Oral Presentation

MATHEMATICAL LESSONS IN A NEWSPAPER OF PORTO IN 1853 (PRIMARY EDUCATION)

Hélder Pinto

Centre for Research & Development in Mathematics and Applications (CIDMA), University of Aveiro

The Industrial Association of Porto was founded in 1849. Industrial education was one of its first priorities, having created the Industrial School of Porto in 1852. In 1853, António Luís Soares, a professor of this school, published several arithmetic lessons for primary education in the Newspaper of the association - Lecture of the arithmetic elements in the Association's primary school class. These lessons were addressed to teachers who should reproduce them to their students. The author includes several considerations about the importance of propagating basic math instruction, either for industry or for the trade workers, two important activities for the city. In this paper, we will present these classes and show how this text may be used in the classroom (primary source in the history of mathematics).

INTRODUCTION

> *As vias férreas são a escola primária da indústria, a escola primária popular é a via férrea de toda a civilização*. (Castilho, 1864)
>
> [The railways are the industry's primary school; the popular primary school is the railroad of the whole civilization]

The *Associação Industrial Portuense* (Industrial Association of Porto) was founded on May 3, 1849 (approved by royal law only on August 26, 1852) in the city of Porto (Portugal), still subsisting today under the name *Associação Empresarial de Portugal* (Portuguese Entrepreneurial Association).

> "(...) The Industrial Association of Porto aims to develop the national industry – instructing the industrial classes and particularly the industrial workers in elementary arithmetic, geometry, drawing, and mechanical, chemical and physical arts; and especially in the study of machines, equipment and processes, which are being successively invented or perfected so that the Portuguese industry can be placed at the same level as the most advanced nations (...)."
>
> (Statutes of the Industrial Association of Porto; Chapter II, Article 4, 1852)

In fact, the industrial education was one of the first priorities of the association, having created the Industrial School of Porto almost immediately (December 30, 1852). Even before that, on December 6 of that year, a course on «reading and writing» began, which was attended by 117 students. This group included 25 individuals who attended these classes in order to propagate this knowledge to several villages around the city of Porto. António Luís Soares (1805 - 1875), a professor of the Polytechnic Academy of Porto since 1836 and of the Industrial School of Porto since 1852, was intended to address these teachers in order to "present some works on the teaching of arithmetics". Just a few could "attend the invitation" and so the alternative was to publish such works in the Newspaper of the Industrial Association of Porto - Lecture of the arithmetic's elements in the Association's primary school class (*Da exposição dos elementos da arithmetica na aula de instrucção primaria da Associação*). This newspaper usually published news about the association as well as others concerned with the city and the surrounding region (they were mostly about industrial and commercial events but there were also others about political and cultural activities).

Thus, arithmetic lessons for primary education were published in several numbers (usually once a month, from April to December of 1853). These were addressed to teachers who should reproduce them to their students.

These arithmetic classes were divided into sections, namely: 1. Formation of the numbers; 2. The first arithmetic operations. After these, several tables of units' conversions were also published (linear, area, capacity and weight).

As an introduction to these classes, António Luís Soares did a brief but interesting analysis of how primary teaching was in Portugal (particularly in mathematics), as well as several considerations about the importance of propagating basic math instruction, either for industry or for the trade affairs workers, two very important activities for the economy of the city of Porto at that time.

In the next pages, we will present António Luís Soares and a brief history of the superior schools in Porto. We will also reproduce some parts of these arithmetic classes and show how this publication may be used in the classroom (primary source in the history of mathematics) and how it may be interesting from the perspective of consolidating very basic notions of the discipline of mathematics.

ANTÓNIO LUÍS SOARES, A PROFESSOR IN TWO SCHOOLS (THE HISTORY OF SUPERIOR SCHOOLS IN PORTO)

The Royal Academy of Navy and Trade Affairs of the City of Porto was the first academy in the city of Porto. It was created in 1803 (laws from February 9 and July 20) and had three mathematical years, classes of Rational and Moral Philosophy, Trade Affairs, Drawing (this class existed in the city since the year 1779), the English Language and the French Language. Note that the three years of mathematics were a copy of what was practiced in the Royal Academy of Navy of Lisbon.

This academy had a Master of Navigation, responsible for the practical teaching of navigation; this master was subordinated to the professor of the third mathematical year and was, somehow, the continuation of a Nautical class established in the city of Porto in 1762. This Nautical class was created to provide officers required for the two war ships that the city used to protect the goods that were destined to Brazil (former

Portuguese colony). This class, as well as the two ships, were paid for by the city itself through city taxes and its customs. This class was under the supervision of the General Company for Agriculture of the Upper Douro Vineyards, a monopolistic company created by the Marquis of Pombal, with great privileges in the Douro Valley and the city of Porto. In 1878, this company started to fund this Nautical class through the dividends of their shareholders. This financing scheme would be continued during the creation of The Royal Academy of Navy and Trade Affairs of the City of Porto in 1803, which made it a unique case in the higher education in Portugal at that time. In fact, the civilian character of this academy was present since its inception and its goal was linked to what was important to the city of Porto: forming good sailors and good merchants was fundamental to the city's economy, which was focused on commercial activities with Brazil and northern Europe.

Finally, note that the scientific production of this academy was quite poor but the implementation of upper level mathematical classes (as is the case of the differential and integral calculus) was an important step in Porto, somehow facilitating the possibility of establishing a school with polytechnic characteristics in the city.

In 1837, the city of Porto witnessed this academy of navy and trade affairs becoming the Polytechnic Academy of Porto, a school dedicated to industrial sciences (law of January 13, 1837). Its creation brought about a new paradigm for the higher education that existed in the city, highlighting the three engineering courses (Mining Engineering, Construction Engineering, Bridges and Railroads Engineering) that were then implemented. Despite the importance of the engineering courses, the Polytechnic Academy also presented some courses that were not typical in a polytechnic school, but, certainly, originated in its predecessor - for example, the course of Navy Officers, Ship Pilots and Trades Affairs people.

In addition to the new courses, it should be noted that there was a significant increase in the number of mathematical and scientific disciplines, established with the creation of the new academy; the eleven disciplines then implemented were divided into four sections: Mathematics (5), Rational Philosophy (4), Drawing (1) and Trade Affairs (1). However, this new academy, unlike its predecessor, was administered and funded directly by the Ministry of the Kingdom (central government in Lisbon), *i.e.*, the city of Porto lost control of its academy. It should be also noted that in 1837 the socio-economic context of Porto was substantially different because Brazil had become independent in 1822. By this time, the Liberal Passos Manuel was in power (who reformed many areas like, for instance, the higher education) and Portugal was in a process of some industrialization and in an upgrade of the communications' network.

During its existence, the polytechnic academy gradually evolved into a university level institution, originating a lack of proper education for mid-level employees (especially for the industry). So, the city of Porto, through the Industrial Association of Porto, created a new school in 1852: *Escola Industrial do Porto* (the Industrial School of Porto). It is important to note that the Industrial Association of Porto was formed by influential individuals of the city, with important connections with the economy of the city. In June of 1853 (*Jornal da Associação Industrial Portuense*, 20, June 1, 1853, p. 307), this association was composed by 192 artisans/artifices, 36 manufacturers, 174 traders, 48 goldsmiths, 32 landowners, 30 medical doctors and chemists, 84 public servants, 5 farmers, and 7 militaries (total: 608 members). In this manner, the city once again controlled an important school that could be managed in such a way as to fulfil the effective needs of the city. When this school was created, there was only one single graduation divided in two types: *Habilitação simples* (simple graduation) and *Habilitação plena* (full graduation). After a few years, in 1864, the school changed its name to *Instituto Industrial do Porto* (Industrial Institute of Porto), with a more complete educative offer: factory director, overseer of public works, overseer of machines (steam engines), overseer of mines, telegrapher, master of public works and master of chemistry. There was also a factory worker graduation that was preparatory for all of the other courses.

The Polytechnic Academy of Porto and the Industrial School of Porto were connected in various ways like, for instance, many professors taught simultaneously at both schools and both shared the same building until 1933 (about 80 years…). There were also proposals to fuse both into one single institution: in 1864 by José Maria de Abreu (a political proposal from Lisbon) and in 1882 by Rodrigues de Freitas (professor from Porto); however, there was a major Reform of the Polytechnic Academy of Porto in 1885 (in 1884 the Academy received the most important Portuguese mathematician at that time: Gomes Teixeira) and, from that point, the difference between the two institutions was definitely settled: one more theoretical (high level studies) and the other more technical (intermediate studies). One of the most important characteristics that both schools shared was the fact that both were created to attend the effective necessities from the city (sailors, trade men, engineers, industrial workers,…) and both were funded by the initiative of private institutions of the city.

António Luís Soares (Porto, 1805 - 1875) was a professor of the Polytechnic Academy of Porto since 1836 (First discipline: Arithmetic, Elementary Geometry, Trigonometry and Elementary Algebra) and a professor of The Industrial School of Porto since 1852 (Arithmetic, Algebra and Geometry). There is a lack of information about this professor and, except the text that we present here, there is only a reference (Scipião, pp. 9-10) to another text (*Exposição dos elementos de Aritmética para uso dos estudantes do Colégio de Santa Bárbara na cidade de Pelotas*, Pelotas, Brazil, 1849), an arithmetic textbook published in Brazil; no copy of this text has been found (Scipião says that António Soares was in Brazil between 1847 and 1851, but no justifications for his stay are pointed out).

LECTURE OF THE ARITHMETIC ELEMENTS IN THE ASSOCIATION'S PRIMARY SCHOOL CLASS

The Industrial School of Porto was created in 1852 (December 30) by the Industrial Association of Porto. Even before that, although primary education was not their main focus, on December 6 of that year, a course on «reading and writing» opened, which was attended by 117 students (many of them were destined to the Industrial School). This group included 25 individuals who attended these classes in order to propagate this knowledge to several villages around the city of Porto. António Luís Soares was intended to address these 25 “teachers” in order to present some works on the teaching of arithmetic. Just a few could “attend the invitation” and so the alternative was to publish such works in the Newspaper of the Industrial Association of Porto, a biweekly newspaper whose first issue came out on August 15, 1852 (fig. 1).


JORNAL DA ASSOCIAÇÃO INDUSTRIAL PORTUENSE.

NUMERO 16. SEXTA FEIRA 1 DE ABRIL. ANNO 1853.

Figure 1. Header of the Newspaper of the Industrial Association of Porto (number 16, April 1, 1853).

Thus, arithmetic lessons for primary education were published in several numbers (usually once a month, from April to December of 1853). These were addressed to teachers who should reproduce them to their students. The text published could be divided in two parts: one that should be replicated to students and another (presented in italics) that was dedicated to professors. The italic part aimed to provide guidance to future professors (for example: how many times should the exercise be repeated; what should the student know at the end of each lesson). These Arithmetic classes were divided into sections, namely:

– Formation of the numbers (in which a study of the metric system is included and compared with the usual Portuguese measures);

– The first arithmetic operations (only addition and multiplication).

After these, several tables of units’ conversions were also published (linear, area, capacity and weight). See the following table highlighting the structure of these classes (table 1).

<table>
<tr><th>Date</th><th>Number
(pages)</th><th>Sections</th><th>Lessons</th><th>Observations</th></tr>
<tr><td rowspan="2">April, 1</td><td rowspan="2">16
(pp. 244-248)</td><td rowspan="7">Section 1:
Formation of
the numbers</td><td>Introductory observations
1. numeration system</td><td></td></tr>
<tr><td>2. spoken numbering</td><td></td></tr>
<tr><td rowspan="2">May, 1</td><td rowspan="2">18
(pp. 277-281)</td><td>[3.] written numbering</td><td></td></tr>
<tr><td>4. observations about quantities</td><td></td></tr>
<tr><td rowspan="2">June, 1</td><td rowspan="2">20
(pp. 307-312)</td><td>5. metric system (units, multiples and submultiples)</td><td></td></tr>
<tr><td>6. metric system (written abbreviations)</td><td></td></tr>
<tr><td rowspan="2">July, 1</td><td rowspan="2">22
(pp. 339-345)</td><td>7. the “big” numbers; roman numerals</td><td></td></tr>
<tr><td rowspan="2">Section 2:
The first
arithmetic
operations</td><td>1. addition</td><td></td></tr>
<tr><td rowspan="2">July, 31</td><td rowspan="2">24
(pp. 374-383)</td><td>addition (cont.)
[2.] multiplication</td><td>Why did he not publish anything about subtraction and division?</td></tr>
<tr><td colspan="2" rowspan="2">Tables of units conversions</td><td></td></tr>
<tr><td>August, 1</td><td>1 (T2)
(pp. 2-3)</td><td>Just the continuation of tables from the previous text.</td></tr>
<tr><td>December, 1</td><td>9 (T2)
(pp. 138-141)</td><td colspan="2">How to convert the ancient Portuguese units to the metric system and vice versa</td><td>At the end of this text the indication “To be continued” appears, but this was the last text in this newspaper by António Soares.</td></tr>
</table>

Table 1. Structure of the classes.

António Luís Soares, as an introduction to these classes, did a brief but interesting analysis of how the primary teaching in Portugal was (particularly in mathematics), as well as several considerations about the importance of propagating basic math instruction, either for industry or for the trade affairs workers, two very important activities for the economy of the city of Porto at that time.

In the next pages, we will explore some aspects of these classes in detail. The first section is denominated "Formation of the numbers"; in the first lesson the author explains how to increase and decrease a set of objects and to compare two small sets of objects (table 2).

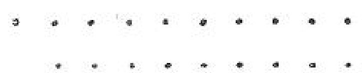
	Which set is bigger? It is easy to see that it is the upper "line" (it is easy to compare small sets of objects… just align all of the objects, one set above the other…)

Table 2. Example from Lesson 1 ("Formation of the numbers").

In the second lesson, the spoken numbering until 10 was taught (table 3).

Um e um fazem dous. Dous e um fazem tres. &c. &c.	One and One does two; Two and One does three; …

Table 3. Example from Lesson 2 ("Formation of the numbers").

In the same lesson, he taught the numbers between 10 and 10000 (table 4).

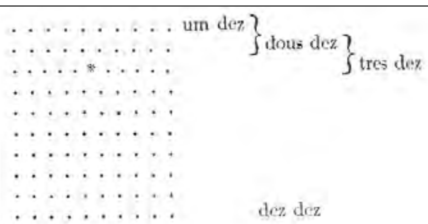
	One ten two tens three tens … ten tens (= hundred)

Table 4. Second example from Lesson 2 ("Formation of the numbers").

At this stage, the author did not use words like twenty, thirty, forty, etc; he also did not use the words eleven, twelve, thirteen, fourteen, etc. For example, 14 is one ten and four; 47 is four tens and seven; 30 is three tens; 328 is three hundreds, two tens and eight. Note that, until now, the author did not introduce the writing digits (1, ..., 9) nor the digit zero (0).

In the third lesson, the writing digits (symbols to use in writing) were presented, from 1 to 9. After that, the author said that it is possible to write the numbers above nine without creating anymore new symbols (he did remark that if each number had its own symbol, it would be impossible to remember them all…). There is only the need for the value in each digit to vary depending on its position (table 5).

um dez e um 1 1	dous dez e um 2 1	One ten and one	1	1
um dez e dous 1 2	dous dez e dous 2 2	One ten and two	1	2
um dez e tres 1 3	dous dez e tres 2 3	One ten and three	1	3
&c.		…	…	
nove dez e nove 9 9		Nine ten and Nine	9	9

Table 5. Example from Lesson 3 ("Formation of the numbers").

At this point, the digit 0 was presented (used when there was a "gap" in the "units", to distinguish, for example, the numbers 2 and 20) and a generalization of this construction for numbers up to 1000. The author also remarked that each digit has two (possibly) different values: a proper or depending on the form (7) and a local or relative to its position (7, 70, 700, …). For example, the digit 7 represents 7 hundreds in the number 723; the digit 7 represents 7 tens in the number 273. The author said that this arrangement was "an ingenious agreement, the best abbreviation that was possible to think".

In the fourth lesson, the author made several observations about quantities. He observed that the numbers ("a number is a set of things with the same name") learned are useful to count, for instance, men and trees. But what about a big quantity of grains of wheat? It was impossible and inutile to count each single grain… There are things that we cannot measure the quantity using only numbers; so, it is necessary to have another type of measurement units for dry volumes (grains and beans, for instance), surfaces (lands and fields), liquid volumes (milk, wine), weights (reference to the scale of two arms), time and money (coins). At this point, the author only did a first introduction to this subject and a reinforcement of the need of other measurement units for everyday life/industry/commerce (the numbering system alone is not sufficient…).

The metric system is formally presented in lesson number 5 (units, multiples and submultiples). The author explained that, historically, the first measurement units were, naturally, the *Palmo* (hand) and the *Pé* (foot), but this kind of units had proven to be difficult to work with and a source of problems and errors over time… Afterwards, he made reference to France and to the difficulties in the implementation of the metric system. But he thought that the context in 1853 was different:

> "However, there was hope that this time was possible to introduce the metric system in the plenitude. The artisans did not fight against this system anymore, because they are familiarized with the new units by the visits of the academic/theoretic people to their shops."
>
> (*Jornal da Associação Industrial Portuense*, 20, June 1, 1853, pp. 307-308)

The next step, in his opinion, was to propagate these units in the common retail trade, because it would facilitate commercial transitions in everyday life. Then, he presented some common old Portuguese units in such a way as to highlight two major problems: first of all, it is difficult to memorize all of the relations between them. For example, he presented the following units: length units – 1 *braça* = 2 *vara*; 1 *vara* = 5 *palmo*; 1 *palmo* = 3 *pollegada*!!!!!; weight units – 1 *quintal* = 4 *arroba*; 1 *arroba* = 32 *arratel*; 1 *arratel* = 16 *onça*; 1 *onça* = 8 *oitava*!!!! It is also very difficult to operate with them (for instance, what is the relation between *quintal* and *oitava*?).

Afterwards, the author finally presented some metric units: the meter (linear); the are (surface; 100 square meters; note that the square meter is too small to measure fields…); the liter (capacity) and the gram (weight). Then, the multiples of these units were presented («Deca» means 10 primitive units, «Hecto» means 100 primitive units and «Kilo» means 1000 primitive units) and submultiples («Deci» means 1/10 primitive units, «Centi» means 1/100 primitive units and «Milli» means 1/1000 primitive units). The author then stated an important warning: for surfaces ($1\ m^2 = 100$ deci-m^2) and capacities ($1\ m^3 = 1000$ deci-m^3) we must be very careful when working with multiples and submultiples.

Lesson number 6 was the continuation of the presentation of the metric system. He taught the written abbreviations and presented various tables comparing the old Portuguese units with the “new” metric system (units for big lengths, small lengths, agricultural, small surfaces, liquid volumes, dry volumes, solid volumes like wood, weights and small weights).

In lesson number 7, the author returned to the numbering system, expanding what he had already taught regarding the “big” numbers (thousands, millions, billions, trillions,…). On the other hand, it is only at this point that he teaches the formal name of some numbers like 11 (eleven instead of ten and one), 12, 13, 14, 15, 20 (twenty instead of two tens), 30, 40, 50, 60, 70, 80 and 90.

Curiously, at the end of this lesson, he presented the Roman numerals (using the history of mathematics in the classroom…). As an introduction to this subject, he said that the digits that were presented in these classes did not come from antiquity and many people used the alphabetic letters to write numbers, namely the Greeks and the Romans. The author explained the roman numerals because they were still in use at the time (ancient coins, opening date of buildings and to numerate items in texts) and it is also a positional system (the same symbols could represent different numbers; for example, IV is different from VI)

The second section is denominated “The first arithmetic operations”. In the first lesson of this new section, the author explained addition, beginning with a table with all sums up to 10 (fig. 2).

	2	3	4	5	6	7	8
2	4	5	6	7	8	9	10
3	5	6	7	8	9	10	
4	6	7	8	9	10		
5	7	8	9	10			
6	8	9	10				
7	9	10					
8	10						

Figure 2. Addition table.

Afterwards, he taught how to add small numbers (up to 10). Next, to explain how to add bigger numbers, he presented a scheme resembling an abacus (though he did not use this designation) with an example: 326 plus 172 (table 6).

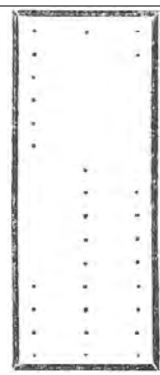
 326	 Total: 498 (just count the dots in each column)

Table 6. Example from Lesson 1 (“The first arithmetic operations”).

Of course, after this example, the author explained what we should do when the sum of two digits in a column is bigger than 9. Afterwards, he presented another example, but now with no abacus scheme (table 7).

78.564. 3.2895 73468 7.3452 258379 ’ ’ ’ ’ ’ ’	First right column: 4 + 5 + 8 + 2 = 19. And 1 (’) goes to the next column. Second right column: 6 + 9 + 6 + 5 **+ 1** = 27. And 2 (’ ’) goes to the next column. …

Table 7. Second example from Lesson 1 (“The first arithmetic operations”).

Afterwards, he presented a very pedagogical example to emphasize the fact that we should always do the columns from right to left and not the other way around (table 8).

354 213 401 ——— 968	In this first example is indifferent.	632 765 528 852 ——— 26	But not in this second example… If we start on the left column we need to erase the 6 that will be corrected with the value that goes from the mid column (**1**6).

Table 8. Third example from Lesson 1 (“The first arithmetic operations”).

Then, he noted that adding numbers with decimal parts is basically the same and presented several examples (highlighting that this is not more difficult than adding integers). The decimal points just need to be vertically aligned and the method is exactly the same. In fact, this is an important advantage from the metric system: it is easier to work (in this case, add) with the sub-units of the metric system than with the old Portuguese subunits. And to emphasize this point of view, he presented a very difficult example (table 9) using linear units (note that: 1 p(olegada) = 12 l(inha); 1 P(almo) = 8 p; 1 B(raça) = 10 P).

B.	P.	p.	l.	
30	3	7	5	5 l. + 11 l. = 16 l. = 1 p. + **4 l**.
12	8	2	11	7 p. + 2 p. + 1 p. = 10 p. = 1 P. + **2 p**.
				3 P. + 8 P. + 1 P. =12 P. = 1 B. + **2 P**.
13	2	2	4	30 B. + 12 B. + 1 B. = **43 B** (there is a typo).

Table 9. Fourth example from Lesson 1 (“The first arithmetic operations”).

For the students’ homework, the author suggested more examples with the old Portuguese units, even more complicated, in order to convince everyone that it was confusing to work with the old units and it was necessary and easier to adopt the «new» metric system.

The second lesson of this section concerned multiplication that was presented as a particular case of addition, in which all of the numbers to add are equal. First of all, he presented all of the products between two numbers lower than 10, explaining how to construct the following multiplication table (fig. 3).

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Figure 3. Multiplication table.

Afterwards, he explained how to multiply bigger numbers recurring to an example (table 10).

Suppunhamos que se quer juntar 4 vezes o numero 237, se escrevessemos como para achar a somma ter-se-hia .... 237 Mas devemos resumir a escripta, e vê-se que seria bastante assentar 237, e indicar por baixo o numero de vezes que era necessario escrever este numero, assim .... 237 237 237 Ora pela columna das adicções sabemos que a somma é de 4 vezes 7 unidades........ 4 — 28 4 3 dezenas........ 12 4 2 centenas........ 8 —	237+237+237+237 = 237x4 7+7+7+7 = 7x4 units = 28 units 3+3+3+3 = 3x4 tens = 12 tens 2+2+2+2 = 2x4 hundreds = 8 hundreds So, the final result is 28 + 120 + 800 = 948

Table 10. Example from Lesson 2 ("The first arithmetic operations").

After this first example, he provided other examples, including some numbers with zeros because, in that situation, there are simplifications. Unfortunately, the example that he gave has several typos and, consequently, it was not very clear for the reader what he meant (table 11).

Multiplicando. . 376864 Multiplicador. . 27020 Prod. por 20........ 753728 » 7000........ 2638048 » 20000........ 753728 1018286528	Intermediated lines are not aligned properly; A zero is missing at the end of the final result.

Table 11. Second example from Lesson 2 ("The first arithmetic operations").

This passage is an example of several typos in this text that, probably, caused many difficulties for the readers who intended to replicate these classes. Similarly to other places of this text, he also suggested that the students should practice a lot by doing many exercises (in this case, many multiplications).

Once again, the author pointed out that there is no problem in working with numbers with decimal parts; it is not more difficult than the multiplication of integers – we just need to be careful with the position of the decimal point.

Finally, these classes ended with the presentation of several tables connecting the old Portuguese units with the metric system. These tables were very complete and covered all important economic areas of that time: length (big and small); area (big and small); dry and liquid volumes; weight (big and small, including specific tables for chemical and pharmaceutical drugs, gold, silver and diamond; to see the difficulties of operating with the ancient units note that 1 *Marco* is equal to 1152 *Oitava* if it is silver and 1 *Marco* is 768 *Oitava* if it is gold). See, as an example, fig. 4.

PEZOS

Systema metrico								
Centigram. 4,97969618	Grão							
Grammas 1,195127083	24	Scropulo						
Grammas 3,58538125	72	3	Oitava					
Grammas 28,68305	576	24	8	Onça				
Grammas 458,9288	9216	384	128	16	Arratel			
Kilogrammas 14,6857216	294912	12288	4096	512	32	Arroba		
Kilogrammas 58,7428864	1179648	49152	16384	2048	128	4	Quintal	
Kilogrammas 793,02896640	15925248	663552	221184	27648	1728	54	13,5	Tonellada

Figure 4. Table connecting the old Portuguese weight units with the metric system.

Later that year (December), António Luís Soares published similar tables in this newspaper again, under the title "How to convert the ancient Portuguese units to the metric system and vice versa". This new text was now dedicated to industrial and trade workers and should be used as a guide table by those professionals. In fact, all of the texts published by this author in the Industrial Association's Newspaper had a professional teaching agenda, *i.e.*, to instruct the low and middle working classes.

APPLICATION IN THE CLASSROOM

There are various possibilities for using this ancient publication in the classroom:

- Put older students teaching arithmetics to younger students;
- Give small parts of the text to the students for interpretation and to compare the «old» mathematics and units with the present time;
- Ask to write a similar text (classes of mathematics in a public newspaper) but adapted to current times;
- Research work, as it could be an interesting interdisciplinary theme: History (Portuguese history, scientific history, implementation of the metric system;…); Chemistry/Physics (the measurement units, the industrial applications of these disciplines); Biology (some units used in agriculture) and Portuguese language (reading and interpretation of a text, comparing the old language with the current one).

Note that this text is in Portuguese, a fact that constrains its utilization to natives of this language (for instance, Portugal and Brazil). However, this is a good example that it is possible to find educative and attractive texts in many different contexts. This text is particularly interesting because it is strongly linked with the history of the city of Porto and it is very rich in contents and examples. In fact, with the study of this text, it is possible to:

- Review some basic arithmetic's notions;
- Understand and emphasize the importance of our decimal numbering system (connection with the metric system), showing that it is a good system to operate and easy to work with;
- Recall some basic properties of the decimal system (always omnipresent in everyday math classes, but not always noted): positional (the value of each digit depends on its position), economical (just needs ten digits/symbols) and the importance and significance of the digit zero in a number (how to distinguish 202 from 2002 without the zero?);
- Emphasize the difficulty of doing basic calculations in other eras (when the technologies were too rudimental);
- Understand that the metric system was a major breakthrough that came to facilitate measurements and calculations (avoiding many errors and complications in calculations).

CONCLUSION

The classes presented here are intimately connected with the socio-economic context of the city of Porto: an industrial and commercial city and the second city of the country. On the other hand, these classes were sponsored by an Industrial Association, which explains the fact that they were classes with a very practical goal (teaching the basic arithmetic always with the aim of using it in the industrial/commercial trade). It is also relevant to note that the metric system was implemented officially in Portugal in 1852 (the first attempt was in 1814 but with no success), so it was a very important and new subject when these classes were published. All of these factors explain the (excessive?) focus on the metric system and its relation with the old Portuguese units. However, this text has a very rich content in basic arithmetic and could be applied in the classroom (primary source in the history of mathematics) in various ways, for example, as an interesting tool for consolidating very basic notions of the discipline of mathematics.

ACKNOWLEDGEMENTS

This work was supported by Portuguese funds through the CIDMA - Center for Research and Development in Mathematics and Applications, and the Portuguese

Foundation for Science and Technology ("FCT–Fundação para a Ciência e a Tecnologia"), within project PEst-OE/MAT/UI4106/2014.

REFERENCES

Alves, Jorge Fernandes (1996). *O emergir das associações industriais no Porto (meados do século XIX)*, in Análise Social, vol. 31, n.º 136/137, pp. 527-544. Lisboa: Instituto de Ciências Sociais.

Alves, Luís Alberto Marques (2006). *ISEP: Identidade de uma Escola com Raízes Oitocentistas*, in Sísifo, Revista de Ciências da Educação, n.º 1, pp. 57-70. Lisboa: Universidade de Lisboa.

Carvalho, A. Scipião G. (1937). *A Matemática na Academia Politécnica do Pôrto*, in O Ensino na Academia Politécnica, pp. 1-31. Porto: Universidade do Porto.

Fernandes, Rogério (1993). *Génese e Consolidação do Sistema Educativo Nacional (1820-1910)*, in O Sistema de Ensino em Portugal, Séculos XIX-XX (coord. Maria Cândida Proença), pp. 23-46. Lisboa: Edições Colibri.

Jornal da Associação Industrial Portuense (1853). n.º 16, 1 de Abril, pp. 244-248; n.º 18, 1 de Maio, pp. 277-281; n.º 20, 1 de Junho, pp. 307-312; n.º 22, 1 de Junho, pp. 339-345; n.º 24, 31 de Julho, pp. 374-383; n.º 1, 1 de Agosto, pp. 2-3; n.º 9, 1 de Dezembro, pp. 138-141; Porto.

Pinto, Hélder (2013). *A Matemática na Academia Politécnica do Porto*; Tese de Doutoramento. Lisboa: Universidade de Lisboa.

Serra, António Dias da Costa (1989). *História do Instituto Industrial do Porto convertido no Instituto Superior de Engenharia do Porto, em 1974*; Porto.

Torgal, Luís Reis (1993). *A Instrução Pública*, in História de Portugal, o Liberalismo (1807-1890), Vol. V (dir. José Mattoso), pp. 609-651. Lisboa: Círculo de Leitores.

Torgal, Luís Reis e Vargues, Isabel Nobre (1984). *A revolução de 1820 e a instrução pública*. Porto: Paisagem Editora.