bores became more efficient, and higher furnace temperatures achieved provided cast iron[23] and bronze, much stronger metals, so that by the end of the 17th century both ammunition and the bore of the gun became more consistent and the sale of these armaments abroad led to furhter standardisation[24]. A new kind of cannon ball began to be produced in these furnaces, made of wrought iron or cast bronze. This innovation resulted in a revolution. Cast iron cannon balls of a relatively low calibre had a much higher capacity for penetration than those made of stone. Stone balls often disintegrated and crumbled on impact with the target, and only had a potential for destruction if they were very large and in free fall, while the low calibre cast-iron cannon balls finally made it possible to use smaller and lighter artillery that was easier to transport and cheaper to produce. This innovation led to a significant increase in the velocity of projectiles, which established artillery as essential to the art of war, such that it spread to a hitherto unimaginable extent. Over the following decades, the calibre and type of artillery were produced to the setting of standards, each of which was valid within at least one single country or princedom. There was a change in the role of the gunners who from this time became recognised as vital to the Renaissance army.

2.2 From the art of gunnery to ballistics

Aristotelian mechanics was concerned with natural and violent motion which, according to Tartaglia, failed to provide a sufficient answer to the fundamental question concerning the curvilinear segment of a trajectory. This question led to the formulation of an idea of "mixed motion," and thus formed the basis for a *concept of compositions of motions*. This idea is

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[23] The English Iron foundries such as that in Horsmonden, Kent (1574 – 1685) set the highest quality for cast iron guns that were used by our ships and armies and were sold abroad until the development of steam power moved the industry to the Midlands in the 18th century. See http://www.horsmonden.co.uk/history/furnace/

[24] English canon were sold to the Dutch during their war of independence from Spain (1568-1648).

expressed in Digges' final chapter of *Stratioticos* (1579) and appears in the 1578 edition of *Nova Scientia.*

The gunner's problem was knowing how to hit a target with precision from as far away as possible, so during the 17th century gunners' manuals began to encourage the recording of firing tables to record the fall of shot for different angles of elevation. This meant that the gunner would find it easier to use fewer adjustment shots each time the target changed. Tartaglia's firing table (promised but not produced) would have been particularly relevant for the gunner, and the method for calculating his own table based on the data he would have obtained from a single shot would have been useful. As we have seen, firing tables began to appear in a wide range of publications, although there were great discrepancies in the values they showed. Generally, these results could only be useful to the person who had written them since only he was able, on the basis of his memory and accumulated experiences, to see their significance. But, believing that they would have some practical use, many people began to produce these tables *and in doing so gunners and others began to learn to read these tabular forms obtained through experiment*, and a series of records of angles of elevation, relating to one specific piece of artillery, firing similar projectiles and maintaining the same quality and quantity of gunpowder, would amount to a firing table. However, it took quite a long time for all these different conditions to be consistently achieved. While the accumulation of this data was of a very particular nature, it would have remained quite local and individual. While there was no organised collection of such data there was plenty of material to be found in the many published works that included 'gunnery'. There were too many varied parameters, but it gradually became possible to formulate *professional habits*, writing down the information became a custom, *it became what gunners do*, as part of their training, and from these general, practical rules the use of a more or less inductive method developed. The popularity of gunners' manuals during the 17th century to some extent depended on the promise that the next manual would have even better ways of solving the problem of consistent shooting.

Many other tables of measures appeared that provided early 'ready reckoners' for matching shot diameter and weight to canon; estimating weight of shot by size of stone, iron or mixed material; shot and powder for the length of a given canon; proportions for mixing components of gunpowder etc. Despite all of this data the problem of predicting the flight of a projectile remained unsolved. Matthew Bourne, whose picture of the trajectory was pretty well a complete curve (Fig. 1.) had clearly adopted the idea of 'mixed motion' while others like Samuel Sturmy (Fig. 3) and the Anonymous author of the *Compleat Gunner* …(1672) (Fig. 4.) were still clinging on to the old theory. Robert Anderson's trajectory of 1674 was the *cumulative result of labour* and the best collection of results before the eighteenth century.

Surveying techniques improved with better designed instruments; the rudimentary theodolite appeared, methods of measuring distances by triangulation became common, better surveys provided information on how defenders were positioned, and the efficiency of the artillery battery was considerably improved.

The quadrant, gunners level, and 'professional habits'

As we have seen, the quadrant found new applications, *randons* became *degrees* and a considerable amount of empirical data was accumulated by recording elevation in more accurate measurements. This became the first step in a process of abstraction, the *beginning of a theoretical reflection on the gunner's own actions*.

The quadrant, the gunners level, firing tables, and more accurate surveying, provided the physical instruments and records for the *intellectual milleu that became the epistemological spur that initiated a process of theoretical abstraction*, leadting to a better formulation of the gunners' question. During the seventeenth century, the accumulation of these data, formed the empirical basis from which the theory of ballistics emerged. In the early 17th century, special artillery schools for training gunners had appeared which created centres for the newly emerging science of ballistics. These advances in technology were attributable to the accumulation of small improvements, essentially empirical, collaborative and democratic, which were used by Francis Bacon (1561-1626) to demonstrate the manner in which intelligent application could lead to economic progress and intellectual advancement in his system of Natural Philosophy.

3 Social contexts, learning and epistemology

In the study of the practice of mathematics in 16th and 17th Century England, the epistemological varieties concern the views of the nature, use, and processes of the formation of the mathematics and its uses that were espoused by the different agents. As far as the authors of these texts are concerned, their views on the formation of mathematics depended on their ideologies, and were inevitably involved in the production of their texts. The purposes of mathematics that were perceived, and the uses to which it was put, to some extent determined the kind of mathematics that an individual was motivated to use and develop, and these needs were felt in different ways in particular sections of society. The mathematical practitioners of this period were influenced not only by the social context of enterprise, but also the belief that there was no particular barrier between practical mathematics and theoretical mathematics. The principal characters cited above and others like John Wallis, Thomas Harriot, and William Oughtred were all at some time or another involved in practical enterprise. This *Community of Practice*[25]became the basis of the generation of all kinds of specialised knowledge in the sixteenth and seventeenth centuries. With regard to the texts studied here, the aim of the authors of these books may be to produce an organised system of knowledge, and this assumes that their text, when read in the 'right' way provides a reader with a clearer understanding of the nature and purpose of their subject. This involved a process of investigating phenomena to establish new facts by developing the *method of induction*.[26] An important practical aspect of these investigations was introduced, and that was the process of *thinking with objects.* (Meli 2006; Rogers 2015).

By the seventeenth century, there were many instruments available for measuring distance and ranging the canon, and an important part of most gunnery manuals involved a

[25] The idea of a *Community of Practice* has been developed in sociology by Wenger (1999) and applied in mathematics education by Adler (2000)
[26] In particular as we know, in the case of Recorde, Dee and Digges.

section on the basics of arithmetic, geometry and surveying, with the use of proportional reasoning to solve the problems by organising range tables. The tables that we find in these gunnery manuals brought home the important idea that, if we could accumulate and refine enough data, we should be able to find an answer to our questions. The accumulation of this data formed the epistemic background, and in this atmosphere, new variations of the objects, the canons, the gunpowder and the instruments were experimented with. There is a reflexive relation between working with the object (material or theoretical) that provides new affordances (Gibson 1997) enabling new ideas to emerge and come into practical and theoretical use. The learning that takes place is shared in a community which emphasizes socialization, spreading values with not only the acquisition of skills and participation in activities, but a third stage where individual and collective learning goes beyond mere information given, and advances knowledge and understanding by a collaborative, systematic development of common objects of activity into shared *knowledge-creation.* (Paavlova & Hakkarainen 2005) Some of these attitudes, contexts and processes we learn from history, can be applied by reflective contemplation and adaptation to our current situations.

The next real innovation in the context considered here, will be the discoveries leading to the application of the new fluxional mathematics to the trajectory of the bullet in Benjamin Robins' *New Principles of Gunnery* (1742).

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