THE APPLICATION OF HPM VIDEO CLIPS IN MATHEMATICAL TEACHING IN MIDDLE SCHOOL

Teaching the application of linear equation with one unknown

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ABSTRACT

The history of mathematics is of great importance to mathematical teaching. To give full play to its educational value and to solve the dilemma faced by middle school teachers, the authors shoot some video clips with math-related historical materials and bring them into classroom. It proves that these video clips not only play a guiding role in the classroom, but also broaden students' horizon of knowledge. It's constructive to reach aim of teaching in the syllabus. The goal of teaching is to master knowledge and skills, it also helps to understand the process and methods, attitude and values are to be judged. Meanwhile, it provides a new way of thinking for the study on integrating history of mathematics into mathematical teaching.

1 Introduction

One of the major studies in contemporary mathematics education focuses on the relations between History and Pedagogy of Mathematics (HPM). Recent years have witnessed a great number of researches on applying history in mathematical teaching. The HPM study group at East China Normal University believes that the historical materials selected for mathematics teaching must comply with the five principles of being interesting, scientific, effective, learnable, and innovative (Gao & Hu, 2014). The group classifies all HPM teaching methods as the approach of complementation, replication, accommodation, and reconstruction (Wang, 2012, see table 1). In his idea, there is no good or bad between these four methods. We may choose one or more methods according to the curriculum standards and students' aptitude. They also set up a standard teaching process, i.e. choosing a teaching subject → investigating related history → selecting suitable materials → analyzing classroom requirements → designing classroom activities → implementing teaching design → evaluating the course.

Table 1. Approaches of using history of mathematics in teaching

Approaches	Description
Complementation	Display mathematicians' pictures, give an account of related stories, etc.
Replication	Directly using historical problems, methods, etc.
Accommodation	Problems adapted from historical ones or based upon historical materials
Reconstruction	Genesis of knowledge based on or inspired by the history of mathematics

The improvement of HPM related theoretical researches and the development of related cases have caught the attention of middle school math teachers. They would like to have a try in their own teaching practice. However, they often feel perplexed about how to acquire reliable historical materials and how to effectively integrate these materials into class teaching. There's a seemingly insurmountable valley between academic researches and their application in middle school classrooms (Zhang & Wang, 2009).

As information technology advances rapidly, multimedia video learning has been witnessing a sound momentum in terms of development and utilization. Among which, the video clip teaching, with the characteristics of short and exquisite, lively and convenient, and dynamic and repeatable, has been applied in many of the education and teaching levels, and is gradually becoming the hotspot and frontier of education and teaching reform.

High School Mathematics Curriculum Standards (Provisional) promulgated by the Chinese Ministry of Education states that promoting the integration of information technology and mathematical teaching would have a great impact on course contents, teaching and learning, and enable students to understand the nature of mathematics (He,2012). The purpose of this research is to make fully use of video clips, to resolve teachers' perplexities, and to provide a freer and broader platform for integrating the history of mathematics into practical teaching. The authors took math-related historical materials in some video clips which has educational values. It helps and expands the students' knowledge and reach aim of teaching in the syllabus. The goal of teaching is to master knowledge and skills, it also helps to understand the process and methods, attitude and values are to be judged. These short clips are called HPM video clips.

2 The teaching application of HPM video clips in *the Practical Problems and Linear Equations with one unknown*

The Practical Problems and Linear equations with one unknown is a teaching content in Mathematical Textbook for the 7th graders, they are between the ages of 12-14. Prior to this course, the students have learnt algebraic expressions, simple equations and solutions to linear equations with one unknown. This course is aimed at solving matching problems between the number of people and workload, which not only consolidate the knowledge students have learnt, but also serves as an extended application of integrating theory with practice.

In 2014, the Chinese Ministry of Education issued an *Opinion on Comprehensive Deepening Curriculum Reform and Implementing the Fundamental Task of Setting High Moral Values and Cultivating Persons*, in which it states clearly that too much emphasis on subject contents in the current curriculum standards should be gradually changed and the educational mode also needs to shift. As to this teaching practice, the concept of equation is first introduced in primary education. Some teachers try to enlighten their classrooms by integrating the history of mathematics into practical teaching. However, due to either unfamiliar with or not profoundly understanding of the historical materials, their classrooms are still lack of liveliness. Similarly, they can't display the cultural charms of the history of mathematics or the values of moral education.

To further the 7th graders' understanding of and love for mathematics in cultural dimension, and to strengthen their perception and application of equations, the authors incorporate some video clips into the teaching design of this course and deliver it in classroom.

2.1 Selecting and processing math-related historical materials

Going back in history, we can find the problems of linear equations with one unknown in ancient Egyptian papyri and Babylonian clay tablets. And in China the term equation first appeared in *the Nine Chapters on the Mathematical Art*. This book endows great historical and cultural connotations to linear equations with one unknown. It echoes the idea of *from life to mathematics and from mathematics to life* advocated by the new curriculum standards. It is also helpful to cultivate students' exploring spirit, practical ability and application awareness. It is different from the modern definition of *equation*, but they have the same ideas of solving the problem. Therefore, all historical materials in this teaching design are chosen from *the Nine Chapters on the Mathematical Art.*

2.1.1 Linear equations with one unknown recorded in *the Nine Chapters on the Mathematical Art* (Wang,2007a;2007b)

In *the Nine Chapters on the Mathematical Art*, there are altogether 246 mathematical problems, among which are five types of linear equations with one unknown, i.e., the problems of four basic arithmetical operations, travel, cooperation, fixed sum, and remainder (see figure 1).

Word Problems	Description	Original Question
Four Basic Arithmetical Operation	Linear equations with one unknown can be written as $x+a=b$ or $ax=b$.	Now given a rectangular field whose width is 1 ½ *bu*. Assume the area is 1 *mu*. Tell: what is its length?
Travel Problem	Meeting and catching-up word problems.	Now a wild duck flies from the south sea to the north sea in 7 days, and a wild goose flies from the north sea to the south sea in 9 days. Assume the two birds start at the same moment. Tell: when will they meet?
Cooperation Problem	A number of people, n, accomplish a task individually need $a_i(i = 1,2,\cdots,n)$ days. Find out how many days they need if they cooperate to finish the task.	Now given a cistern which is filled through 5 canals. Open the first canal and the cistern fills in ⅓ day. With the second, it fills in a day; with the third, in 2½ days; with the fourth, in 3 days; with the fifth, in 5 days. Assume all of them are opened. Tell: how many days are required to fill the cistern?
Fixed Sum Problem	Find out the given number which, after	Now given a person carrying gold through 5 passes. At the first pass he

	adding some times of its own, fractions of its own and a certain known number, is equal to another known number.	pays a tax of one part in 2. At the second pass, one part in 3; at the third, one part in 4; at the fourth, one part in 5; at the fifth, one part in 6. Assume the total tax at these five passes is just 1 *jin*. Tell: how much gold is carried originally?
Remainder Problem	Find out the given number whose remainder, after subjecting several fractions, is a known number.	Now given a person carrying cereal through 3 passes. At the outer pass, one-third is taken away as tax. At the middle pass, one-fifth is taken away. At the inner pass, one-seventh is taken away. Assume the remaining cereal is 5 *dou*. Tell: how much is carried originally?

Figure 1. Word Problems of linear equations with one unknown recordedin *the Nine Chapters on the Mathematical Art*

2.1.2 Selection of math-related historical materials and production of HPM video clips

In order to introduce the matching and work problems, the original exercise is revised and divided into two questions with the same background. This approach employs the method of accommodation to introduce the history of mathematics, which is in line with the principles of being innovative and scientific for selecting historical materials. As the main thread, the historical materials chosen from *the Nine Chapters on the Mathematical Art* are used through the whole teaching process, which is consistent with the current textbook, well-matched with students' cognitive levels, and also in agreement with the principle of being learnable. With the history of mathematics as background, the teaching enhances the reaching of teaching goals through arousing and resolving doubts, which is compatible with the principle of being effective.

To ensure a smooth teaching process, the authors use screen recorder software to make three short video clips of mathematical history. The first video is a three-minute introduction about *the Nine Chapters on the Mathematical Art*, lively and emotionally displaying the wisdom and achievements of the ancient mathematicians. It enables the students to get an idea about this ancient and most complete Chinese mathematical classic. In the situation creation section, the second video clip lasting for three minutes tells about the history of linear equations with one unknown. It also borrows the original problems from the book to deepen the concept of equation in students' mind. And the last video clip, two minutes in length, is used to exhibit the original solutions to the problems in *the Nine Chapters on the Mathematical Art*. By comparing the ancient and modern solutions, it aims at assisting students to imperceptibly understand the development and evolution of mathematics.

2.2 Teaching Sections

2.2.1 Introduction

The teacher started the class not with the typical pedagogy, but by playing a video clip introducing the book *Nine Chapters on Mathematical Art*. The author noticed that all students were drawn to the screen. They watched the original book of *the Nine Chapters on Mathematical Art.* They learned that it's the Chinese who first introduced the concept of negative numbers and the addition and subtraction algorithm. They got to know that the solutions to linear equations are almost the same as the elimination method in Algebra, which is 1500 years earlier than the Europeans. Their eyes were filled with wonder and endless longing.

The video clip conveys a lot of information within a short time in a lively way. It helps the students immersing in an information setting to feel the need for personality growth and the power of spiritual growth. It's a wonderful prelude to the intensifying and expanding of the teaching process.

2.2.2 Lecturing

Why should we study equation? Upon the end of the video, this question raised by the teacher brought the students back to reality. They started whispering.

Student A: Equation can solve problems.

Teacher: Why can they solve problems? What problems can they solve?

Student B: We have learnt that an equation is created by two expressions set equal to one another. And we only need to solve the unknown.

Student C: If we set the unknown, every problem can be solved.

Teacher: Yeah. An equation is a mathematical model created by two expressions set equal to one another. It is a useful approach to describe social and natural phenomena. That's why today we are going to learn how to create equations and find solutions to word problems.

The teacher wrote the topic for this lesson on the blackboard, i.e. practical problems and linear equations with one unknown. And here came the next part, playing the video about the history of the linear equations with one unknown, and the problem recorded in the book *Nine Chapters on Mathematical Art* appears in the video. (Shen Kang-shen,1999) *Now one person makes 38 prostrate tiles or 76 supine tiles a day. Assume he makes an equal number of both kinds of tiles a day. Tell: how many tiles of each kind can he make?* (see figure 2).The video showed and explained the pictures of supine tiles, prostrate tiles, and related ancient buildings. To leave suspense which set the stage for the following teaching, there was not give solution to the problem in this video.

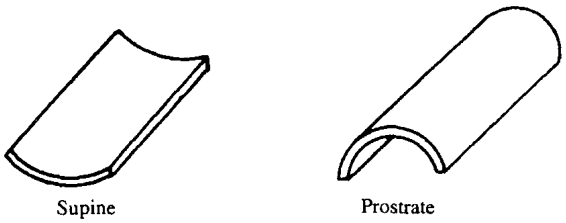

Figure 2. The pictures of Supine tile and Prostrate tile

When the video was over, the teacher displayed the following problem in multimedia courseware.

Problem A: Now there are 34 people. One can make 38 prostrate tiles or 60 supine tiles a day. Assume some tiles are needed, among them the supine tiles are two times of the prostrate tiles. Tell: how many people are needed to make prostrate tiles and supine tiles respectively?

The textbook offers a matching-up problem as follows. Now there're 22 workers in a workshop. One worker can make 1200 screws or 2000 nuts a day. Assume a screw goes with two nuts to make a set. Tell: in order to produce exact sets of screws and nuts in everyday, how many workers should be assigned to make screws and nuts respectively?

By analyzing the given information, the teacher led the students to discover that the ratio of prostrate tiles to supine tiles was 1:2. Assume x as the number of people needed to make prostrate tiles, and the equation would be $2\times 38x=60\times(34-x)$. It's easy to work out x=15. Therefore the people needed to make supine tiles were 18. The teacher gave a detailed solving process on the blackboard.

Then, the teacher presented the second problem for the students.

Problem B: Now a number of people are needed to make prostrate and supine tiles. It will take one person 10 days to finish the work. Assume in the first period some people make tiles for 2 days, and then one person joins them to finish the work together in 2 days. Tell: how many people with the same working efficiency and working style are needed in the first period?

The problem in the textbook goes like this. Now it takes one person 40 hours to arrange some books. Assume some people arrange books for 4 hours and then 2 more people join them. And they work together to finish the job in 8 hours. Tell: how many people with the same work efficiency are needed in the first place?

The teacher suggested that the total amount of the work be regarded as 1. With the equation that workload = productivity $\times$ the number of people $\times$ time, the students assumed that there were x who started the work in the first place and had the equation $\frac{2x}{10}+\frac{2(x+1)}{10}=1$.

Hence x=2.

These two examples, adapted from the original problems in *the Nine Chapters on Mathematical Art*, shared the same problem solving strategy with the matching problems in the textbook. The author noticed in class that solving the mathematical problems under the

same historical background made the teaching environment lively and natural, and greatly raised students' learning initiative. This was also backed up by the post-class survey.

2.2.3 In-class exercises

In the next section, the teacher displayed the original problem recorded in *the Nine Chapters on Mathematical Art* and required the students to solve it.

Teacher: Please think about it and discuss with your tablemates.

Student C: Teacher, this problem is simpler than what you have just taught us. It can be solved by creating an equation related to the time which will take.

Teacher: OK. You may have a try and tell me the result.

Student C: It's weird. The result is not an integer.

Student D: 25.33333 tiles! How can they make it?

Student E: Teacher, how did the ancient people solve it?

……

Teacher: Great. You've done it correctly. Now, let's watch a video clip to see how the ancient people solve it.

And the video was played, displaying the solution used in *the Nine Chapters on the Mathematical Art.*

Student F: I See. The ancient people were smart, but the current solution seems much simpler and clearer.

……

Those three HPM video clips were integrated into the practical teaching perfectly, just as *the wind sneaked into the night and moistened everything silently* that is from the Tang poem by DU Fu.

2.2.4 Induction and expansion

Amid the ancient charms of the mathematical history, the class came to an end. The teacher asked the students to generalize the basic process of solving practical problems by creating linear equations with one unknown. And then they came up with a process as: set an unknown→create an equation→solve the equation→check the result→finalize the answer (see figure 3). The teacher stressed repeatedly that the key to solve the problem was to find *the two expressions equal to one another*.

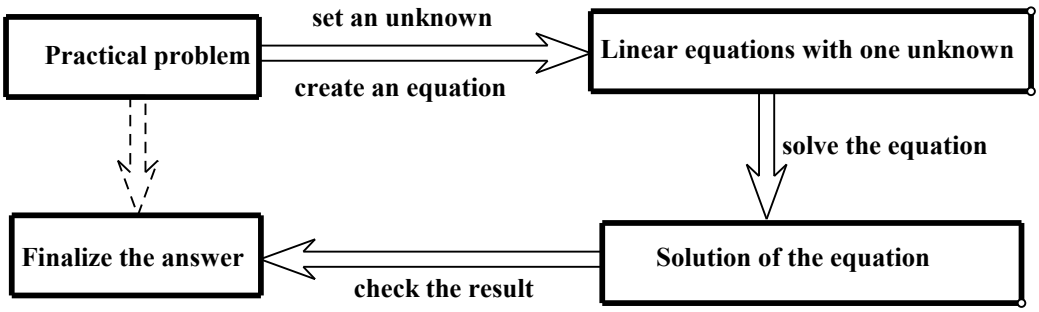

Figure 3. Class generalization

In the end, the teacher asked the students to go over the problems in the textbook and assigned an extended word problem in the textbook as after-class practice so as to reinforce their consciousness of applying the linear equations with one unknown. This would enable the students *to learn while practicing* and *to think while studying*.

Exercise:

Please collect some data concerning the important issues like climate, energy-saving and economy, make an analysis and create a problem that can be solved with linear equations with one unknown. State clearly the problem and its solving process.

2.3 Student Feedbacks

This teaching was carried out in a common class with 61% minority students in western China. The researchers conducted a post-class survey among the 42 students who attended the class. 45% of the students said *they completely understood the ancient problems after teacher's lecture*. 22% of the students said *they completely understood the teaching*. 29% of the students said *they partly understood.* 4% of them said *they did not understand*. As to the opinion on *teaching integrated with the history of mathematics*, only 4% of the students said *they did not like it*.

As to the question *which forms you like most to integrate history of mathematics in class,* the students' choice goes like this: HPM video clips, stories told by teacher, courseware display, pre-class reading materials, in-class reading materials, and after-class reading homework.

As to the subjective question *what impressed you most in this course*, nearly all students talked about how they liked the HPM video clips. They said that they learnt a lot about ancient Chinese mathematics, had a great interest in ancient mathematics, and were proud of ancient Chinese mathematicians. Here are some answers from the students.

(1) On *the Nine Chapters on the Mathematical Art*

Student G: This video let us get to know the history of ancient Chinese mathematics.

Student H: The mathematical problems illustrated in the videos interest me a lot and push me to think about them.

Student I: After watching these videos, I wanna have a look at the book.

(2) On Ancient mathematical problems

Student J: What the teacher said about the ancient mathematical problems got my attention. They may be used to solve real life problems.

Student K: The ancient mathematical problems are quite interesting. It's good for our brain, activate it.

(3) On the class integrated with HPM video clips

Student L: This class tells me that our method to solve equations is 1500 years earlier than Europeans. I get to know a lot of mathematicians, learn how to solve some ancient problems listed in *the Nine Chapters on the Mathematical Art*, and have great interest in the history of mathematics.

Student M: This class vividly tells me the history of equations and let me love to solve the practical problems and linear equations with one unknown.

Student N: The HPM video clips are funny. It seems we were back in the ancient society. We, modern people, can solve the ancient problems.

3 Conclusion and inspiration

From students' active in-class performance and simple answers in the survey, we can see that HPM video clips play an important role in guiding the teaching progress, expanding students' horizon of knowledge, and enhancing the achieving of knowledge and techniques, process and methods as well as attitude and value. Besides, HPM video clips also help to solve middle school teachers' perplexities in the following ways:

(1) Presenting historical materials lively, it's time saving time and effort saving.

Within a short time, HPM video clips can display complete mathematical information through rich forms. It enables the teachers who even don't know much about the history of mathematics to teach in a flexible way. It assists teachers to help students enrich their knowledge and understand mathematics better. It is of great support for the integration of mathematical history with practical teaching. If applied properly, it can be conducive to the harmony for the teaching factors.

(2) Exploring the teaching from diversified levels, it arouses students' interest to study.

HPM video clips present the study task to students in a way that suits their cognitive characteristics. It not only creates a lively learning environment for students, but also helpful to interest and motivate them. As the videos can be played repeatedly without the limitation of time and place, it rectifies the traditional teaching approach that what a teacher says is all, creates personal and diversified learning paths for students to explore by themselves, and provides them with more opportunities to think and make a decision.

(3) It catches students' attention and advances their study.

The teaching practice shows that HPM video clips can attract students' attention in a short time. On the one hand, they meet students' curiosity; on the other hand they help to

improve their knowledge construction and application, technique forming and consolidation, and the betterment of study strategies. And what's more, the effect of study and memory lasts longer.

This practical teaching with integration of HPM video clips brings us some inspirations as follows.

(1) Professional production and application promotion

The use of HPM video clips can improve students' understanding and internalization of related knowledge points, and is good for them to carry out complex mathematical research activities. To realize this objective, the producers should not only have a complete knowledge of the history of mathematics, but also master relatively advanced shooting and production techniques. Therefore, it's necessary to organize a technical team specifically for the selection and production of historical materials of mathematics. The HPM video clips produced by this special team may serve as a handy tool for all teachers as natural as the use of paper and pens. This will solve the perplexities felt by middle school teachers lacking proper knowledge of mathematical history, eliminate the limitation brought by techniques, and promote the application and development of HPM video clips.

(2) Learning from others and developing flexibly

At present, the research and application of HPM video clips in China is still in the exploring stage. On the one hand, we need to learn from the foreign experience and search for new ideas and new techniques, on the other hand we should initiate reasonable application blueprint based on our own teaching practice. For example, we may focus on the development of video production techniques from the angles of opening and exploratory, and at the same time build a complementary resource library of mathematical history so as to provide a lasting guarantee for researchers and educators.

All in all, the study on the history of mathematics with orientation of mathematical education and the exploration of HPM video clips will be important research directions in the future (Wang & Zhang, 2006).

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