Oral Presentation

MATHEMATICS TEXTBOOKS, IN PORTUGAL: THE CASE OF THE FIRST UNIQUE MANDATORY ALGEBRA TEXTBOOK (1950)

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This paper gives an overview on Portuguese political and educational systems. After, it attempts to provide an understanding on how textbooks were chosen in previous decades. So, it addresses the Portuguese textbook approval systems, in particular the model that emerged in 1947 (unique textbook) and the contemporary published opinion about it. Regarding mathematics, the first unique textbook was approved in 1950; it was the Algebra textbook for the 3rd cycle of Liceus (upper secondary school). This paper discusses the criticism on the Algebra textbook that was published in the journal Mathematics Gazette. In our study we analysed two versions of the first unique Algebra textbook in order to trace changes in content, in this paper we will present part of that analysis. The paper is based mainly on legislation, newspapers, educational magazines and mathematics textbooks.

INTRODUCTION

When we attempt to visualize a schools' discipline past based on its material supports, textbooks are some of the most relevant elements to the study of that past. Choppin (2004) states that "the conception of a textbook is inserted into a specific pedagogical environment and into a regulated context, which, along with the development of national and regional systems, is, most of times, characteristic of educational productions (state editions, approval procedures, freedom of production)" (p.554). As a written document, textbooks are sensitive to national contexts and can be seen, in this case, as probes of the state and structure of mathematical education, its goals and its organization.

In Portugal, studies have shown that textbooks have been the most common resource in classrooms, representing and structuring mathematics school knowledge (APM 1998, Janeiro 2005). Today we are confronted in Portugal with new syllabuses for all grades of elementary and secondary mathematics education. In what concerns secondary mathematics education the syllabus incorporates contents that have not been taught for more than a decade or have never been taught at this level, so teachers are faced with a novel situation requiring teaching new contents. There is the possibility that many teachers experience difficulties and it may be the case that textbooks will come to influence the mathematical knowledge taught in schools. This situation increases the importance of both textbook production and textbook approval.

As regards political control and policies of approval for textbooks Repoussi and Tutiaux-Guillon (2010) developed a typology of approval systems or models, they distinguished five different models: "one single officially approved textbook; several officially approved textbooks; coexistence of officially approved and non-approved textbooks; officially recommended textbooks; and textbooks only produced by private publishers, without official approval" (Repoussi & Tutiaux-Guillon 2010, p.160).

At the present time, in each Portuguese secondary school a group of teachers analyses the certified textbooks available on market and decides on the adoption of the discipline's textbook to be used in the next six years. This period of adoption may be shortened if the syllabus changes. However, during a time legislation forced the adoption of a unique textbook, i.e., all Liceus (public secondary schools) had to use the same textbook per discipline and cycle; this is usually referred as the unique textbook period.

This paper gives an overview on Portuguese political and educational systems. After, it attempts to provide an understanding on how textbooks were chosen in previous decades. So, it addresses the Portuguese textbook approval systems, in particular the model that emerged in 1947 (unique textbook) and the contemporary published opinion about it. Regarding mathematics, the first unique textbook was approved in 1950; it was the Algebra textbook for the 3rd cycle of Liceus (upper secondary school). This paper discusses the criticism on the Algebra textbook that was published in the journal Mathematics Gazette from the point of view of the development of mathematics teachers' professional knowledge. In our study we analysed two versions of the first unique Algebra textbook in order to trace changes in content, in this paper we will present part of that analysis.

SHORT BACKGROUND ABOUT POLITICAL AND EDUCATIONAL SYSTEMS

In 1910, the Portuguese political system became a republic deposing the monarchy. The Constitution of 1933 established the dictatorship of Estado Novo (literally New State) that persisted until 1974. After the end of Second World War social an economic order began to change, we can observe some development and a drive towards strengthening the industry that needed skilled manpower to succeed. The education system needed to adjust to the new reality and the secondary and technical education reforms of 1947 marked the beginning of this accommodation process. By the reform carried out, in 1947, by the Minister of National Education Pires de Lima, the educational system consisted of a mandatory primary cycle (6-9 years old), followed by parallel branches for secondary education: the Liceus and the Technical Schools. The Liceus course encompassed three cycles: 1st (10-11 years old), 2nd (12-14 years old) and 3rd (15-16 years old), this course, especially the last cycle, was oriented to studies at the universities. Technical School studies were oriented to the work market or to pursue studies at the polytechnic institutes.

On April 25th 1974, the Carnation Revolution restored democracy in Portugal. And, in the years following 1974, the structure of the education system gradually began to change. One of the first major alterations was the elimination of the distinction between the two educational tracks that existed for secondary education and the creation of the Unified Secondary Schooling beginning at 7th grade (12 years old) and ending at 11th grade. The immediate result was the alteration of designation, the Liceus and Technical Schools became Secondary Schools. The unification was considered a means to balance educational opportunities for all students. The secondary schooling encompassed two parts: the lower secondary (7th – 9th grade) and the upper secondary. The latter comprehended a 1st cycle (10th – 11th grade) and a 2nd cycle (12th grade). In 1986, the accession of Portugal to the European Union and the society evolution demanded a major reform at all levels of the education system (e.g., structure, methods, contents). At secondary level, the main alteration was that the upper secondary ceased to encompass two cycles. In the present time, secondary schooling (15 -18 years old) is a three years cycle.

SHORT BACKGROUND ABOUT TEXTBOOK APPROVAL SYSTEMS

With regard to textbook approval systems, one could say that from the Estado Novo regime there are two main periods. In 1947 it was established a formal state textbook approval of only one textbook, in which the approval had a prescriptive status, that progressively ended in the first half of the seventies, and is usually referred as the unique textbook period. The 1974 revolution promoted a change in textbooks approval policy and the adoption of textbooks was then and up to now assigned to the teachers of each school.

During the Estado Novo regime, the policies of approval for textbooks produced some controversy. Before 1947, the schoolteachers' council chose the mathematics textbooks to be used for the following year in each Liceu. Books were chosen amongst the ones previously approved by the Minister of Education, the period of approval of a textbook was five years. Then, the disagreements arose mostly within the members of the scientific commissions nominated for textbook assessment and concerned the criteria for textbooks approval (Almeida, 2013).

In what concerns textbooks, the 1947 Pires de Lima reform, on the one hand, established that a disciplines' textbook would be the same for all the Liceus, on the other hand, formalised a new approval system to choose that textbook. After the approval of a textbook, the Ministry of Education would choose the textbook publisher by contract. The aim of the contract was to obtain the lowest sell price for the book. The emergence of this approval system was not peaceful; some teachers argued in favour of the new system and others stated against it, using teachers' bulletins, as well as, the press (Almeida, 2013). We will resume this particular model later on this paper.

As outcome of the 1974 revolution the choice of the textbook becomes the responsibility of teachers and there was an editorial freedom to their design, on the

assumption that authors follow the curriculum guidelines, so there were several textbooks available for adoption. In the 1980s, given the increasing number textbooks, the task of analysing and decide for one or another textbook became more complex. Moreover, the extent of time allocated to teachers' assess of the textbooks was short. In the next school year the process had to be repeated because the period of adoption was only one year. From one year to the next, the publishers usually improved the textbooks already on the market, this meant, on the one hand, correct text and scientific mistakes, and, on the other hand, adjust to educational objectives or to the age of the students. The textbook's publishers also developed supplements (teacher's guide, slides, etc.) that began to be distributed to the teachers.

A major change concerning legal mechanisms to control the quality of textbooks was set in 1987. Then, new official procedures and criteria for textbooks assessing and textbooks adoption were established. The assessment would take place every year and be performed by an official nominated committee that approved up to three textbooks by level and by discipline, from which teachers should make their choices. It was also established that the period of adoption of a textbook would be three years at least (Decree-Law 57/87). In 1990, the task of the officially nominated committee changes; at that point, the object of the group's work was to build an evaluation table, that is, criteria for textbook choosing. Next, the Ministry of Education would send the evaluation tool to schools yearly, in order to assist them on textbooks adoption. For secondary education the period of adoption of a textbook does not change (Decree-Law 369/90).

More recently, in 2006, the Ministry of Education, intending to increase the quality of education, introduced a new policy for quality control and textbook assessment. And, so Law 47/2006 identified the evaluation of textbooks as a means of improving their quality. Moreover, it established that the period of adoption of a textbook would be six years and also instituted the system for textbook assessment, approval and adoption. One of the central issues of this new law is the establishment of protocols with universities that will constitute teams to evaluate textbooks in areas of knowledge.

Unique textbook: choosing procedures and contemporary outlooks

In 1947, it was established that textbooks needed to be approved by the National Board of Education, a department of the Ministry of Education and would be the same for all Liceus and private high schools. The textbook could have one volume - with sections, one per year - or more than one volume. Teachers and students would use the book the following five years. During this five-year time, the authors of a unique book could propose, in new editions, amendments they deemed important.

A book approval depended on its consonance to the syllabus, scientific rigor and suitability to support teaching. The process of selecting a unique book began with the opening of a call to which the authors presented their textbooks. Next, the National Board of Education appointed two schoolteachers (jury) to look over the books. Then it would get reports back on what these schoolteachers thought about the books.

Finally, the National Board of Education came to a decision about the book to take, i.e., the unique book. And, announced it officially (Decree-Law n.º 36 508, 17 september 1947).

When a unique textbook was published, the words "Officially approved as unique textbook" on the back cover together with an official stamp and a number, guaranteed the textbook's authenticity.

To choose mathematics textbooks the Ministry of Education appointed two mathematics schoolteachers who worked at the Liceus. The textbooks proposed to a call were individually evaluated by the two appointed school teachers. The evaluator reported on the scientific and pedagogical value of the textbook. In the report he could propose amendments he deemed necessary for the approval or he could consider that the book wasn't worth of approval. Each evaluator had to grade the textbooks accordingly to their scientific and pedagogical value.

At time, in articles published in the daily press, teachers' bulletins and mathematics journals, there was a public discussion about the new textbook approval system. Although the article authors were mainly teachers, we can also see a mathematician's opinion:

> This [book approval] system can relegate to oblivion some good books for the work of students and teachers. (…) it is notorious the huge responsibility of authors and jury, the former in writing the books and the latter in evaluating and approving them, for they are endorsed for five years. (Barros, 1950, p.19, tr. M.A.)

The most mentioned advantages of new textbook approval system were, firstly, the low price of the books, which was officially enforced (Editorial, 1947); secondly, a more uniform preparation of students to take the state exams (Russo, 1956), which was granted by the teachers' obligation to follow the unique textbook content. About the disadvantages that this new system of approval faced, the articles spoke mainly on ones regarding the textbook authors and the jury. The time and effort needed to write a good textbook risking it was not chosen as a unique book (Soares, 1956) along with the knowing that it could only be chosen five years later, at the best, were reasons that kept textbook authors away from the calls (Ataíde, 1956; Soares, 1956). According to Ataíde (1956), gather a jury with enough competence for the job was not easy, given that the important teachers were usually textbook authors and so they were not allowed to take part in textbook evaluation. Indeed, at that time there were few certified teachers.

In terms of this approval system, the first mathematics textbook was approved in 1950, it was an Algebra textbook for the first grade of upper secondary school and it was object of dispute in the mathematics journal Mathematics Gazette.

THE FIRST UNIQUE TEXTBOOK APPROVED FOR THE 3RD CYCLE: ALGEBRA BOOK (1950)

Regarding mathematics textbooks to be used during the 3rd cycle of Liceus, they referred to four topics– Algebra textbook, Geometry textbook, Rational Arithmetic textbook or Trigonometry textbook.

By 1949, the process of approval of an Algebra textbook for the 3rd cycle of Liceus was initiated. António Lopes [1] was the only author to submit an Algebra textbook for the 3rd cycle of Liceus to the first call for the approval of the unique textbook of this topic and his textbook was the first mathematics textbook to be chosen (Almeida, 2013).

The 1947 reform had changed the mathematics syllabus for the 3rd cycle of Liceus (grades 10th and 11th). The infinitesimal calculus, removed from the programs in 1936, was reintroduced. The introduction of derivative study prompted debates about the quality of mathematics terminology in the programs and the unique textbook and also about the ways in which its study should be articulated with the study of limits (Matos, 2014).

On the production of mathematics textbooks, in particular for the 3rd cycle, Sebastião e Silva (1951) [2] argued about the complexity on writing textbooks encompassing the study of infinitesimal analysis. It was his belief that the presentation of infinitesimal analysis should as much as possible match intuition and accuracy. So, he underlined the exposition of this topic as a difficulty authors had to cope in what concerned the development of textbooks for secondary education. Moreover, authors were faced with a novel situation requiring writing about contents that had not been taught for twelve years at this level. He considered that the latter as a cause of the imperfections pointed to the Algebra textbook, which was approved as unique textbook for the 3rd cycle. Silva (1951) mentioned that, while reading the textbook

> one gets the impression that the author tried to reconstruct its mathematical culture and, at the same time, keep up with the deadline to present the textbook to the call - and it is very likely that he, author, already has realized the drawbacks of his hurried decision. (Silva, 1951, p.2, tr. M.A.)

Sebastião e Silva (1951) stated that the huge development yielded in order to clarify the analysis concepts helped matching logic with intuition; however, he points out that some attention has to be paid when trying to bring them together:

> What we immediately observe, in the theory of limits, is that the language becomes more intuitive coming to speak of «limit of a variable» instead of «limit of a succession» or «limit of a function». But, it must be kept in mind that it makes no sense to talk about limit of an independent variable, when, actually, we are addressing function limit in all cases: - functions of a natural variable (succession) or functions of a real variable. (Silva, 1951, p.2, tr. M.A.)

We can perceive from the words of Sebastião e Silva (1951) that the Algebra textbook was object of dispute. The largest discussion on this textbook was published in the

Mathematics Gazette and its author was a mathematician Laureano Barros. According to Barros (1950), the textbook lacked quality, so it should not have been approved as unique; he supported this position by posing the question "how is a teacher supposed to teach well by using books that aren't good?" (p. 19). Barros (1950) criticisms were very sharp, he stated that sometimes it seemed the book had been written by somebody who didn't know what he was talking about, that some definitions weren't accurate and that the exposition of some contents was correct and clear, but the explanation of other was very confusing and totally or partially incorrect. After exposing his critics, who mainly concerned scientific issues, he urged the author to make the necessary amendments.

In his examination of the first unique Algebra textbook Barros (1950) pointed out fourteen imperfections that can be allocated in two types, by the one hand, the deficiencies of scientific nature, on the other hand, the ones of pedagogical kind. In the conclusion of is review he says

> what particularly amazes us is the fact that none of the textbook evaluators had felt the gravity of the errors and defects that we point out (and others that are not mentioned here). And there is no doubt that the jury, collectively, did not feel it; otherwise, it had to propose the appropriate amendments to the Author, as required in the law.
> For the good of Mathematics teaching in our country, let these changes appear soon or, at least, in the next edition of this work. (Barros, 1950, p.24, tr. M.A.)

Barros (1950) comment on the bibliography that he thinks the author of the textbook should have consulted:

> we think, for example, that the [book] Algebra and Analysis Lessons of prof. Bento Caraça (that the author does not mention in the bibliography) could provide almost any material for an elementary exposition, correct and easily accessible, of the limits theory. (Barros, 1950, p.24, tr. M.A.)

The previous words relate the textbook's quality with its author expertise on the subject and on the subject teaching. This will allow us to use Shulman's (1986) idea of pedagogical content knowledge on our discussion. At the heart of this knowledge is the manner in which subject matter is transformed for teaching. This occurs when the teacher interprets the subject matter, finding different ways to represent it and make it accessible to learners.

We will attempt to illustrate difficulties authors/teachers have to cope when facing a syllabus transition, particularly, when there are changes in the mathematical content to be taught. In this case the author seemed to have experienced problems, especially on the approach of contents that have not been taught for twelve years. We intend to show by evidence – through the text analysis – that the textbook author has enhanced his pedagogical content knowledge in order to improve the textbook content comprehensibility and altered the parts he considered textbook's 'negative' features.

The Algebra textbook (1950): two different versions

Given the criticism to the Algebra textbook and the possibility of amendments to the approved textbook, we conjectured that the author might have made changes to the original content; and, so we searched in the Portugal's National Library and in private collections trying to find different versions of the Algebra textbook.

Using mainly Barros's (1950) observations as a support to trace changes in content we located two different versions of the Algebra textbook – an original and an altered version. We named the original version as Textbook B1 and the altered one as Textbook B2 (Fig. 1).
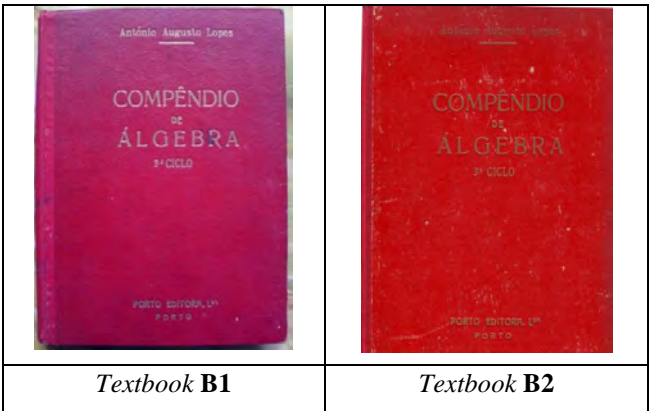

| Textbook **B1** | Textbook **B2** |

Fig. 1: Front cover of the two different textbook versions

The two versions have a similar structure: chapter – paragraph – section. Theoretical concepts that the student must learn are the first to be introduced, followed by examples and some exercises (with solutions). Bold is used to highlight the most important parts. Both textbooks include mathematicians' biographies, historical notes and bibliography. From Textbook B1 to Textbook B2, we observe that the bibliography of B2 has two more books than the bibliography of B1: Bento Caraça – Lessons on Algebra and Analysis (vol.2); Léon Brillouin – Mathématiques. Lib. Masson et Cie. Paris, 1947. We consider particularly interesting that the bibliography of textbook B2 includes a book which had been recommended by Barros in his comments about B1.

We analysed Barros (1950) review of the original version of the Algebra textbook, in terms of, the arguments he used to ground his thoughts, and of the suggestions he gave to the author in order to change the content. In the scope of this paper we will present part of the observed changes, focusing on the most criticised chapter – chapter II, *Limits*. On both versions of the textbook, the organization of Chapter II is quite alike, namely the number of paragraphs and their titles, as well as of sections.

As regards paragraph I, entitled *Infinitely large. Infinitely small*, Barros (1950) firstly points out the confusing way as the notions of *infinitely large* (section 1) and of *infinitely small* (section 2) are presented. He continues by referring that the definition of *infinitesimal simultaneous* (section 3) has no content and mentions that following the author tries to correct this however his effort is not successful, also the examples presented contradict what was previously written. Barros considers that the exposition of the topic addressed in section 3 is not at all accurate.

The analysis performed on textbooks B1 and B2 allowed us to trace differences. In *paragraph I,* we identified small changes regarding the definition of *infinitely large* and of *infinitely small* notions and a clearer explanation of these notions. All the examples included in B1 don't appear in B2. There is, in B2, a new section regarding infinitely small that introduces the concept of neighbourhood of zero, which refers "*x* is an infinitely small*", i.e. "*x approaches zero" or "the limit of x is zero", that in symbolic language becomes: "$x \rightarrow 0$*" or "$\lim x = 0$". In *paragraph I*, the major identified changes occurred in section 3, in which a totally different definition of *infinitesimal simultaneous* is presented and that brought changes to the following content of this section. In B1, the definition of *infinitesimal simultaneous* was as follows:

> Suppose x e y two infinitely small and let ε e δ be two positive numbers, arbitrarily small. We say x e y are *infinitesimal simultaneous*, when to the values of x_n (de x) that verify the inequality, $|x_n| < \varepsilon$ for $n \geq n_1$ correspond values of y_n (de y) in such way the inequality, $|y_n| < \delta$ is verified for $n \geq n_2$. (Lopes, n.d. a), p.47, tr. M.A.)

In B2, the definition of *infinitesimal simultaneous* was as follows:

> Suppose y = f (x) a real-valued function, variable x.

> Definition: If to all positive number δ is possible to correspond a positive number ε (variable with δ, i.e., function of δ) by means that the inequality $|y| < \delta$ is verified to all values of x that satisfy the inequality $|x| < \varepsilon$ we say that y = f(x) is an infinitesimal simultaneous with x. (Lopes, n.d. b), p.49, tr. M.A.)

In B2, we identified changes in two other sections, namely, section 4 – *Theorems related to the product of infinitely small* and section 5 - *Theorems related to the sum of infinitely small*. In both sections, as a result of the new definition, there were adjustments on the statement of theorems and on its demonstration.

Concerning paragraph II, entitled *Limit of variables and of functions*, Barros (1950) states that the errors in its content are a repetition of the ones that were manifested in the previous paragraph and names some defects.

One of the identified changes to paragraph II regards the title of a section. In B1, the title of section 6 was *Limit of an independent variable*, and, in B2 it was altered to *Limit of a variable*. In B1, there was an observation concerning the definition of limit and a graphical interpretation to the definition that were removed in B2. The previous mentioned change relates to one of the defects named by Barros. We also identified some other changes related to the defects referred by Barros. In textbook B1, there

was a section, entitled *Preliminary Theorems*, which no longer appeared in B2. In the remaining sections, some theorems have a different statement and all proofs are altered.

However, there were changes that do not relate to Barros criticisms, namely, some graphical representations enclosed in B2 (Fig. 2).
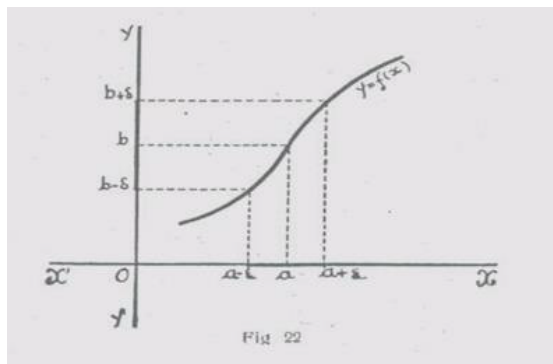

Fig. 2: Graphical representation in the context of the definition of limit of a function at a point (B2, p. 58)

Textbook B1 was text-oriented. The inclusion of images in B2 suggests both, an improvement of the authors' pedagogical content knowledge, and his understanding that the incorporation of visual material in textbooks can greatly enhance students' learning.

In paragraph IV, entitled *Basic notion of continuity of a function*, we traced some differences. The one that stands up occurred in subsection D that addressed the *right-continuous* and the left-continuous notions.

We point out that, in both versions of the textbook, Chapter II ends with an identical historical/biographical note, entitled *A XIXth century mathematician: −Augustin Cauchy*.

CLOSING DISCUSSION

Over the course of time textbooks have been a teaching and learning tool. In Portugal, mathematics teachers rely on the textbook using it as a main source of and tool in teaching. The educational importance of textbooks puts a stress on the quality of textbooks used in schools, which means that the process how these textbooks are approved is of the essence. The policy of textbook approval changed and evolved over time. This text provides an overview on how textbooks were chosen in previous decades. The outline begins in 1947, then it was established a formal state textbook approval of only one textbook, initiating a time that is usually referred as the unique

textbook period. The 1974 revolution promoted a change in textbooks approval policy and the choice of textbooks was then and up to now assigned to the teachers of each school. The control of textbooks quality was the main reason of afterward changes in the policy of textbook approval. The most recent change happened in 2006. For the purposes of the study, we focus on the unique textbook period. Here we characterized the unique textbook approval procedures and show the contemporary opinion on this textbook approval system. The most mentioned advantages of this mode were the low price of the textbooks and the guarantee of a uniform preparation to the final exams. On the other hand, the major disadvantage stated was the life span of five years. In which regards mathematics, we centre our attention on the Algebra textbook for the 3rd cycle of Liceus that was the first unique textbook approved and its approval occurred in 1950. The 1947 reform had changed the mathematics syllabus for the 3rd cycle and the infinitesimal calculus, which was out of the program for more than a decade, was reintroduced. There were some criticisms to the above-mentioned textbook that were published in the Mathematics Gazette and teachers' journals. Our analysis of the criticisms on the Algebra Textbook clarified that they concerned mainly to scientific issues. The analysis of two versions of the Algebra textbook showed that some amendments to the original version were carried out. Some of the identified changes are in line with the criticisms made by Laureano Barros, a mathematician. In the scope of this paper, only the chapter referring to the infinitesimal calculus was discussed. Our analysis showed a change in the textbook content that implied a development of the authors' pedagogical content knowledge. This example illustrates the difficulties authors/teachers have to cope when facing a syllabus transition, particularly, when there are changes in the mathematical content to be taught. Today teachers face new mathematics syllabus that, in the case of secondary education, incorporates contents that have not been taught for more than a decade, as well as, new textbooks in line with the new reality. One can say that there are no perfect textbooks, however there are better or worse textbooks. We believe this study, supported on an historical example, can help teachers' understanding on policies of textbook approval in Portugal and on the role that these policies play on the quality of mathematics textbooks.

NOTES

1. António Augusto Lopes (1917-2015) was a certified Liceus teacher since 1941. Later he became a teacher trainer with responsibilities in the initial formation of teachers and a member of the Commission for the reform of the upper cycle of Liceus. He also was an active participant in the Modern Mathematics reform.

2. The mathematician and university teacher José Sebastião e Silva (1914-1972) would become the best known leader of the Modern Mathematics movement in Portugal.

REFERENCES

Almeida, M. (2013). *Um olhar sobre o ensino da Matemática, guiado por António Augusto Lopes* Unpublished Doctoral dissertation, New University of Lisbon. [*A perspective on mathematics teaching guided by António Augusto Lopes*].

APM (1998). Matemática 2001: Diagnóstico e recomendações para o ensino e aprendizagem da Matemática.[Mathematics 2001: Diagnosis and recommendations for mathematics teaching and learning] Lisboa: APM.

Ataíde, A. (1956). "Livro único". Labor n.º 146, Fevereiro. Aveiro, pp. 172-183. ["Unique textbook"].

Barros, L. (1950). "Crítica de livros. O livro único de Álgebra – 3.º ciclo". Gazeta da Matemática n.º 70/71, Março – Junho de 1958, pp. 44-46. ["Book review. The Algebra unique textbook – 3rd cycle"].

Choppin, A. (2004). História dos livros e das edições didáticas: sobre o estado da arte. Educação e Pesquisa, v.30, n.3. Sao Paulo: USP. pp. 549-566. [History of books and of didatica edition] Retrieved July, 10, 2007, from http://www.scielo.br/pdf/ep/v30n3/a12v30n3.pdf

Decreto-lei n.º 36 508, de 17 de setembro de 1947 [Decree-Law n.º 36 508, 17 september 1947].

Decreto-Lei 57/87, de 31 de janeiro [Decree-Law 57/87, 31 january].

Decreto-Lei 369/90, de 26 de novembro [Decree-Law 369/90, 26 november].

Editorial (1947, September 24). *O Século*, p. 1.

Janeiro, J. (2005). Os manuais de Matemática: O que deles dizem os professores. [The Mathematics textbooks: what do teachers say about them] Actas do ProfMat 2005 (CD-ROM), Évora.

Lei 47/2006, de 28 de agosto [Law 47/2006, 28 august].

Lopes, A.A. (n.d. a)). Compêndio de Álgebra, 3.º ciclo. N.º 2707. Porto: Porto Editora, Lda. [Algebra textbook]

Lopes, A.A. (n.d. b)). Compêndio de Álgebra, 3.º ciclo. N.º 694. Porto: Porto Editora, Lda. [Algebra textbook]

Matos, J.M. (2014). Mathematics education in Spain and Portugal. Portugal. In A. Karp & G. Schubring (Eds.), *Handbook on the History of Mathematics Education*, pp. 291-302. London: Springer.

Repoussi, M., & Tutiaux-Guillon, N. (2010). New trends in history textbook research: Issues and methodologies toward a school historiography. *Journal of Educational Media, Memory and Society*, *2*(1), 154-170.

Russo, A. (1956). "Livros únicos, inconvenientes do seu aparecimento tardio". Labor n.º 157, Abril. Aveiro, pp. 487-488. ["Unique textbooks – disadvantages of their late onset"].

Shulman, L. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.

Silva, J. (1951). "A Análise Infinitesimal no Ensino Secundário". Lisboa: Gazeta de Matemática n.º 49, Outubro, pp. 1-4. ["The Infinitesimal Analysis in Secondary Education"]

Soares, J. (1956). "Mais considerações sobre problemas do ensino liceal". Labor n.º 155, Fevereiro. Aveiro, pp. 328-336. ["More considerations about secondary education problems"].