Oral Presentation

AROUND A BOOK DEDICATED TO CHILDHOOD FRIENDS: *INITIATION MATHÉMATIQUE* BY C.-A. LAISANT

Jérôme Auvinet

Laboratoire Jean Leray, University of Nantes

The purpose of this article is to highlight some of the original thinking found in the book Initiation mathématique *written in 1906 by the French mathematician Charles-Ange Laisant. In this work he offered an innovative approach using varied mathematical ideas for young children, but one can also observe that this work was the result of his lifelong route encompassing his epistemological thinking about the nature of mathematics, his interest in the visualisation of mathematical processes and his convictions about the role of Science in Society. Moreover, it's also interesting to describe the various networks which were at work before and after the publication of this* Initiation.

In 1906, the French mathematician Charles-Ange Laisant (1841-1920) published *Initiation mathématique* his definitive book resulting form a lifetimes work. This work was well recognised at the time and represented the crystallization of its author's goal to propose a new approach for the mathematical education of young children.

It is interesting to look at all the themes that link this book to the varied careers of its author. In fact, by 1906, Laisant's work as a mathematician was almost completed. It even seems quite surprising that this mathematician who followed a highly scientific route should change direction towards the education of children. It is proposed to highlight how this book was actually the result of various scientific experiments and higher mathematical exchanges. It is also intended to show how the book was the result of the production of several mathematical networks and how it enabled Laisant to develop new ones. These two points show how important a book like this was for a mathematician like Laisant during the latter period of his life.

It will be shown how original the role of this book was in the first mathematical education of young children and will highlight its origins and the references that appear in this work. In addition to the teaching tools which are applied to mathematical ideas, the background and implementation of general educational schemes are discussed. Finally it will be explained why Laisant became, following the *Initiation*, a special member of the movement called "new education" at the beginning of the twentieth century.

A BOOK AS A RESULT OF SEVERAL CAREERS

Some biographical facts about Laisant (Auvinet, 2011, 2013) can help us to understand the process that eventually led him to write the *Initiation mathématique*.

Charles-Ange Laisant was born in 1841 near Nantes in France. From 1859, he studied at the École Polytechnique and became a military engineer. He wrote numerous articles first about equipollences and quaternions, then about discrete mathematics (for examples, generalisations of Pascal's triangle or magical squares, Laisant & Arnoux, 1900). He was also an active member of several scientific bodies including the French Mathematical Society for whom he was president in 1888 (Gispert, 1991) and also the French Association for Advancement of Sciences (AFAS, presidency in 1904, Laisant, 1904b). The regional meetings of this association enabled many exchanges with all kinds of people interested by Science (Gispert, 2002).

He was mainly known to be a left-wing deputy from 1876 to 1893, a stormy period that led him to support Général Boulanger in 1889. He eventually came to consider politics as incapable of realising his strong desire to change Society. He subsequently became an anarchist and retained his belief while searching for other ways to realise his plans for popular education.

From 1893, he became a teacher in preparatory class and finally examiner for admissions to the École Polytechnique. Between 1892 and 1897, he published several books of exercises for students of preparatory classes, proposing new themes for preparing entrance exams (Laisant, 1893-1896).

In 1894, he founded the journal *L'Intermédiaire des mathématiciens* with Émile Lemoine (1840-1912), another former student at the École Polytechnique (Pineau, 2006). This original review enabled direct discussions on various mathematical subjects (including mathematical games as "récréations"). Most significantly, he also created in 1899 a true international journal, *L'Enseignement mathématique*, with the Swiss Henri Fehr (1870-1954). This focused for the first time on the progresses made in the teaching of mathematics within secondary schools of all countries (Furinghetti, 2003). This publication was a new medium for exchanges between mathematicians while questions about reforming the mathematical teaching appeared in many countries (see the 1902 reform France for example). These two innovative reviews enlarged his thinking about the mathematical community and the scope of his first work as a mathematician.

The first particularity of the book results through these numerous aspects of Laisant's life. It is a "book outside any curriculum, dedicated to childhood friends" (this is the complete title), written by a well-known mathematician at the time who was used to practicing and teaching higher mathematics. Pedagogy, especially linked to childhood, was a quite recent concern for him with an important political and sociological meaning.

GENESIS OF THE *INITIATION MATHÉMATIQUE*

In 1898, Laisant wrote for the first time a book which dealt with epistemology and included his observations about mathematics. It was entitled *La Mathématique. Philosophie. Enseignement* (2nd edition in 1907, Laisant, 1898). This important work featured some very general and personal ideas about mathematics and their teaching (Auvinet, 2007). He expressly stated that:

> 1° In the current environment, mathematical notions are necessary for everyone;
>
> 2° Anyone of average intelligence is able to acquire these notions, restricted to certain limits [1].

These statements point to the beginning of Laisant's desire to improve mathematical education for children. His goal was to prepare each young child from six years old in order to put him in a good position to receive later (after 12 years) a true mathematical education. This "initiation" was thus the base of a larger scientific culture, required by every citizen in the context of the technological transformations seen at the end of XIXth century.

Other ideas developed in the same work from 1898 were equally important. In addition to the unity of all mathematics, that mathematics is an experimental science. Laisant later explained in his *Initiation* that:

> I consider that all sciences, without exception, are experimental, at least to some extent [...] there is no concept, no idea that could enter our brain without a preliminary contemplation of the outside world and of the facts that the world presents to our observation. [2]

Thus, mathematical notions are the result of an abstract thought from the real objects which surround us. Here, Laisant pointed out that "abstraction" is a simplification. It is not to be explicit for a young child naturally endowed with an instinctive ability for "abstraction". He explained: "we should never try to make him abstractions that he wouldn't make himself" [3].

All these statements and others in *La Mathématique* were consequently fundamental for the processes developed in the *Initiation mathématique*. Here we get another particularity of this book: it is relevant across a wide vision of mathematics. As a result, Laisant was invited to present his educational theories and teaching tools in a series of lectures delivered between 1899 and 1903 at the Institut psycho-physiologique of Paris. These speeches were put together in one volume: *L'Éducation fondée sur la Science* (Laisant, 1904a). The first lectures was entitled "L'Initiation mathématique" (Laisant, 1899). It contained the main principles which should be applied during the mathematical education of children between four and eleven years old. He also strongly criticised teaching methods at the time and explained that this "initiation" remains as essential as the acquisition of reading and writing.

In 1906, the *Initiation mathématique* developed these ideas into a set of 65 lessons plans. As a reflection of its success, it was reprinted (in France) 17 times (Laisant, 1916) and it was well received in Italy too.

It is perhaps of value to state the intended purpose and audience for this work. Laisant explained that this book was not a handbook but more "a guide into the hands of the educator" [4], in particular for mothers. It contained several original educational principles which could be applied to every-day life. As already stated, this work was strongly and explicitly separate from curriculum or dogma and was in contrast with existing exhausting school practices at the time. The "initiation" has also to be presented rationally: it is based on rigorous observations of each child as an individual and the difficulty of the notions taught has to be progressive.

This "initiation" is based on experience and empirical evidence. It takes an physical form with the help of a specific material (cubes and rods for examples). This material was created by Jacques Camescasse (1869-1941), one of Laisant's collaborators and, like him, an Esperantist and a freemason. Laisant explained that the educator has "to bring the first images into child's brain by putting objects in the scope of his senses. The education should be absolutely concrete and only devoted to the contemplation of outside objects" [5]. The result should be to "give the illusion at all levels of education that he [the child] discovers himself the truth that has to be brought into his mind" [6].

Consequently, formalism and symbolism should never precede this stage. For example, the concept of numbers has to be integrated before the introduction of the corresponding notation. It is for this reason that the chapter about digits is not the first lesson (see also fig. 1). Mathematical proofs are therefore prohibited, the word "theorem" is not used instead the child has rather to "feel" things.

This *Initiation* is therefore based on the child's natural curiosity rather than the use of his memory. It attempts to reduce effort and routine. The use of games is actually the only route to learning and the educator has to stop a lesson if any boredom appears. This was why Laisant highlighted experiments such as those of the Swiss educationalist Johann Heinrich Pestalozzi (1746-1827) who was himself inspired by Jean-Jacques Rousseau and who also insisted on giving tangible form to ideas and on the grading of difficulties.

Finally, Laisant summed up:

> We will use entertaining questions as an educational way to attract the child's curiosity and thus effortlessly get to put into his mind the primary mathematical concepts. [7]

From a teaching point of view, Laisant referred to the book *L'Arithmétique du grand-papa* (Macé, 1862) written by Jean Macé (1815-1894), a book which was reissued several times. This tale presented the fundamentals of arithmetic (numeration, the four operations on integers, fractions and decimal fractions, metric system) through a dialogue into which tangible objects were used. Jean Macé was again one of Laisant's friends and founded the League for Education in 1867. The aim of this organisation was to support initiatives for the progress of popular education. However, it was while

he taught in eastern France in the 1850's that he wrote several original works dealing with the teaching of history, anatomy and arithmetic.

Laisant referred to many other sources of inspiration (always from a teaching point of view). In addition to Édouard Lucas, whose case will be discussed later on, Laisant mentioned, concerning the teaching of geometry, a professor from the University of Dijon, Charles Méray (1835-1911). His book, *Nouveaux Éléments de géometrie* (Méray, 1874, reissued in 1903), was based on the experimental aspect of geometry. Méray also led several innovative experimentations into the upper primary schools in Dijon from 1876 to 1879.

Carlo Bourlet (1866-1913), another close collaborator of Laisant's and director of the journal *Nouvelles Annales de Mathématiques* like him, was also cited having also taken this experimental approach in his *Cours abrégé de géométrie* (Bourlet, 1906). Both books illustrated this view during the 1902 reform of secondary school in France (Bkouche, 1991). These aspects of the reform were no strangers to some rules in the *Initiation mathématique*.

SOME INNOVATIVE TEACHING TOOLS

Within the lessons of the *Initiation*, Laisant offered some specific principles for reaching the first mathematical fundamentals and even better for approaching more complex concepts such as magic squares, conicals or harmonic divisions.

The most well-used method is through handling actual objects (sticks, beans, matches …) in order to introduce the concept of counting (fig. 1, this section is very close to Macé's work) but also quickly the theory of negative quantities (with stems put in the two directions of the real line).

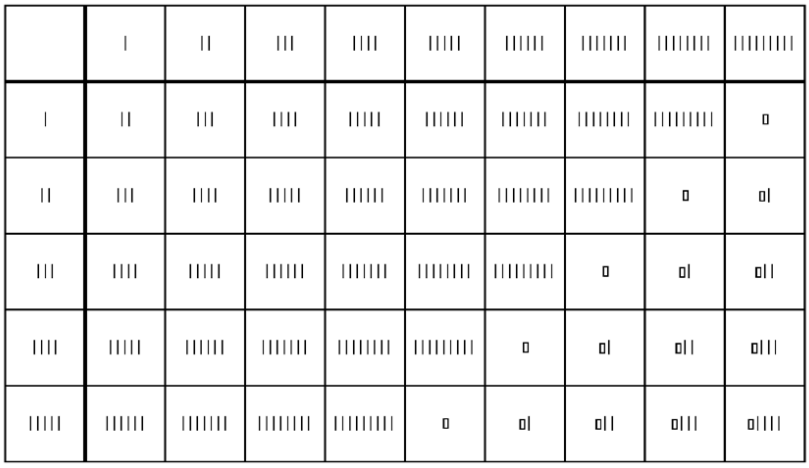

Figure 1: Introducing the addition [8]

The educator can also use graph paper in order to introduce “the concepts of shape, size and position” [9]. André Sainte-Laguë (1882-1950), a professor at the Conservatoire des arts et métiers, also developed these ideas. He was the author of an article “Note sur l’utilisation du papier millimétré” in *L’Enseignement mathématique* (Sainte-Laguë, 1910) where he revealed similar uses (for example, for the sum of odds). He was also the author of *Avec des nombres et des lignes. Récréations mathématiques* in 1937 (Sainte-Laguë, 2001) which took examples from the *Récréations mathématiques* (Lucas, 1882) written par Édouard Lucas in 1882.

Édouard Lucas (1842-1891) was a secondary school teacher and one of Laisant's closest friends. His work on number theory and arithmetic was well-known and influenced Laisant in the 1890's. They both worked on visual ways to present concepts from discrete mathematics. In his *Récréations mathématiques*, he explained to parents how they can help their children to understand mathematics using games and drawings. Some illustrations in the *Initiation* (fig. 3 and 4) are very similar to those proposed by Lucas.

It’s also interesting to make a distinction between recreational games (Barbin, 2007) and the numerous visualisations used in the *Initiation*. In his foreword, Laisant differentiated his work on “initiation” and the games exposed in Émile Fourrey’s *Curiosités géométriques* in 1907 (Fourrey, 2007) or Lucas’ *L’Arithmétique amusante* (Lucas, 1895). Here, there is no theoretical knowledge needed, his goal was not to impart original theories. It is instead humorous and varied but fully ordered questions to be used as teaching tools.

However, the *Initiation mathématique* was linked to these mathematical games that exercised the mind and that were developed in the press at the time. They were both based on visualisation of mathematical processes at different levels. They took their origin from original research (see for example Lucas’ *Théorie des nombres*, Lucas, 1891) and were discussed in new mathematical journals like *L’Intermédiaire des mathématiciens* or in the congresses of the AFAS, two places where Laisant was deeply involved and where exchanges fed his thinking. As evidence of this interest, he was also vice president of the Society of Recreational Sciences in 1894 with several of his collaborators (Delannoy, Arnous Rivière for examples).

Laisant also insisted on the importance of drawing and on the use of the compasses in order to learn geometry, believing that by exercising the hand, drawing can be a way to train the mind and to approach mathematical objects like curves. He offered that Carlo Bourlet’s courses could be used to study geometry after the “initiation period”.

One of the most important principles in the *Initiation* is the use of various visualisations. Below are shown numerous examples including tables for sums and products (with graph paper), visualisations of binomial expansions (fig. 2), of the sum of integers (fig. 3), of the sum of odds, of the sum of the square or the cube of integers or even visualisations of permutations based on Lucas’ idea (fig . 4).

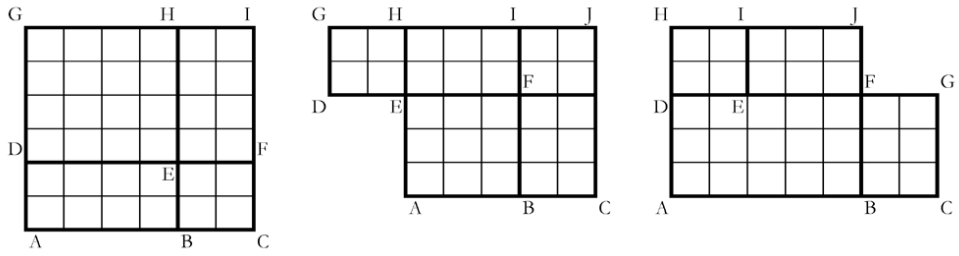

Figure 2: Visualisations of binomial expansions $(a + b)^2$, $(a – b)^2$ and $(a + b)(a – b)$ [10]

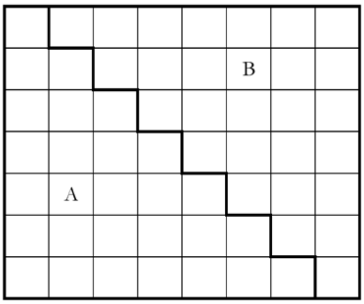

Figure 3: Visualization of the sum of integers [11] (see also Lucas, 1884)

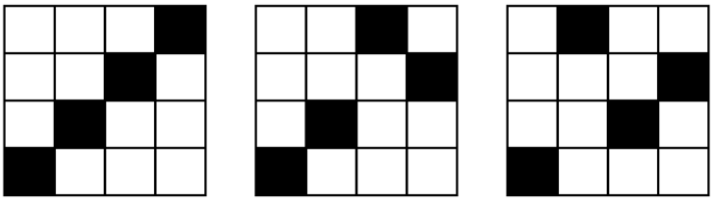

Figure 4: Visualisation of permutations *abcd*, *abdc*, *adbc* [12]

Most of all, the use of curves especially illustrates the concept of function. The first lesson about functions is even entitled “algebra without calculation”. One of the corresponding exercises is from Lucas and the question, perhaps proposed after a congress of the AFAS, is:

> Each day, a boat leaves Le Havre to New York and another leaves New York to Le Havre. How many boats will encounter the first during the journey? [13]

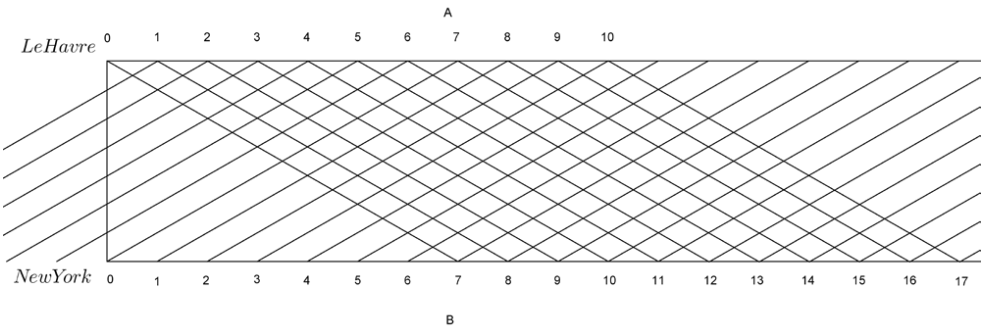

Figure 5: Solving Lucas' problem [14]

Laisant explained that, with his drawing (fig. 5), one can see the solution, without reasoning. He also used many graphics extracted from newspapers and magazines as a help for his "initiation". To sum up, he wanted to "educate by the eyes". A similar idea concluded an article about magical squares written in collaboration with a mathematics amateur, Gabriel Arnoux (1831-1913): "when one can simply say, "See," the proof approaches perfection. One could almost say that the art of exposing is to make diagrams" [15].

Educators may also play on pupils' natural curiosity. That is why Laisant computed the number of digits in 9 to the power 9 to the power 9; he was therefore the first to give the number j_9 = 369 693 100 of the Joyce's sequence. This surprising result was given with several modern remarks about computability and times of calculation in order to arouse the child's interest.

Laisant also used historical references. For example, he recounted the principle of multiplication by gelosia of which he even conceived a method of automating after an exchange with Henri Genaille during a congress of the AFAS. He presented also an ancient proof of Pythagoras' theorem (fig. 6, Fourrey also gave 24 historical proofs). Additionally, Laisant criticised Legendre's presentation in the "Euclidian style" which was very common in the nineteenth century.

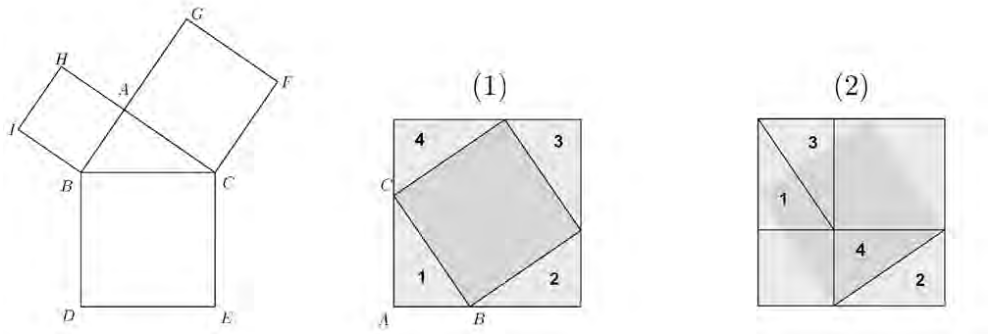

Figure 6: Proof of Pythagoras' theorem [16]

Finally, Laisant insisted on various visualisations of processes and on practical and continual examples to avoid any “artificial abstraction”. Many of these themes were already effective in Laisant’s previous handbook and even in his pure mathematical works (about equipollences for examples).

WITH THE *INITIATION*, A LAST INVOLVEMENT

With this small book, Laisant wished to establish a collection of similar *Initiations* about other scientific domains. This project implied that he engaged with a new community of authors who shared his points of view about young children’s education. It proves also that Laisant always kept in mind a far larger vision of scientific education since his lecture on “L'Initiation à l'étude des sciences physiques” (Laisant, 1901). Camille Flammarion (1842-1925) of the Astronomical French Society like Laisant was the first to join him through the publication of an *Initiation Astronomique* (Flammarion, 1908). The *Collection des initiations scientifiques* included also principles on the teaching of chemistry (1909), mechanics (1909), zoology (1910), botany (1911) or physics (1913) with many reissues for each of them.

Furthermore, Laisant became quickly involved in a journal published from April 1908 to November 1909: *L’École rénovée* (Mole, 2011). This review was founded by the Spanish educationalist Francisco Ferrer (1859-1909), who also created a school in Barcelona named the “Modern School” and who quickly became Laisant's friend and close collaborator (Ferrer, 1962).

The journal contained some anarchists’ ideas about education but was in the first place the organ of the International League for the Rational Education of Children, which Ferrer developed with Laisant's help. In this review whose complete title was “Review for developing a modern education plan”, one could discuss the main themes from the movement called “the new education”, that is to say: a sharp criticism of the traditional education, the children's specific psychology, the development of a liberating education for children and for the whole society, beyond the social origins of each of them (see Laisant's article, "The Purpose of Education" in 1909). These main themes were studied by many libertarian thinkers like Laisant who had arrived to these ideas since his political retreat (Lamandé, 2010).

The journal was a totally independent publication and enabled a broad debate between educational players and thinkers from several countries like the Dutch F. Domela-Nieuwenhuis, the Belgian J. F. Eslander, the Swiss S. E. W. Roorda van Eysinga and Pierre Kropotkine. Many teachers were also present and Laisant established many relationships with them. For Ferrer and for Laisant, the objective was to overthrow the school system and to free children from any dogma (all this is a point of view close to positivism: the child has to reach a rational state). This is also the final message of the *Initiation mathématique*. When Laisant wrote “the solution of the problem of education will determine the future of human society” [17], he highlighted a common interest for all the players of this heterogeneous movement called “the new education”: the link between school evolution (in a pedagogical sense) and a social revolution. His

final political convictions appeared in his article “The Purpose of Education” in which he described education as “the power to get out of their class [Children's social class], any class, and one day to get rid of the oppression of the ruling classes” [18].

After the execution of Ferrer which Laisant had always condemned, the journal stopped. A second journal was founded in October 1910: *L’École émancipée*. The editorial line was nearly the same as the former one but this new review was attached to the national federation of teachers unions. Many collaborators from *L’École rénovée* were still present in this project: Laisant published several articles between 1910 and 1914. He appeared thus as a militant linked to a larger community, including teachers unions, till the end of his life.

Laisant’s desire to bring new guidance to educators about young children’s mathematical training relied on new approaches to the first mathematical notions. Thanks to his work on visualisation in discrete mathematics, he succeeded in delivering various processes aimed at reducing off-putting formalism. His modern approach based on the use of graphs and diagrams was the result of many exchanges with mathematicians from various horizons. However the *Initiation mathématique* was also the work of an accomplished mathematician convinced by the positivism, driven by his thinking about the origin and the nature of the mathematics. He became one member of a community which included other famous mathematicians concerned with addressing the problem of education, at the beginning of the twentieth century. Poincaré, Klein or Borel (Borel, 2002) also presented their thoughts on mathematical training following the recent reforms. Experimentation is one of the themes studied by all of them (Gispert, 2013; d’Enfert, 2003).

The *Initiation mathématique* marked a new and last route for Laisant. He subsequently wrote two books entitled *La Barbarie moderne* (Laisant, 1912) and *L’Éducation de demain* (Laisant, 1913) within these he extended his ideas from childhood right through to adulthood. He dealt with a complete and popular education which is lifelong and necessary for anyone in a social sense. He always kept in mind this principle: “mathematical initiation is essential to every child, without distinction of wealth, social status, gender” [19].

NOTES

1. (Laisant, 1898), p. 187.

2. (Laisant, 1899), p. 358.

3. (Laisant, 1899), p. 366.

4. (Laisant, 1916), p. 6.

5. (Laisant, 1899), p. 360.

6. (Laisant, 1899), p. 362.

7. (Laisant, 1916), p. 7.

8. (Laisant, 1916), p. 18.

9. (Laisant, 1899), p. 365

10. (Laisant, 1916), p. 82-83.

First figure: $(AB + BC)^2 = DE^2 + BC^2 + AB.DE + EF.FI = AB^2 + 2.AB.BC + BC^2$, that is to say:

$(a + b)^2 = a^2 + 2ab + b^2$.

2nd figure: $AB^2 = \text{area}(ACJGDE) - 2.\text{area}(BCJI) \Leftrightarrow (AC - BC)^2 = AC^2 + BC^2 - 2.AC.BC$, that is to say:

$(a - b)^2 = a^2 - 2ab + b^2$.

3rd figure: $\text{area}(ACGD) = AC.AD = (AB + BC).(AB - BC) = \text{area}(ABJH) - \text{area}(DEIH) = AB^2 - BC^2$, that is to say:

$(a + b)(a - b) = a^2 - b^2$.

11. (Laisant, 1916), p. 87.

12. (Laisant, 1916), p. 118.

13. (Laisant, 1916), p. 140.

14. (Laisant, 1916), p. 140.

15. (Laisant & Arnoux, 1900), p. 36.

16. (Laisant, 1916), p. 79.

17. As cited in Mole, 2011, p. 20.

18. As cited in Mole, 2011, p. 20.

19. (Laisant, 1916), p. 173.

REFERENCES

Auvinet, J. (2007). *C.-A. Laisant viewed through his book* La Mathématique. Philosophie. Enseignement. Session presented at ESU-5, Praha, Czech Republic.

Auvinet, J. (2011). *Charles-Ange Laisant. Itinéraires et engagements d'un mathématicien, d'un siècle à l'autre (1841-1920)* (Unpublished doctoral dissertation), University of Nantes, France.

Auvinet, J. (2013). *Charles-Ange Laisant. Itinéraires et engagements d'un mathématicien de la Troisième République*. Paris: Hermann Éditeurs.

Barbin, É. (2007). Les récréations: des mathématiques à la marge. *Pour la Science, 30*, 22-25.

Bkouche, R. (1991). Variations autour de la réforme de 1902/1905. *Cahiers d'histoire et de philosophie des sciences, 34*, 181-213.

Borel, É. (2002). Les exercices pratiques de mathématiques dans l'enseignement secondaire. *Gazette de la SMF, 93*, 47-64.

Bourlet, C. (1906). *Cours abrégé de géométrie. Géométrie plane*. Paris: Hachette.

d'Enfert, R. (2003). Inventer une géométrie pour l'école primaire au XIX[e] siècle. *Tréma*, 22, 41-49. Retrieved may 15, 2015, from http://culturemath.ens.fr/content/inventer-une-geometrie-pour-lecole-primaire-au-xixe-siecle-2088#_ftn1.

Ferrer, S. (1962). *La vie et l'œuvre de Francisco Ferrer. Un martyr au XX[e] siècle*. Paris: Librairie Fischbacher.

Flammarion, C. (1908). *Initiation astronomique*. Paris: Hachette.

Fourrey, É. (2007). *Récréations mathématiques* (2nd ed.). Paris: Vuibert.

Furinghetti, F. (2003). Mathematical instruction in an international perspective: the contribution of the journal *L'Enseignement mathématique*. in Coray, D. (Ed.), *One Hundred Years of L'Enseignement mathématique. Moments of mathematics education in the twentieth century* (pp. 21-46). Geneva: L'Enseignement mathématique.

Gispert, H. (1991). La France mathématique. La société mathématique de France (1872-1914). *Cahiers d'histoire et de philosophie des sciences, 34*, 11-180.

Gispert, H. (Ed.). (2002). *Par la Science, pour la Patrie. L'Association française pour l'avancement des sciences (1872-1914): un projet politique pour une société savante*. Rennes: Presses universitaires de Rennes.

Gispert, H. (2013), Une conférence pour une réforme. *Image des maths*. Retrieved may 15, 2015, from http://images.math.cnrs.fr/Une-conference-pour-une-reforme.html.

Laisant, C.-A. (1893-1896). *Recueil de problèmes mathématiques, classés par divisions scientifiques* (Vols 1-6). Paris: Gauthier-Villars et fils.

Laisant, C.-A. (1898). *La Mathématique. Philosophie. Enseignement*. Paris: G. Carré et Naud.

Laisant, C.-A. (1899, mars 25). L'Initiation mathématique, *Revue scientifique*, (4) 11, 358-368.

Laisant, C.-A., & Arnoux, G. (1900). Applications des principes de l'arithmétique graphique: congruence, propriétés diverses. *Compte-rendu du congrès de l'AFAS de Paris* (Vol. 2), 36-59.

Laisant, C.-A. (1901). L'Initiation à l'étude des sciences physiques. *Revue scientifique, (4) 15*, 289-294.

Laisant, C.-A. (1904a). *L'Éducation fondée sur la Science*. Paris: Félix Alcan éditeur.

Laisant, C.-A. (1904b). Le Rôle social de la Science. *L'Enseignement mathématique, 6*, 327-362.

Laisant, C.-A., (1912). *La Barbarie moderne*. Paris: Bataille syndicaliste.

Laisant, C.-A. (1913). *L'Éducation de demain* (2nd ed.). Paris: Les temps nouveaux.

Laisant, C.-A. (1916). *Initiation mathématique* (15th ed.). Paris: Hachette.

Lamandé, P. (2010). Une personnalité du monde de l'éducation nouvelle: Charles Ange Laisant (1841–1920) et son combat politique pour une éducation rationnelle fondée sur la Science. *Paedagogica historica*, 1-19. doi: 10.1080/00309230.2010.509908.

Lucas, É. (1882). *Récréations mathématiques* (Vol. 1). Paris: Gauthier-Villars.

Lucas, É. (1884). Le Calcul et les machines à calculer. *Compte-rendu du congrès de l'AFAS de Blois* (Vol. 2), 111-141.

Lucas, É. (1891). *Théorie des nombres* (Vol. 1). Paris: Gauthier-Villars.

Lucas, É. (1895). *L'Arithmétique amusante*. Paris: Gauthier-Villars.

Macé, J. (1862). *L'Arithmétique du grand-papa*. *Histoire de deux petits marchands de pommes*. Paris: J. Hetzel.

Méray, C. (1874). *Nouveaux éléments de géométrie*. Paris: F. Savy.

Mole, F. (2011). "L'École rénovée": une revue d'éducation nouvelle entre anarchisme et syndicalisme (1908-1909). *Carrefours de l'éducation*, *31*, 5-8.

Pineau, F. (2006). *L'Intermédiaire des mathématiciens: un forum de mathématiciens au XIX^{e} siècle* (Mémoire de DEA). University of Nantes.

Sainte-Laguë, A. (1910). Note sur les usages du papier quadrillé. *L'Enseignement mathématique*, *12*, 5-17.

Sainte-Laguë, A. (2001). *Avec des nombres et des lignes. Récréations mathématiques* (2nd ed.). Paris: Vuibert.