THE DEVELOPMENT OF VIEWS ON MATHEMATICS EDUCATION AS SEEN IN MATHEMATICS EDUCATION CONTROVERSIES IN JAPAN

Naomichi MAKINAE
University of Tsukuba, 1-1-1 Ten-nou-dai, Tsukuba-city, Japan
makinae@human.tsukuba.ac.jp

ABSTRACT

It can be said that mathematics education in Japan was started in 1872 when the school system was established. Since that establishment era, controversies have emerged time and again in mathematics education in Japan. Through these controversies, debates have been held on views on mathematics education such as how mathematics ought to be taught and what constitutes knowledge concerning numbers, quantities, and shapes that is desirable for students to acquire. In this paper, I shall look back at how views on mathematics education in Japan have developed since the Meiji era from the perspective of such controversies on mathematics education. As the controversies on mathematics education, the four phases are picked up. The first is Theoretical Mathematics and the Enumeration Principle. The second is Controversy over Formal Building. The third is Conventional Teaching of Mathematics and the Creation of Mathematics. The forth is Relationship between Daily Life and Mathematics.

1 Introduction

The purpose of this paper is to identify the historic process of mathematics education in Japan, and thereby focus on the controversies emerged time and again in mathematics education. As recent studies that introduced the history of mathematics education in Japan, we can mentioned Baba et al. (2012), Ueno (2012), Kota (2012) and so on. In this paper, I focus on another view point from these previous studies. Through the educational controversies, the debates have been held on views on mathematics education in elementary and lower secondary level about its subject objectives and curriculum structure, teaching methods etc. When we look back history of mathematics education in Japan, we can pick up four phases for these controversies. In following sections, I will illustrate such controversies.

2 Theoretical arithmetic and the enumeration principle

2.1 Beginning of the modern school education system and mathematics education

The beginning of the modern school education system in Japan can be traced back to the school system in 1872. Aiming at building a modern state, the Meiji government began incorporating the technology and culture of Europe and the United States, and the school system came to be developed in this process. In mathematics education, a policy was adopted in which Western mathematics based Arabic numerals and the base-10 positional notation system would be taught in addition to traditional Japanese mathematics. More time was devoted to the teaching of the four operations in written calculation, and such textbooks as "*Hissan Kunmo* (Learning Written Calculation)," "*Yozan Hayamanabi* (Quick Learning of Western Arithmetic)," and "*Shogaku Sanjutsusho* (Elementary Arithmetic Textbook)" were published. In the decade between 1877 and 1886, "*Sugaku Sanzendai* (Three Thousand Math Problems)" by Seikyu Ozeki who was an Aichi Prefecture samurai, was published (first edition in 1880). "Three Thousand Math Problems" is a textbook in the format of a collection of problems. Although it returns to the principle of seeking answers since "*wasan* (Japanese mathematics)," the textbook became widely used at the time. An explanation on calculations appears at the beginning of the textbook, followed by 3,000 problems in accordance with its title.

2.2 Theoretical arithmetic

Firstly, "*Chuto Kyoiku Sanjutsu Kyokasho* (Secondary Education Arithmetic Textbook)" (published in 1888) by Hisashi Terao who was a professor of Tokyo University, can be cited as a textbook that is representative of theoretical arithmetic. The basic idea of theoretical arithmetic is stated at the beginning of the textbook.

> Observing the method of teaching arithmetic at the school where I am in charge of secondary education, everyone basically neglects theory, seemingly simply solving problems...(passage omitted)...Arithmetic is essentially a type of science, and it is simply not art, regardless of what they call it. Even if I were to yield an inch and call arithmetic an art, this art, like medicine and architecture, cannot form a firm foundation unless it is based on theory. Therefore, trying to give a lecture on arithmetic while neglecting theory is like trying to teach surgery without learning about anatomy. (Hisashi Terao 1888, p. 10)

It is explained here that arithmetic is not simply calculation skills but rather that there is a legitimate theory behind it. Terao asserts that calculations hold true because they are supported by theory. He emphasizes that it is necessary to teach theory in mathematics education. Under this concept, such items as "definition," "principles," "laws," and "caution" are listed in the textbook to explain the theory of arithmetic. After the explanation, calculations using specific numbers are given as examples. In the explanations in this textbook, an approach is taken to lead to new contents based on definitions that have already been learned rather than simply describing calculation.

A characteristic of theoretical arithmetic is the concept of not regarding mathematics education as simply learning calculation methods but rather as teaching the underlying theory. It is based on mathematics education based on a strict French-style theory that Terao learned while studying abroad. In his theoretical arithmetic, Terao pursued arithmetic as a discipline and looked toward theory as its cornerstone.

2.3 Enumeration principle

It was Rikitaro Fujisawa, a professor of Tokyo University, who criticized theoretical mathematics and expounded a different argument. "*Sanjutsu Jomoku Oyobi Kyoju-ho* (Mathematics Rules and Teaching Methods)" can be cited as a work in which Fujisawa straightforwardly expounded his views on mathematics education. Here, he lists the two following points at the purpose of mathematics education.

> The purpose of mathematics education is to provide phased preliminary mathematical

knowledge and to nurture mathematical philosophy, i.e., spiritual discipline (Fujisawa, 1895, p. 2)

The first point refers to providing knowledge of mathematics that will serve as the necessary foundation for advancing to higher grades and studying mathematic. Fujisawa mentions an aspect of substantial discipline in mathematics education. The second point refers to strengthening mathematical thinking and philosophy through mathematics. Fujisawa mentions an aspect of formal building in mathematics education. Thereupon, Fujisawa rejects the argument of theoretical arithmetic, asserting that there are definitions, laws, and explanations in arithmetic, and arithmetic is not biased toward only seeking answers. He goes on to distinguish between arithmetic and algebra, asserting that the theory of arithmetic is algebra and proof should not be taken up in arithmetic. Moreover, he argues that in view of the current situation of school education and the developmental stages of children, theory should not be taken up.

In criticizing theoretical arithmetic, the enumeration principle is what Fujisawa advocated as a replacement guidance principle. Among textbooks based on the enumeration principle is "*Sanjutsu Sho Kyokasho* (Arithmetic Textbook for Elementary Schools)" (published in 1898). Fujisawa was introduced to the enumeration principle by his teacher L. Kronecker at the time of his return from studying in Germany. It is based on the principle advocated by R. Knilling and others. The enumeration principle is based on the fundamental idea that acquisition of the concept of numbers becomes possible by counting. For this reason, textbooks based on the "enumeration principle" start with the introduction of numerals and how to count numbers. The concept of numbers is said to be acquired by associating numbers that are abstract with numerals that are the names of the numbers when counting and also by counting those numerals out aloud.

2.4 Views on mathematics education seen in the conflict between theoretical arithmetic and the enumeration principle

With regard to the conflict between theoretical arithmetic and the enumeration principle, debates were held over which policy to adopt in the process of editing the first government–designated textbook "*Jinjo Shogaku Sanjutsusho* (Ordinary Elementary School Arithmetic Textbook)," which was published in 1905. The conflict was ultimately settled when it was decided that Fujisawa's enumeration principle would be adopted. Attention must be paid to the fact that this conflict was taking place in an era when it was questioned what constitutes mathematics education in introducing the modern education system in Japan.

Those who advocated theoretical arithmetic were concerned that mathematics education would get buried in simple teaching of calculations and seeking answers; they placed priority on the underlying theory. Asserting that calculations are possible by using the rules of calculations that have been proven by theory, they sought the essence of mathematics education in the underlying theory rather than in the calculations themselves. In contrast, those who advocated the enumeration principle placed priority on the concept of numbers and the understanding of the rules of calculations. While asserting that it is necessary to take up theory in studying more advanced mathematics, they sought the essence of mathematics education not only in the understanding of mathematics itself but also in disciplining one's mind and acquiring a way of thinking by recognizing the concept of numbers and through specific calculations.

Although people get the impression that the theory of mathematics and the concept of mathematics are the same thing, there is a big difference. The two viewpoints take on a completely different appearance with regard to developing teaching methods and textbooks/teaching materials. Theory refers to axiomatic development in accordance with mathematics as a system of learning. It can be considered a difference between the standpoint of those who want students to acquire this theory and the standpoint of those who assert that even if the students forget what they learned about mathematics, they would not forget the concept of mathematics, which they acquired through studying, and this is what they want the students to acquire.

3 Controversy over formal building

3.1 Period of adjustment of school education and review toward formal building

After the school education system was established, mathematics education in Japan entered a period of adjustment under the influence of Taisho democracy and the liberalism movement and as the mathematic education reform movements in Europe and the United States were conveyed. Ever since the Meiji period, mathematics education was promoted for the purpose of both substantial discipline and formal building, but in actual teaching, lessons were held centering on teaching calculations. What was actually sought of children in mathematics education was for them to be able to solve problems in textbooks, and it was a period when this kind of teaching was criticized and improvements were sought. Amid calls for education centering on children that is at the foundation of the improvements, a major controversy occurred in mathematics education.

During this period, Fujisawa retained his influence with regard to the purpose of mathematics education. The following phrase remains intact in the Elementary School Ordinance Enforcement Regulations: "The gist of arithmetic is achieving proficiency in daily calculations, providing knowledge required in life, and at the same time making thinking accurate." The concept of formal building that aims at making thinking accurate had been placed at the basis of mathematics education. However, at the time, this idea has contained a misunderstanding that lead to a so-what attitude in teaching mathematics. In other words, there spread an idea that solving mathematic problems would train thinking, resulting in acquiring the ability to think logically. As a last resort when it was not possible to respond to the question of whether mathematics taught in school is useful or what meaning there is in learning mathematics at school, the concept of formal building was used as the theoretical background in answering that mathematics taught in school was for training the process of thinking. Teaching mathematics was justified simply asserting that as long as students solved mathematical problems, they would be able to train their thinking process.

In response to mathematics education that took comfort in the concept of formal building, doubts that were presented against formal building triggered the controversy over formal building.

3.2 Arata Osada's disavowal of the formal building theory

In 1922, a meeting of the "*Zenkoku Chutogakko Sugaku Kyoju Kenkyu-kai* (National Research Society of Secondary School Mathematics Teachers)" was held in Hiroshima. At this meeting, Arata Osada, a professor of Hiroshima Higher Normal School gave a lecture titled "Recent Controversy over Formal building." Here, he totally disavowed the traditional theory of formal building, going on to introduce a doctrine that it is harmful to approve of the formal building theory. Osada described formal building as follows:

> In other words, formal building tries to discipline mental capacity itself by using mathematics teaching materials as an expedient. When we study not only mathematics but other things as well, it can be said that the effects are all universal. (Osada, 1923, p. 61)

For example, when solving a geometry problem, some kind of psychological effects remain in the student. Osada says these effects are beneficial in solving other geometry problems, other mathematical problems, or even problems other than mathematical problems. This is known as *transference* in psychology. When we think this way, the power of reasoning is trained by studying mathematics, and, therefore, mathematics is worth learning.

3.3 Kinnosuke Ogura's "Fundamental problems in mathematics education"

"Fundamental Problems in Mathematics Education" by Kinnosuke Ogura who was a researcher of Siomi Institute of Physical and Chemical Research, mathematician, was published in 1924. This is a book that criticized mathematics education at the time, asserted that improvements were necessary. Here, Ogura also supported disavowal of the formal building theory advocated during the 5th annual meeting of the Secondary Education Mathematical Society of Japan, and made a similar argument in the book.

Ogura asserts reform of the traditional mathematics education with regard to pre-war mathematics education. In addition to morals, religion, and the arts, science is necessary for humans to create a lifestyle. What can be learned from science include various matters such as the facts of biology and physical and chemical phenomena, but the most fundamental thing is to learn about scientific views, scientific concepts, and the scientific spirit. When there are more than two phenomena, a close examination of their causes is conducted on the basis of empirical fact, and a determination is made as to whether there are any cause-and-effect relationship between the phenomena, and if so, what kind of relationship that is. The efforts made and spirit for discovering that is scientific spirit. Development and creation of human lifestyles and human ideals require nurturing this scientific spirit. Similar to the development of natural sciences, the development of mathematics was born from nature, and mathematics has the similar aim as the natural sciences. For this reason, "the significance of mathematics education lies in the development of the scientific spirit" (Ogura, 1924/1953: 159). The "concept of functions" speaks of the scientific cause-and-effect relationship, and at the same time, it is most widely and deeply related to human life.

Ogura expanded the grounds and structure of his argument against formal building in a similar manner as the aforementioned lecture by Osada. After describing formal building, he disavowed formal building by negating the existence of the ability, which is the prerequisite of formal building. From the standpoint of disavowing formal building, Ogura has pointed out the need for the "concept of functions." Although the contents of teaching pure mathematics are an ad hoc, unrealistic event, functions are related to everyday life. Functions are learned not simply from formulas and graphs but also by linking them with actual phenomena and experiences, and that must be done. Although general formal building cannot be approved, transfer is allowed when identical elements are included, and therefore, teaching material must be socialized. When socializing mathematics teaching materials, teaching materials that are related to actual daily life shall be selected. This is because the selection of teaching material from events in daily life in which transfer is valid and is most likely to occur is derived from disavowing formal building. What links daily life with mathematics as such is the "concept of functions."

3.4 Rebuttal by Motoji Kunieda

In 1924, Motoji Kunieda who was a professor of Tokyo Higher Normal School, presented a counterargument against Osada and Ogura in a lecture at the 6th annual meeting of the Secondary Education Mathematical Society of Japan. At the time, Kunieda was vice chairman of the Secondary Education Mathematical Society of Japan, and as an expert in mathematics education, he had taken a position to recognize formal building. First, he says that arguments by Osada and Ogura introduce only parts of the debates on formal building, and although it may appear that formal building has been completely disavowed in Europe and the United States, that is not true. He went on to assert that many psychologists in the United States admit the significance and effects of formal building.

While reviewing the results of experiments and the theories of eight psychologists, Kunieda introduced the fact that formal building was recognized. Specifically, he reported such matters as that investigating formal building of memorization abilities from a survey on changes in the time it takes to memorize a book revealed that some effects were found with children, and that the habit of writing answers neatly was transferred not only to the relevant subject but also to other subjects. In addition, as Chairman Tsuruichi Hayashi who was a professor of Tohoku University, stated in his opening speech, a survey report on formal building was released in the United States, Kunieda stressed that even if there were some differences the effects of formal building were recognized[1].

3.5 Views on mathematics education seen in the wake of the conclusion of the controversy over formal building

In the controversy over formal building, both the opponents and proponents based their

[1] In his speech, Hayashi stated that the argument against formal building cannot be taken at face value, that the importance of mathematics education would not disappear as a result of the argument against formal building, and that the effects of formal building are recognized in "The Reorganization of Mathematics in Secondary Education," a report edited by J. W. A. Young and released by the Mathematics Committee on Mathematics Requirements. (Hayashi, 1923)

arguments on overseas theories and survey results. They did not state their view based on their own survey results. The controversy was not settled in the form of one view being adopted while the other view was discarded in the education policy and the editorial policy concerning government-designated textbooks. Regardless of the pros and cons regarding formal building, assertions regarding the objectives of mathematics education and what is desirable for children to acquire are at the basis of the arguments. The arguments against formal building served as criticism against teachers who taught mathematics as is as a discipline in mathematics education and became stepping stones toward improvements.

Ogura advocated learning the scientific spirit, while Kunieda advocated learning mathematical common sense adapted to the culture of the times. In either case, regardless of the pros and cons regarding formal building, they asserted the significance of studying mathematics. Here, the arguments include assertions concerning improving mathematics education to train the thinking process rather than regarding the ability to solve math problems as mathematics education. In fact, Kunieda also asserted incorporating experiments and actual measurements in geometry, incorporating intuitive handling, and incorporating teaching materials on functions. Here, we can sense their desire to see children being able to understand mathematics and being able to think.

At the time, the Secondary Education Mathematical Society of Japan was organized in 1918 for the purpose of "studying matters related to mathematics and the method of teaching mathematics in secondary schools and taking initiatives to achieve progress and make improvements," and research in mathematics education began to be promoted in mathematics education circles in Japan. The controversy over formal building was the first of an academic controversy here, and an issue that started the sprouting of academic research on mathematics education. In addition, it can be said that at its root, it was an issue calling into question the goal of mathematics education.

4 From conventional teaching of mathematics to the creation of mathematics

4.1 Mathematics education reconstruction movement

Affected by mathematics education reform movements in Europe and the United States, mathematics education in Japan progressed gradually in 1930's and 40's. In particular, in elementary education, government-designated textbooks were drastically revised by Naomichi Shiono who was a compiler of national textbooks in Ministry of Education, and the effects of the reform movement were clearly evident at the government-level educational administration. Reform of mathematics education in secondary schools was inevitably sought when children who received this new mathematics education advanced to the secondary school level. A movement that occurred in response to this was the mathematics education reconstruction movement – a movement to drastically reconstruct mathematics education ranging from the objectives of teaching to the contents of teaching and the methods of teaching. This movement culminated in the revision of the syllabus of teaching for secondary schools and the compilation of "Mathematics," an authorized textbook.

In the mathematics education reconstruction movement, proponents asserted the introduction of new teaching contents such as analytic geometry, calculus, descriptive geometry, statistics, and dynamics, as well as teachers' own curriculums and teaching methods, without being bound by the constraints of the system of mathematics as a discipline. A study group was established centering on the Secondary Education Mathematical Society of Japan, and the research contents were reported in journals and at conventions. In response to these research results, the Ministry of Education carried out a revision of the syllabus of teaching in 1942.

4.2 Revision of the syllabus of teaching for secondary school

With regard to the revision of the syllabus of teaching for secondary schools, various teaching contents and the curriculum were subject to studies from the idea that mathematics education is intended for students to discover and create mathematics on their own rather than for teaching conventional mathematics. The guidelines for this revision are shown below.

(I) Without adherence to the existing academic system, adopt the appropriate system for extending the students intellectual abilities.

(II) Carefully select teaching materials across the board, and in particular, adopt them by taking the points at the left into consideration.

1. Matters that are effective and appropriate for the daily lives of the people

2. Matters that are for industries and national defense

3. Matters that should contribute to long-term culturing of insights

(III) Designate specific operations such as observation, actual measurement, and construction as the foundation of learning, and along with training for knowledge and action, make efforts to nurture the ability to discover and create.

(IV) In addition to placing priority on intuition, make efforts to train the capability to think abstractly, analyze, and integrate.

(V) Make teaching matters as precise as possible, while adding the teaching policy and adopting a system that clarifies the objective of teaching

(Ministry of Education, 1942, p. 2)

Compared with the conventional syllabus of teaching, there were the following changes. Firstly, the teaching contents were separated into Type 1, which concerns quantity, and Type 2, which concerns diagrams. Secondly, although only mathematical contents such as "ratio," "quadratic equations," and "similar figures" were listed in the conventional syllabus, what kind of teaching is to be conducted was written in the new syllabus before listing the teaching contents. In other words, mathematical contents and the objectives and activities in the teaching process were combined as the teaching content.

4.3 Government-designated textbook: "Mathematics"

Following the revision of the syllabus of teaching for secondary schools, publication of

“Mathematics,” a new mathematics textbook created under this spirit, started in 1943. Corresponding to the syllabus of teaching, two volumes – Class-1 and Class-2 – were issued for each grade from 1st grade to 5th grade. The editorial prospectus of this textbook contains the following editorial guidelines, which plainly indicate the characteristics of the textbook.

1. Stop introducing conventional mathematics and guide the students so that they can discover mathematical principles on their own in line with events.
2. Incorporate many concrete materials in problems, mathematize events, and give priority to training for processing them.
3. Ensure that students sufficiently understand mathematical principles in line with specific examples; then abstracted and formalize them and train so that they can be freely applied to concrete events.
4. Give priority to the handling of similarities in mathematical expressions of events.
5. Make it a rule to give the definitions of terms and symbols after their concepts have been developed.

(Secondary School Textbook Co., Ltd., 1943, p. 2-3)

Textbook materials are introduced from the scene of specific problems. Through the course of processing them through mathematization, a procedure has been taken in which the concept of mathematics and the method of processing are abstracted and formalized. Students are expected to write between the lines on the textbooks what they notice here, or definitions are written in small type so that they are not conspicuous. In this way, the textbooks reflect the intention to have the students create mathematics on their own rather than teaching them mathematics that has already been completed.

Experiments, actual measurements, and functions are often used for thinking about such scenes of specific problems. Data is actually collected through experiments and actual measurements, and quantitative relationships concerning events are regarded as functions. They are processed by expressing them as graphs, drawing graphs, and expressing them as numerical formulas. New formulas are derived and new rules are found through this process. In this way, the textbooks are designed so that new mathematical contents intended as the teaching contents are created. It is the aim of the textbooks to have the students conduct such activities on their own, and here in lies an argument that leads to the “syllabus residue theory” advocated during debates on the syllabus of teaching.

4.4 Views on mathematics education seen in criticisms of the syllabus of teaching and textbooks

However, the syllabus of teaching and textbooks of the time were not necessarily fully accepted at the field of education and in mathematics education circles. As a social situation, the effects of World War II extended to school education itself, and teaching requiring such time and effort had become impossible. In addition, under such circumstances, the aim and intentions that substantially differed from those of conventional teaching was not accurately transmitted to the field of education and thus not understood.

As a more fundamental problem, there existed conflicts over how mathematics education ought to be. With regard to teaching material that were introduced from scenes of specific problems, mathematicians of the time, while acknowledging their diversity, pointed out that the contents were too wide-ranging. In the backdrop of this situation was the concept that the utilization and application of mathematics to specific scenes rested on the understanding of the basics of mathematics. While not denying the utilization and application of mathematics, the fact that this has become the center had become the problem.

5 Relationship between daily life and mathematics

5.1 Postwar educational reforms and empirical approach to mathematical education

Right after the end of World War II, occupation by the Allies and the fundamental social reformation took place in Japan. The reform also took place in the field of education. The educational system, contents, and instructional methods were modified in conformity with the idea of pacifism and democracy. In this process, education centered on the needs of children was sought instead of prewar teacher-centered uniform instructional method. Especially, the unit learning method that was contextualized in relation to children's experience or day-to-day activities was widely adopted. Curriculum structure that incorporated daily life, not just math as a subject, was also introduced to math education. The following chart from the draft of the Course of Study for Lower and Higher Secondary Schools, which was published in 1951, describes mathematics instructional content based on "living experience."

This curriculum model reflects the criticism against pre-war mathematics education, which overly emphasized teaching mathematics as a discipline and did not correspond with the children's real learning conditions. It was claimed that in order to incorporate children's needs in mathematics education, examples and problems discussed needed to be driven from everyday situations so that children could learn mathematics through solving those problems. The Allied Forces made this claim to the Japanese Ministry of Education, and this idea was also supported by a group of core curriculum proponents who had been promoting research in the field of pedagogy regarding the curriculum in the United States. In addition, it was claimed that the standard of mathematics education did not match the children's comprehension ability, and after 1948, standards were lowered by one level in all grades, and instructional contents that met the new standards were implemented.

5.2 "Mathematics for middle school students," a model textbook for unit learning

In 1949, the Ministry of Education published its "Mathematics for Lower Secondary School Students," a model textbook for unit learning based on empirical education discussed above. The textbook uses examples from children's daily lives and aims to teach mathematics through problem solving. The problem takes up a scene in daily life and unfolds in such a way that relevant mathematical contents are taken up. Taking the structure of daily objects into consideration, it can be acknowledged mathematical concept and knowledge.

The textbook was a collection of these materials and it showed how to teach in classroom. The areas covered in this textbook were the number and calculation, quantity and geometry, the textbook picked up the children's daily life experience that corresponds to each.

5.3 Criticism against unit learning

These initiatives to promote unit learning came under criticism after the conclusion of the (San Francisco) Peace Treaty, along with the objection to the occupation policy, and declined rapidly. The strongest point of the criticism was the falling of children's academic standards. Compared to surveys on computational problems conducted before the war, the results of the survey conducted at the time showed a clear drop in the academic standards among children. Of course, there certainly must have been many other factors responsible for the fall in academic standards such as the period of turmoil in education during and after the war, reduction in teaching contents, and changes in the grade system. However, unit learning solely was targeted as the cause of the fall in children's academic standards.

In reaction to the criticism, teachers and schools asserted the significance of student-centered education and the necessity of solving problems. In 1958, however, on the occasion of the revision of the course of study, the instructional contents were presented based on the system of subjects, and since then, systematized learning was adopted.

5.4 Perspective on mathematics education during post war educational reforms

If based on empiricism, priority is placed not on the assumption that the contents of mathematics that ought to be learned will become useful someday in the future, but rather on the fact that they are necessary in the situation that students are currently facing. Since this is often mixed up with the notion of practicality in the familiar sense, caution should be exercised. Here, practicality refers to a condition in which something is operating and functioning on the spot, and it is asserted that students should learn mathematics that is actually useful in this sense. It does not mean simply mastering mathematics that students can utilize and apply. Furthermore, it does not mean that if students take a mathematical approach, they can discover relevant mathematics in their daily lives. Rather, it means to capture mathematics that exists in our daily lives, or on the contrary, to capture our daily lives that exist because of the existence of such a function. This is about learning mathematics that exists on top of people's experiences rather than mathematics that exists as an abstract idea in our minds. The objective here is totally different from the contents as in the case of criticism against unit learning, and it is also not restricted to the principle of teaching that humans first gain recognition and understanding by experiencing. It is an issue of how to look at mathematics itself as a subject of learning.

Through four phases of controversies, we can see the changes in views on mathematics education in Japan. In establishment era, it was important to introduce western mathematics to school. It meant they regarded mathematics as discipline. In controversies, the debates have been held on views on mathematics education such as how mathematics ought to be taught. At that time they became to regard mathematics as school subject. We can recognize the history of mathematics education in Japan as development of educational philosophy of school mathematics.

REFERENCES

Baba, T., Iwasaki, H., Ueda, A., & Date, F. (2012). Values in Japanese mathematics education: Their historical development. *ZDM – The International Journal on Mathematics Education, 44*, 21–32.

Chuto Gakko Kyokasho Kabushikigaisha (1943). *Sugaku Hensan Shuisho (Mathematics Editorial Prospectus)*. Tokyo: Chuto Gakko Kyokasho Kabushikigaisha. (in Japanese).

Fujisawa, R. (1895). *Sanjutsu Jomoku oyobi Kyojuho (Arithmetic Rules and Teaching Methods)*. Tokyo: Dainippon Tosho Publishing. (in Japanese).

Fujisawa, R. (1898). *Sanjutsu Sho Kyokasho (Arithmetic Textbook)*. Tokyo: Dainippon Tosho Publishing. (in Japanese).

Hayashi, T. (1924). Kaikai no ji (Opening Speech), *Nippon Chuto Kyoiku Sugakukai-shi (Journal of the Secondary Education Mathematical Society of Japan), 6*(4, 5), 177-181. (in Japanese).

Knilling, R. (author), & Sasaki, K. (translation and commentary) (1906). *Sanjutsu Kyoujuho Shinzui (Essence of Teaching methods for Arithmetic (Die Naturgemasse Methode des Rechen-Unterrichts)*. Tokyo: Dobunkan (in Japanese).

Kota, O. (2012). Teaching and learning of functions in modern Japan. *HPM 2012 The Satellite Meeting of ICME-12 Proceeding Book 2*, 659-670.

Kunieda, M. (1924). Sugaku Kyoiku Zakkan (Miscellaneous Thoughts on Mathematic Education). *Nippon Chuto Kyoiku Sugakukai-shi (Journal of the Secondary Education Mathematical Society of Japan), 6*(4, 5), 131-150. (in Japanese).

Ministry of Education (1942). Chugakko Kotojogakko Sugaku oyobi Rika Kyoju Yomoku Kaisetsu Yomo (Commentary for Syllabi of Mathematics and Science in Secondary Schools). *Monbu Jiho (Bulletin of Ministry of Education), 759*(2), 1-25. (in Japanese).

Ministry of Education (1949). *Chugakusei no Sugaku (Mathematics for Lower Secondary School Students)*, Tokyo, Chuto Gakko Kyokasho Kabushikigaisha. (in Japanese).

Ministry of Education (1951). *Chugakko Kotogakko Gakushu Shido Yoryo Sugaku-hen (Shian) (Course of Study for Lower and Upper Secondary Schools: Mathematics (Tentative version))*. Tokyo: Chubu Tosho Publishing. (in Japanese).

Ministry of Education (1958). *Chugakko Shido Yoryo (Course of Study for Lower Secondary Schools)*. Tokyo: Printing Bureau, Ministry of Finance. (in Japanese).

Ogura, K. (1924). *Sugakukyoiku no Konpon Mondai (Fundamental Problems of Mathematics Education)*. Tokyo: Idea Shoin. (in Japanese).

Osada, A. (1922). Keishikitoya ni Kansuru Saikin no Ronso (Recent Controversies over Formal Building). *Nippon Chuto Kyoiku Sugakukai-shi (Journal of the Secondary Education Mathematical Society of Japan), 5*(2), 60-74. (in Japanese).

Ozeki, S. (1880). *Sugaku Sanzendai (Three Thousand Math Problems)*. Tokyo: Narumido Publishing. (in Japanese).

Secondary School Textbook Co., Ltd. (1943). *Sugaku Hensan Shui Sho (Compilation Prospectus for Mathematics)*. Tokyo: Secondary School Textbook Co., Ltd. (in Japanese).

Terao, H. (1888). *Chuto Kyoiku Sanjutsu Kyokasho (Secondary Arithmetic Textbook)*. Tokyo: Keigyosha. (in Japanese).

Ueno, K. (2012). Mathematics teaching before and after the Meiji Restoration. *ZDM – The International Journal on Mathematics Education, 44*(4), 473-481.