Oral Presentation

DECIPHERING THE DOODLINGS OF THE "SHOEBOX COLLECTION" OF THE PAUL A.M. DIRAC PAPERS

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Florida State University Libraries Special Collections and Archives hold the complete papers of Paul Adrien Maurice Dirac (1902-1984). In this workshop we shared an overview of the digital scans of 72 documents initially identified for study as part of a project to digitize and preserve the "shoebox papers" in the collection. In particular, we discussed the process of making sense of mathematical problems found on a variety of repurposed paper. During the workshop we focused on the reconstruction of two investigations: (1) work related to solving polynomial equations of degree n = 3, 4, 5, and 6, and (2) a selection of combinatoric problems. Finally, we sought to connect Dirac's doodlings to well-known sources from the history of mathematics and their treatment of similar problems.

INTRODUCTION

Paul Adrien Maurice Dirac was a Professor of Physics at Florida State University from 1972 until his death in 1984 (although different resources report different "start dates"). Among other discoveries, he formulated the famous Dirac equation, which describes the behavior of fermions, and he predicted the existence of antimatter. Dirac shared the Nobel Prize in Physics for 1933 with Erwin Schrödinger, "for the discovery of new productive forms of atomic theory." Dirac is buried in Roselawn Cemetery, in Tallahassee, Florida: "It was his family's wish that he should rest where he left the world" (Pais, 2005, p. 28).

Whereas those familiar with Dirac's work would readily connect him to his famous equation or his shared Nobel Prize with Schrödinger (to whom he referred as "Schröd" in the "shoebox papers"), we approached the collection at FSU with an eye on the lookout for mathematical investigations. The initial motivation to investigate the collection in this way was that Dirac was "wooed…to do a full mathematics degree free of charge" at Cambridge beginning in September 1921. Furthermore, we believed the time period of the identified doodlings is between 1921 and 1923, though due to the scrap paper nature of the papers (that is, Dirac often repurposed paper) used in our study, almost none of the pages are dated.

This paper is organized in the following manner. First, we present a brief biographical sketch of Paul Dirac. Next, we discuss the acquisition of the Dirac Papers at Florida State University (FSU) and describe the current efforts of the FSU Digital Library (FSUDL). Related to this work, we also describe our contribution of identifying sets of related pages for digitization and entry into the FSUDL. Finally, we present several examples of pages from the "shoebox collection" that fall within two broad mathematical investigations: solving polynomial equations and combinatorics.

BRIEF BIOGRAPHICAL SKETCH: PAUL A. M. DIRAC

Paul Adrien Maurice Dirac was born on 8 August 1902 in Bristol, United Kingdom, to Charles Adrien Ladislas Dirac (a Swiss-born French teacher) and Florence Hannah Holten (a library clerk). From 1908 to 1918, Dirac attended Bishop Road Elementary School and the Merchant Venturers' Technical College, which was a secondary school where Dirac's father taught. In 1918 Dirac went on to study engineering at the University of Bristol, and finished a degree there in 1921. Although admitted to study at St. John's College at Cambridge, Dirac was unable to due to lack of sufficient funds to live on. Consequently, he stayed on to study mathematics at the University of Bristol, and he completed a first class honours degree in 1923.

From 1923 to 1926 he studied physics at St. John's College, under the direction of Ralph A. Fowler. While there, he published his dissertation (in Quantum Mechanics) and several technical papers. He also travelled to notable European centres where physics was being investigated, including Copenhagen and Göttingen. Dirac enjoyed a prolific career at Cambridge. In the years 1927 to 1933 he progressed rapidly, being named to numerous research positions. He was elected Fellow at St. John's College of Cambridge University in 1927, Praelector in Mathematical Physics in 1929, Fellow of the Royal Society in 1930, and Lucasian Professor of Mathematics in 1932. In 1933 he was awarded the Nobel Prize for Physics, which he shared with Erwin Schrödinger.

Dirac remained at Cambridge until 1969 (when he reached mandatory retirement age in England). However, from 1933 until 1969 he travelled widely in order to continue his research. Dirac frequently taught and lectured in Europe, Asia, and North America and he held short-term lecturing positions at major universities until his retirement from Cambridge in 1969.

Paul Dirac married Margit ("Manci") Wigner in 1937. Dirac met Margit when she visited her brother, the chemical engineer (and mathematician and physicist) Eugene P. Wigner, at Princeton's Institute for Advanced Studies. Dirac adopted Margit's two children (Judith and Gabriel) from her previous marriage, and they also had two daughters, Mary Elizabeth and Florence Monica.

In 1971, Dirac accepted a physics faculty appointment at Florida State University and the family moved to Tallahassee. Dirac continued to do research and travel until almost the end of his life. He died in Tallahassee on 20 October 1984, and was buried there in Roselawn Cemetary. Margit died in 2002 and was laid to rest with her husband. On 13 November 1995, "a plaque was dedicated in Westminster Abbey commemorating Paul Dirac" (Goddard, 2005, p. ix). Stephen Hawking, who delivered the Dirac Memorial Address in 1995 stated that:

> Dirac has done more than anyone in this century, with the exception of Einstein, to advance physics and change our picture of the universe. He is surely worthy of the memorial in Westminster Abbey. It is just a scandal that it [took] so long. (Hawking, 1995, p. xv)

ACQUISITION OF THE DIRAC PAPERS

The Paul A. M. Dirac Papers are housed in the Special Collections and Archives Division of Florida State University (FSU). The physical collection resides in the Robert Manning Strozier Library on the FSU campus. However, efforts are underway to digitize the full collection of the Dirac Papers. (A brief description of these efforts is given in the next section.)

The Special Collections and Archives Division (SC&A) at FSU acquired the Dirac Papers over a period of 29 years. The first portion of the papers, which were mostly Dirac's work papers and artefacts, were given to SC&A in 1985 by Dirac's widow, Margit ("Manci") Dirac. Further significant portions were given in 1992 and 1997. Finally, between 2012 and 2014, Monica Dirac (the Dirac's only living child) donated the remainder of Dirac's papers – which were mostly of a personal nature – to SC&A.

The Dirac "Papers" are extensive in that there are professional papers, journal articles and their drafts, books, calculation books, lab books, and school work from Dirac's student days, photographs, recordings, awards, and so on. At one point an archivist stated that there were over 190 linear feet of materials, but storage of the collection is currently undergoing modification and this number may no longer be accurate (after September 2014) due to the methods in which the collection is stored.

FSU DIGITAL LIBRARY (FSUDL) EFFORTS

Previous Efforts

The initial work toward processing, digitizing, and preserving the Dirac Papers began in 1999 when SP&A applied for and received a processing grant. This work led to a digitization grant (also in the late 1990s). Unfortunately, the early efforts did not produce high-quality records that documented the work; at the same time, there was a high turnover of administrative personnel within the FSU libraries that also contributed to the lack of sufficient documentation of processing and digitization efforts. The preservation work that is taking place now, which includes reviewing, reordering, and re-boxing materials in the Dirac Papers collection, is somewhat complicated, since idiosyncrasies resulted from previous efforts. Additionally, server failures after the early digitization of some of the materials in the Dirac Collection meant that any previously scanned materials were lost and needed to be digitized again.

Current Efforts

In February 2014, FSU launched the new Florida State University Digital Library, which "provides online access to Florida State University's rich and unique historical collections of photos, pamphlets, maps, manuscripts, and rare books" (Florida State University Libraries, 2014). The digital library uses an open source platform available state-wide and the primary aim of the FSUDL is to provide a central database for FSU's unique digital materials, of which the Dirac Papers is a centrepiece. Since the goal is to facilitate collaboration on mutual use of the Dirac Papers, an important goal of the workshop we conducted at the Seventh European Summer University (ESU-7) was to introduce participants to the collection as a whole and to the example of how we are using the materials that will become part of the FSUDL presence of the Dirac Papers.

"SHOEBOX COLLECTION" EXAMPLES

A portion of the Dirac Papers was originally identified as part of a "shoebox collection," presumably because they arrived at SC&A in shoebox-type containers (or, actually shoeboxes!) from either the Dirac home or Dirac's FSU office. We approached the overall Dirac Papers with an eye on the lookout for mathematical investigations. The initial motivation to investigate the collection in this way was that Dirac was "wooed…back to the lecture theatres in the mathematics department [at Bristol]…to do a full mathematics degree free of charge" at Cambridge beginning in September 1921 (Farmelo, 2009, p.47). This investigation began with the first author reading entries in the former finding aid for SC&A, for clues of which document folders to search. Series 1 ("Family Papers, Student Papers, and Photographs") held the most promise for finding items that we were interested in. According to the former finding aid, Series 1 included "materials associated with [Dirac's] student days at the Bishop Road primary school, the Merchant Venturers' secondary school, the University of Bristol (1918-1921), and at St. John's College at Cambridge University (from 1921 through the awarding of his doctorate in 1926)." Of particular interest were the boxes (and corresponding folders) with dates from 1921 until 1926, as items focused on pure mathematics were more likely to appear during these years because of the influence of the course work that Dirac took when he first arrived at Cambridge. The materials in the following folders were searched first:

Box 10

Folder 6: Merchant Venturers' School (Later, The Cotham School), 1918-1931(?); Physical Description: 4 items

Folder 9: Merchant Venturers' Technical College, Bristol (Calculations: Apparently Text Exercises), 1914-?; Physical Description: 171 items

Box 11

Folder 4: Exercise Book: 'Functions of a Complex Variable'. (Inserted second title: 'Surface Waves'), 1920(?) (21 miscellaneous leaves inserted, about 40 pages); Physical Description: 1 item

Box 12

Folder 1: Calculations, 1920(?); (62 leaves); Physical Description: 5 items

Folder 2: Calculations. Booklet from 3rd year Electrical Technology, 25 Oct 1920 - 8 Feb 1921; others, 1920(?); Physical Description: 2 items

Folder 3: Calculations. Includes Drawings, 1920(?); (38 leaves); Physical Description: 26 items

Folder 4: Calculations. No Context, 1920(?); Physical Description: 10 items

Folder 5: Calculations. School Related Items, 1909-1933; Physical Description: 25 items

In the workshop, we shared an overview of the digital scans of 72 documents initially identified from the folders described above, which was part of the project to digitize and preserve the "shoebox papers" in the collection. The examples discussed in our workshop came from Box 12 of Series 1, and are part of the reconstruction of two investigations found in what we called Dirac's "doodlings" (that is, mathematics problems that seemed to have been worked out on whatever paper Dirac could find): (1) a systematic approach for solving polynomial equations of degree *n* = 3, 4, 5, and 6, and (2) a selection of combinatoric problems, for which we are currently trying to determine the underlying purpose.

First Investigation: Solving Polynomial Equations

The first example presented to participants is the first example that appears on document 3 of item 1e of folder 12, or document "FSUDIRAC_12_1e_0003" [1] (Figure 1).

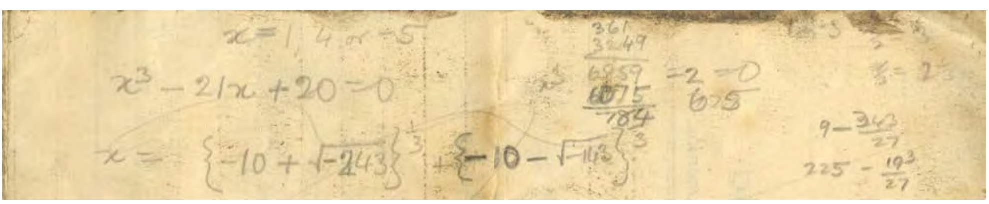


Figure 1: First example found on document FSUDIRAC_12_1e_0003.

In the exercise, it appears that Dirac first lists the three solutions of a desired cubic equation. Then, with no intervening mathematical work, he produces the equation $x^3 - 21x + 20 = 0$. (For the purposes of this paper, we will refer to this as Eq. 1.) Finally, the solution that appears is in the form one may obtain if Cardano's "cubic formula" was employed. That is, for the equation, $x^3 + qx + r = 0$, one solution is given by:

$$= \left(-\frac{r}{2} + \sqrt{\left(-\frac{r}{2}\right)^2 + \left(\frac{q}{3}\right)^3}\right)^{\frac{1}{3}} + \left(-\frac{r}{2} - \sqrt{\left(-\frac{r}{2}\right)^2 + \left(\frac{q}{3}\right)^3}\right)^{\frac{1}{3}}.$$

Thus, for the equation $x^3 - 21x + 20 = 0$, this solution process would yield:

$$= \left(-\frac{20}{2} + \sqrt{\left(-\frac{20}{2}\right)^2 + \left(\frac{-21}{3}\right)^3}\right)^{\frac{1}{3}} + \left(-\frac{20}{2} - \sqrt{\left(-\frac{20}{2}\right)^2 + \left(\frac{-21}{3}\right)^3}\right)^{\frac{1}{3}}$$

$$x = \left(-10+\sqrt{(-10)^2+(-7)^3}\right)^{\frac{1}{3}} + \left(-10-\sqrt{(-10)^2+(-7)^3}\right)^{\frac{1}{3}}$$

$$x = \left(-10+\sqrt{100+(-343)}\right)^{\frac{1}{3}} + \left(-10-\sqrt{100+(-343)}\right)^{\frac{1}{3}}$$

$$x = \left(-10+\sqrt{-243}\right)^{\frac{1}{3}} + \left(-10-\sqrt{-243}\right)^{\frac{1}{3}}.$$

The final value is indeed the form of the solution given on FSUDIRAC_12_1e_0003. The remainder of the work associated with solving Eq. 1 appears to be an incomplete derivation of the remaining two roots to the equation (Figure 2).

Figure 2: Determining the additional roots to Eq. 1 of FSUDIRAC_12_1e_0003.

The remainder of the document FSUDIRAC_12_1e_0003 contains two additional cubic equations and their solutions; or rather, three integer solutions, the resulting cubic equation, and some form of a Cardano's "cubic formula" solution, along with work toward the remaining two solutions similar to that which is seen in Figure 2. As we discussed with participants in the workshop, we do not know the context for why or how the solutions to cubic equations of the form $x^3+qx+r=0$ were of interest to Dirac, particularly since this document was undated. The examples were written on the back of a former student report card form from the school where Dirac's father taught (and, that Dirac also attended, "Merchant Venturers' College"), though this does not help identify a precise date. Some workshop participants offered the idea that perhaps this was an example of Dirac planning for tutoring or instruction of some sort that he participated in during his time at either the University of Bristol where he studied both engineering (degree in 1921) and mathematics (where he was awarded first class honours in 1923). However, there is no evidence that Dirac taught in any capacity in the years leading up to 1923. It is possible that Dirac was investigating mathematics of personal interest to him – and for which he would have possibly doodled about on random sheets of paper – given his talent for mathematics. Dirac stated:

> I consider myself very lucky in having been able to attend [Merchant Venturers']. ... I was rushed through the lower forms, and was introduced at an especially early age to the basis of mathematics, physics and chemistry in the higher forms. In mathematics I was studying from books which mostly were ahead of the rest of the class. (Dirac, 1980)

Thus, if Dirac's penchant for solving cubic equations was not motivated by the need to plan instruction, perhaps it was just for personal interest in either revisiting Cardano's solution methods or comparing them to more modern conceptions. The latter makes sense, as there are numerous examples of such equations and their solutions within the documents that we targeted in our investigation. Furthermore, there are examples of degree 4, 5, and 6, as well as attempts to generalize solutions (see Figure 3 for an example of degree 4).

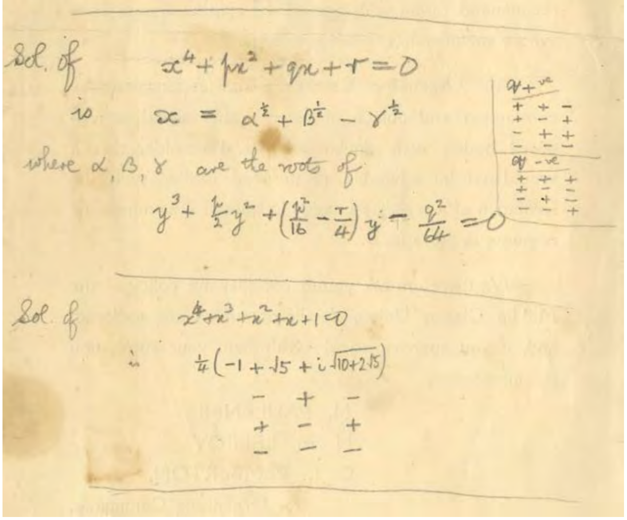


Figure 3: Final two examples of FSUDIRAC_12_1d_0004.

We received helpful advice for how to connect the particular polynomial equation examples to content that Dirac would have been exposed. For example, Bjarne Toft suggested that we try to determine the textbooks that Dirac would have used while attending the University of Bristol. Toft believed that a popular algebra text (*Theory of Algebraic Equations*, by Julius Petersen) would have contained content similar to what we found in the "shoebox collection."

Second Investigation: Combinatorics

The second investigation for which we found numerous related pages we have categorized as "combinatorics." The pages that we identified appeared to have lists of permutations of the letters *a*, *b*, *c*, *d*, and *e* (see the excerpt from FSUDIRAC_12_1a_0018 in Figure 4), as well as evidence of Dirac trying to make sense of patterns of these permutations (see Figure 5), often with tables or other organizational means.

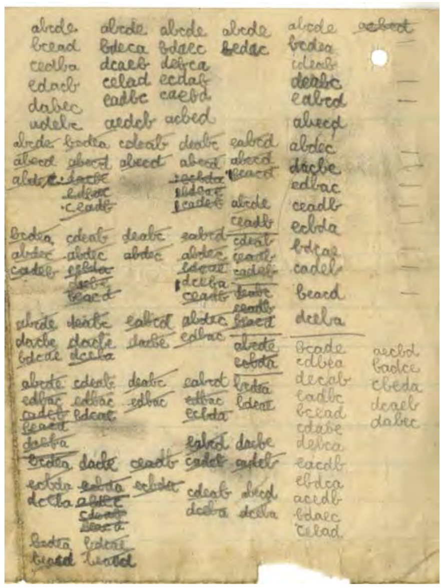


Figure 4: Excerpt from FSUDIRAC_12_1a_0018.

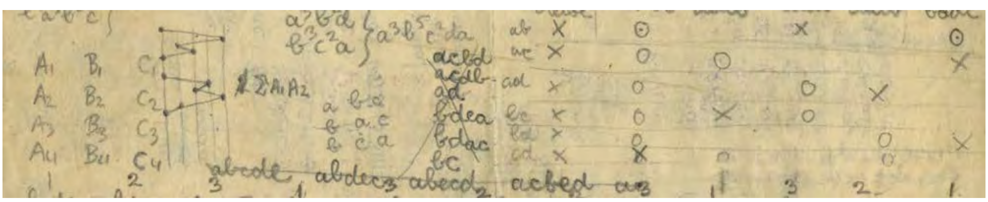
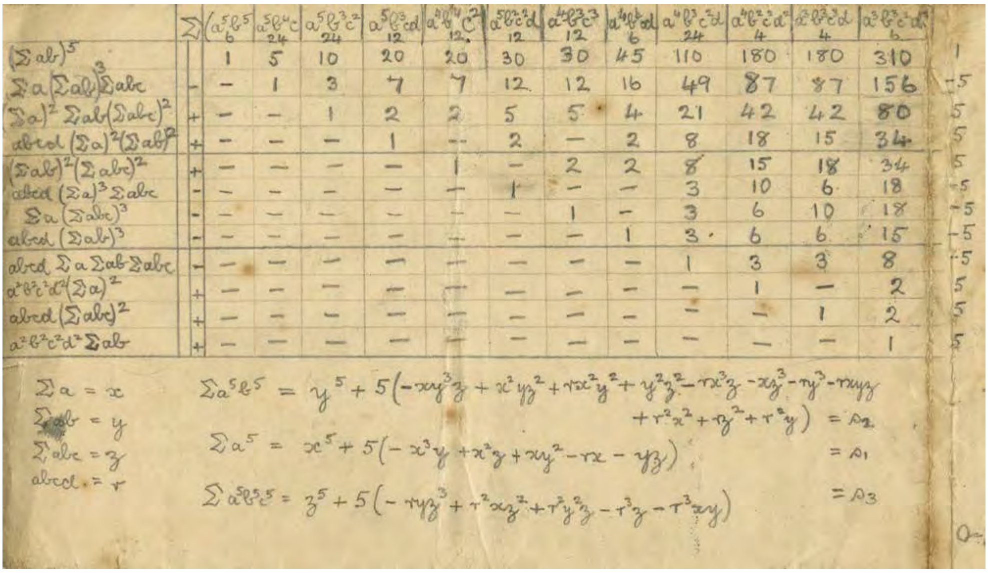


Figure 5: Excerpt from FSUDIRAC_12_1e_0021.

Unfortunately, we had less time to spend on the combinatorics investigation with the workshop participants. The handout we distributed during the workshop prompted Annie Michel-Pajus to provide us with a resource from Leibniz, which may prove helpful with our analysis of what Dirac may have been thinking about. The resource, *Mathesis universalis* (1694-1695) [GM VII, 53-76], shows permutations of powers of *a, b, c, d,* and *e,* as well as the different forms of various degree.

NEXT STEPS

While we intend on following up on the lead from Annie Michel-Pajus while we continue our analysis of what Dirac was attempting in these various mathematical doodlings, we also anticipate that there are connections between two investigations (solving polynomial equations and combinatoric patterns). For example, the excerpt from FSUDIRAC_12_1d_0011 (Figure 6) appears to display such a connection.



	Σ	a^5b^5 6	a^5b^4c 24	$a^5b^3c^2$ 24	a^5b^3cd 12	$a^4b^4c^2$ 12	$a^5b^2c^2d$ 12	$a^4b^3c^3$ 12	a^4b^4cd 6	$a^4b^3c^2d$ 24	$a^4b^2c^2d^2$ 4	$a^3b^3c^3d$ 4	$a^3b^3c^2d^2$ 6	
$(\Sigma ab)^5$		1	5	10	20	20	30	30	45	110	180	180	310	1
$\Sigma a(\Sigma ab)^3\Sigma abc$	-	–	1	3	7	7	12	12	16	49	87	87	156	-5
$(\Sigma a)^2\Sigma ab(\Sigma abc)^2$	+	–	–	1	2	2	5	5	4	21	42	42	80	5
$abcd(\Sigma a)^2(\Sigma ab)^2$	+	–	–	–	1	–	2	–	2	8	18	15	34	5
$(\Sigma ab)^2(\Sigma abc)^2$	+	–	–	–	–	1	–	2	2	8	15	18	34	5
$abcd(\Sigma a)^3\Sigma abc$	-	–	–	–	–	–	1	–	–	3	10	6	18	-5
$\Sigma a(\Sigma abc)^3$	-	–	–	–	–	–	–	1	–	3	6	10	18	-5
$abcd(\Sigma ab)^3$	-	–	–	–	–	–	–	–	1	3	6	6	15	-5
$abcd\Sigma a\Sigma ab\Sigma abc$	-	–	–	–	–	–	–	–	–	1	3	3	8	-5
$a^2b^2c^2d^2(\Sigma a)^2$	+	–	–	–	–	–	–	–	–	–	1	–	2	5
$abcd(\Sigma abc)^2$	+	–	–	–	–	–	–	–	–	–	–	1	2	5
$a^2b^2c^2d^2\Sigma ab$	+	–	–	–	–	–	–	–	–	–	–	–	1	5

$\Sigma a = x$
$\Sigma ab = y$
$\Sigma abc = z$
$abcd = r$

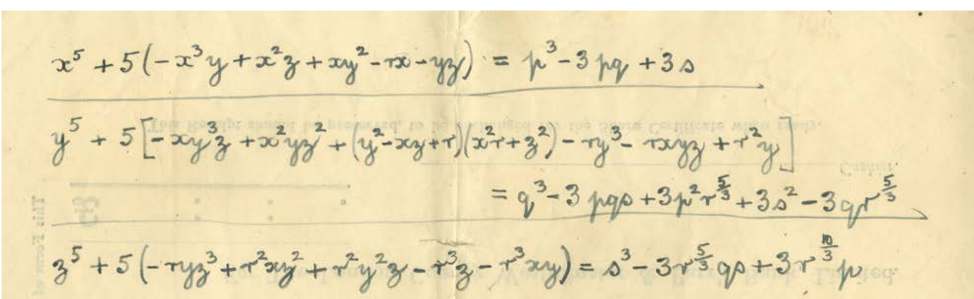
$$\Sigma a^5b^5 = y^5 + 5(-xy^3z + x^2yz^2 + rx^2y^2 + y^2z^2 - rx^3z - xz^3 - ry^3 - rxyz + r^2x^2 + rz^2 + r^2y) = s_2$$

$$\Sigma a^5 = x^5 + 5(-x^3y + x^2z + xy^2 - rx - yz) = s_1$$

$$\Sigma a^5b^5c^5 = z^5 + 5(-ryz^3 + r^2xz^2 + r^2y^2z - r^3z - r^3xy) = s_3$$

Figure 6: Excerpt from FSUDIRAC_12_1d_0011.

The table in Figure 6 displays some subset of variable combinations (each term of 10th degree), and below the table, a collection of fifth degree equations. Additional documents contain similar versions of the same equations that appear in Figure 6. For example, in FSUDIRAC_12_1d_0012 (Figure 7), we see the same equations, though in a slightly different order and now set equal to different expressions.



$$x^5 + 5(-x^3y + x^2z + xy^2 - rx - yz) = p^3 - 3pq + 3s$$

$$y^5 + 5[-xy^3z + x^2yz^2 + (y^2 - xy + r)(x^2r + z^2) - ry^3 - rxyz + r^2y] = q^3 - 3pqs + 3p^2r^{\frac{5}{3}} + 3s^2 - 3qr^{\frac{5}{3}}$$

$$z^5 + 5(-ryz^3 + r^2xz^2 + r^2y^2z - r^3z - r^3xy) = s^3 - 3r^{\frac{5}{3}}qs + 3r^{\frac{10}{3}}p$$

Figure 7: Excerpt from FSUDIRAC_12_1d_0012.

Again, since each of the 72 documents we identified do not contain dates or labels (by Dirac’s hand), and the pages were not bound together but in a mostly-random collection, it is difficult to determine in which order the documents FSUDIRAC_12_1d_0011 and FSUDIRAC_12_1d_0012 were produced. This is further complicated by the method of writing: mathematical expressions squished together wherever there was room on a page, often written in 180-degree orientations of each other.

We hope that conference papers such as this one for ESU-7 will generate interest in the mathematical doodlings that we have identified and that will soon be made available as the first project in the Dirac Papers of the FSUDL, and will prompt readers to contact us to contribute to the work. Such collaboration is critical, as we do not possess the mathematical expertise needed to determine the best way analyse Dirac's doodlings. In closing, we welcome partners into this exciting project.

NOTES

1. All of the images ("FSUDIRAC…") are used by permission of the SC&A of the Florida State University Libraries. We are grateful to Kathleen McCormick and Krystal Thomas of SC&A for their assistance and support of this project.

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