THE COGNITIVE VALUE OF HISTORICAL CASE STUDIES FOR TEACHING AND LEARNING

The case of leibniz's text *de quadratura arithmetica* (1675-1676)

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With the aim to provide students with deeper understanding of the processes involved in doing mathematics, some scholars (Pengelley 2011) suggest to ground teaching activities in the of use of primary historical sources as it is common practice in the humanities. However, given the great variety of primary historical sources the selected texts need to be scrutinized in the context of the corresponding mathematical cultures where they originated. Moreover, the required contextualization of primary sources to be used in mathematics education brings to light further issues such as the question about the way in which the research mathematician organizes the presentation of his results at a given moment in history within the standards of the community of experts, a concern which is also at the center of current debates in the philosophy of mathematical practice. According to the standard view, western cultures of mathematics ever since Euclid have been setting the emphasis on axiomatic-deductive models for organizing the presentation of research results. This idea crystallizes in the modern axiomatic conception of rigor which requires that only those aspects concerning the deductive structure underlying the validity of results ought to be made explicit in textbook presentations. Accordingly, the specificity of problem solving activities underlying other ways of articulating results have been neglected with a view to attaining higher levels of abstraction required by systematic theory construction. From a historical perspective, however, some mathematical texts composed by leading mathematicians bring to light a different way of articulating results. In our presentation we shall consider a different style of textbook presentation in mathematics, a tradition which for many scholars started only with 17th century mathematical analysis. In this tradition the mathematician aims to make explicit some of his most fundamental cognitive strategies as well as the design of innovative working tools and procedures for problem-solving. This is the case with Leibniz's *Quadrature Aritmétique du Cercle, de l'Ellipse et de l'Hyperbole* (from now on, DQA), a text composed towards the end of his mathematical studies in Paris (1672-1676), in which Leibniz aims to provide a general method to solve quadrature problems for curvilinear figures at the same time that he analyzes some fundamental notions concerning the origin of infinitesimal calculus. The style of presentation proposed by Leibniz (DQA) organizes results following the specific cognitive strategies he conceived for his research so that the study of this text ought to provide the reader with interesting details about the newly designed working tools and the pattern of reasoning the author followed. We find this case study particularly interesting as Leibniz intends to engage the reader with the complexity of his reasoning, even if the organization of results and the arguments displayed may be not evident at all for the modern student.

In this paper we aim to provide a study of the style of argumentation present in DQA that sheds light upon the methodological and epistemological aspects underlying the presentation of results with a view to motivate its inclusion in the classroom. With this in mind, we follow current suggestions by Grosholz (2007) and Chemla (2003) concerning different styles of argumentation in mathematical texts to be found throughout history which appear to be guided by two different "ideals" or "epistemic preferences". The first one points to a preference for "abstraction" which guides the "axiomatic exploration of truths" leading to axiomatic presentation, the second one is best described as the search for "generality" of algorithms and procedures on the basis of "paradigmatic problems" finding expression in more historical presentations. We argue that Leibniz organized his text following the "ideal of generality" showing that DQA is designed so as to provide a general method to solve quadrature problems for curvilinear figures where the results obtained are organized around two fundamental problems, the squaring of the circle and the hyperbola. The intermediary reasoning steps involved exhibit a procedure that aims to precise determination of the area of the circle and the hyperbola, at the same time that it indicates how the procedure might be extended to a new family of problems, the squaring of "transcendental curves". Leibniz displays the reasons why the results presented are correct but also shows the way successful procedures can be extended. We discuss the significance of Leibniz´s text presentation for teaching and learning, some of its difficulties as well as requirements to contribute to make the selected primary source accessible to the student.

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