Workshop

GEOMETRY, TEACHING AND PUBLISHING IN THE UNITED STATES IN THE 19TH CENTURY: A STUDY OF THE ADAPTATIONS OF LEGENDRE'S GEOMETRY

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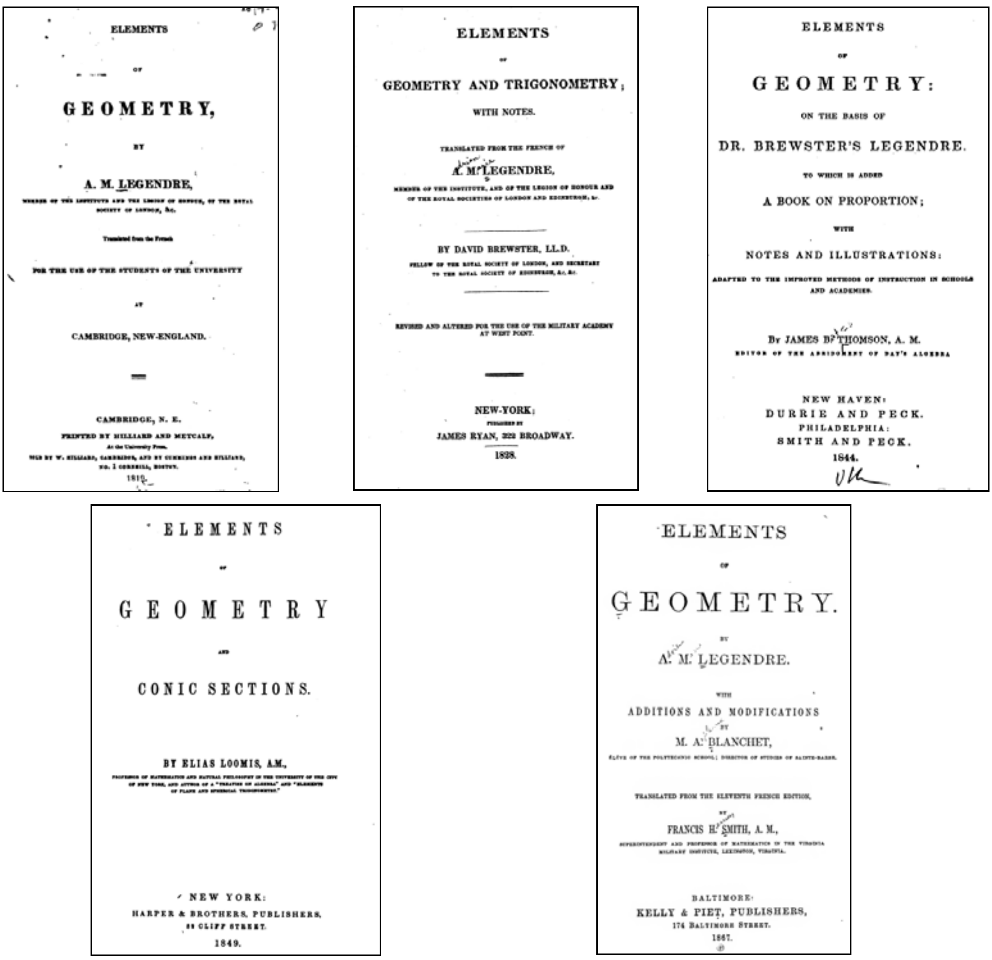
In the late 1810s, Harvard College sought to reform and modernize its curricula which had failed to keep up with the developing needs of scholars and scientists. To achieve this, Professor John Farrar broke with traditional educational practices informed by English and Scottish contents, pedagogical methods and media of diffusion and sought inspiration from French textbooks and pedagogical methods. For the teaching of geometry, he translated Legendre's Éléments de géométrie, an in-between textbook which broke with the inductive methods of Clairaut but also with traditional Euclidian textbooks such as Scottish Elements of Geometry by John Playfair. However, the introduction of these new practices represented a rupture in learning methods. To minimize this change, American mathematicians who adapted Legendre after Farrar adapted it, deeply altered the French textbook and made it more relevant to local uses. These transformations brought about new and original knowledge - a result of the combination of specific pedagogical needs and tendencies of American textbooks publishing.

INTRODUCTION

In the first half of the nineteenth century, the practice and the diffusion of mathematics within the United States were unprecedented in its transformed. The question of the teaching of geometry in colleges strengthened, as it was introduced as a separate college subject around 1790 (Ackerberg-Hastings, 2000, p. 7). In the early years of American Republic, the teaching of geometry leant on practical geometry treatises inspired by English authors and, later, on English-written versions of Euclid's *Elements* – such as *Elements of Geometry* (1795) by Scottish mathematician John Playfair - as shown in (Karpinski, 1940) and in (Cajori, 1890). For many teachers and educationalists, the teaching of geometry had to match what appeared to be two opposite requirements. On the one hand, it had to train learned minds to rhetoric and deductive reasoning, but on the other hand, it had to avoid useless and time-consuming speculations.

In the 1820s, Adrien-Marie Legendre's *Elements of Geometry* (published in France in 1794) seemed to match these expectations, since it was first translated at Harvard by John Farrar (1819) and then subsequently at West Point Military Academy, by Charles Davies (1828). The two translations became widespread and used in several university courses during the first half of the nineteenth century (Preveraud, 2014, p. 217). After 1840, in addition to these two textbooks reprints, three adaptations of *Legendre's*

Geometry were designed in the United States within three different educational contexts: Elias Loomis (1849), for civil higher education; Francis H. Smith (1867), for Virginia Military Institute and James Thomson (1847), for high schools students. As Davies's (especially the 1834 edition), Loomis's and Thomson's textbooks somehow transformed the original French book, this article will refer to the corpus of the "adaptations" of *Legendre's Geometry*. Although more faithful, Farrar's as well as Davies's 1828 translations can also be called adaptations because they both altered the French book.


ELEMENTS OF GEOMETRY, BY A. M. LEGENDRE, Translated from the French FOR THE USE OF THE STUDENTS OF THE UNIVERSITY AT CAMBRIDGE, NEW-ENGLAND. CAMBRIDGE, N. E. PRINTED BY HILLIARD AND METCALF, 1819.

ELEMENTS OF GEOMETRY AND TRIGONOMETRY; WITH NOTES. TRANSLATED FROM THE FRENCH OF A. M. LEGENDRE, BY DAVID BREWSTER, LL.D. REVISED AND ALTERED FOR THE USE OF THE MILITARY ACADEMY AT WEST POINT. NEW-YORK: PUBLISHED BY JAMES RYAN, 322 BROADWAY. 1828.

ELEMENTS OF GEOMETRY: ON THE BASIS OF DR. BREWSTER'S LEGENDRE. TO WHICH IS ADDED A BOOK ON PROPORTION; WITH NOTES AND ILLUSTRATIONS: ADAPTED TO THE IMPROVED METHODS OF INSTRUCTION IN SCHOOLS AND ACADEMIES. BY JAMES B. THOMSON, A. M. NEW HAVEN: DURRIE AND PECK. PHILADELPHIA: SMITH AND PECK. 1844.

ELEMENTS OF GEOMETRY AND CONIC SECTIONS. BY ELIAS LOOMIS, A.M., NEW YORK: HARPER & BROTHERS, PUBLISHERS, 82 CLIFF STREET. 1849.

ELEMENTS OF GEOMETRY. BY A. M. LEGENDRE. WITH ADDITIONS AND MODIFICATIONS BY M. A. BLANCHET, TRANSLATED FROM THE ELEVENTH FRENCH EDITION, BY FRANCIS H. SMITH, A. M., BALTIMORE: KELLY & PIET, PUBLISHERS, 174 BALTIMORE STREET. 1867.

Fig 1. Front pages of Farrar's (1819), Davies's (1828), Thomson's (1844), Loomis's (1849) and Smith's (1867) adaptations.

This article aims to analyze American adaptations of *Legendre's Geometry*, relying on a systematic comparison of the continuities and the changes in the successive textbooks contents. After having introduced the American editorial background of the teaching of geometry the adaptations were produced within, the study will then question the reason why five adaptations of the same text were published within only four decades. Thus, the article will proceed to the compared and time-changing analysis of the books with the French original, focusing on several characteristics of geometry textbooks: the arithmetization, the use of the *reductio ad absurdum*, the list of axioms, the statement of the propositions and the proofs. These examples will seek to highlight the combined influence of the targeted readership with the American publishing context upon the writing and the adaptation in the translation process.

TEACHING GEOMETRY IN THE EARLY CENTURY (1800-1819)

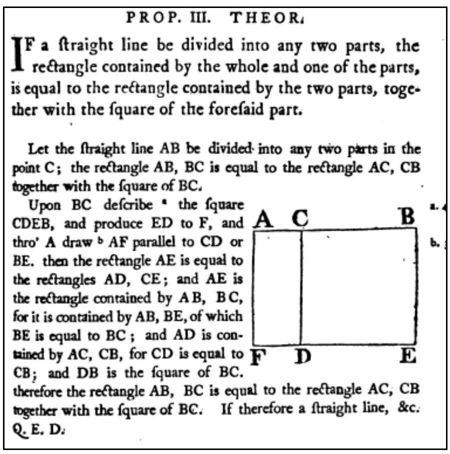
Compendia and Euclidean Scottish textbooks

In the very early years of 19th century America, geometry was taught essentially in colleges [1]. College students were trained mainly with *compendia*, books that covered arithmetic, algebra, surveying, geometry and other subjects related to mathematics. Two famous compendia used in the United States were *Mathematics, Compiled from the Best Authors,* written by Harvard's professor of mathematics Samuel Webber (1759-1810) in 1801, and *A Course of Mathematics*, an English book written by English Charles Hutton for English Military Academy and revised for an American version by Columbia's professor of mathematics Robert Adrain (1775-1843) [2]. The geometry exposed in those compendia was essentially practical. After the definitions, the authors solved problems, giving instructions to complete geometrical constructions [3].

Nevertheless, soon, the compendia in which geometry was too briefly introduced were not appropriate enough to match the requirements of changing curricula in American colleges. Most of them needed geometry not only to perform constructions but, above all, to train students in the art of reasoning more rigorously. American scholars turned to Euclidean geometry, which referred to the Greek book *The Elements.* Two textbooks were predominantly used, as new versions of *Euclid's Elements*, produced in Scotland in the 18th century. *Elements of Euclid*, by Robert Simson, was published in 1756 and offered a restored edition of the previous 16th century Latin versions of Euclid's text. It was almost immediately used in colleges and academies in Scotland and Great Britain because scholars and professors appreciated the logical structure of Euclid, which help students to learn the useful and lifelong skill of reasoning. Throughout the 18th and 19th centuries, Euclidean geometry – taught together with other subjects as Latin, Greek or rhetoric - shouldered the essential role of training up gentlemen at all levels of education from primary schools to university.

Indeed, *The Elements* consisted in an organized arrangement of geometrical propositions, proven through purely deductive reasoning. Each proposition was stated using the definition, the axioms and the previous propositions. In *Euclid's Elements*, most of the solutions of the proposed questions were first laid down and afterwards demonstrated to be true, in order to emphasize the logical process of deductive demonstration. Those so-called "synthetic" demonstrations did not give hints on how the solutions were found, but only on why they were conclusive. For example, to achieve some of his demonstrations, Euclid used *reductio ad absurdum*, a mathematical proof by contradiction, arguing that the denial of an assertion would result in a logical contradiction. *Euclid's Elements* were also known for a specific method of presentation. Each proposition was first stated in the most general way, as the proposition III of Book 2 in *Simson's Geometry*:

> If a straight line be divided into any two parts, the rectangle contained by the whole and one of the parts, is equal to the rectangle contained by the two parts, together with the square of the foresaid part (Simson, 1762, p. 45).



PROP. III. THEOR.

IF a ſtraight line be divided into any two parts, the rectangle contained by the whole and one of the parts, is equal to the rectangle contained by the two parts, together with the ſquare of the foreſaid part.

Let the ſtraight line AB be divided into any two parts in the point C; the rectangle AB, BC is equal to the rectangle AC, CB together with the ſquare of BC.

Upon BC deſcribe a the ſquare CDEB, and produce ED to F, and thro' A draw b AF parallel to CD or BE. then the rectangle AE is equal to the rectangles AD, CE; and AE is the rectangle contained by AB, BC, for it is contained by AB, BE, of which BE is equal to BC; and AD is contained by AC, CB, for CD is equal to CB; and DB is the ſquare of BC. therefore the rectangle AB, BC is equal to the rectangle AC, CB together with the ſquare of BC. If therefore a ſtraight line, &c. Q. E. D.

Fig. 2. Proposition III, Book 2 from *Simson's Elements* (Simson, 1762, p. 45)

This general statement of the proposition, the *protasis*, was then immediately followed by the particular statement of the same proposition related to a particular diagram: "Let the straight line Ab be divided into two parts in the point C; the rectangle AB, BC is equal to the rectangle AC, CB together with the square of BC" (Fig. 2). Then came the proof and the conclusion. The Euclidean geometry was also a geometry without numbers, and Euclid dealt with magnitudes through proportions.

In 1795, Natural philosopher John Playfair, also from Scotland, revised the labor of Simson in *Elements of Geometry*. He did so because the past editions of Simson's book were deteriorated, especially the illustrations. Moreover, even if he closely followed Simson's structure, he modernized his work, including recent developments of mathematics and appealing to algebra techniques and symbols [4]. Simson's and, later on, particularly Playfair's textbooks, were largely used in American colleges during the 19th century, for the same reason they were used in English colleges. *Playfair's Geometry* was even published in the United States with an American edition in 1806, by Francis Nichols. This American version was printed 39 times between 1806 and 1871.

The success of Legendre in France for the teaching of geometry

At the end of the 18th century, an alternative of *Euclid's Elements* was written in France. *Elements of Geometry* by Adrien-Marie Legendre, first published in 1794, was widely used in French schools throughout the 19th century. The textbook was translated into many languages and used in different countries, such as Italy, Brazil, Greece, Sweden or England (Schubring, 2007).

In France, the publication of *Legendre's Geometry* came after two centuries of criticisms of Euclid. With *Nouveaux éléments de géométrie* (1683) and *Elémens de géométrie* (1741), authors Antoine Arnauld and Alexis Clairaut claimed for a geometry easier to read, in which the propositions were arranged in a more evident order. Designed for the teaching of geometry in 18th century French colleges, Clairaut's textbook emphasized on pedagogical ambitions: "Although geometry is abstract by itself, it is nevertheless admitted that the difficulties [students] have to face come most of the time from the way geometry is taught in ordinary Elements" (Clairaut, 1765, p. i). In Euclidean textbooks, definitions, axioms and propositions, provided a teaching of geometry that Clairaut considered nor meaningful nor interesting for a beginner. As a consequence, the proofs in his textbook were less rigorous, they had to highlight the evidence of the geometrical truth. Mostly because he thought that it was more relevant, for a reader, to understand how the geometrical knowledge was established rather than to be taught unquestionable truths, Clairaut also offered a problematized geometry and methods to solve problems (Barbin, 1991).

But after the French Revolution, the establishment of a general and national system of education reinforced the values of knowledge and reasoning (Schubring, 2007, p. 38). The new structures of French higher education, as *École polytechnique*, for the training of engineers, *École normale de l'an III* for the training of teachers, and *lycées* for the training of future graduate students, all offered very theoretical curricula with high levels of excellence. The very demanding entrance examination for *École polytechnique*, prepared within the *lycées* or the *classes préparatoires* by most of the candidates, required a complete training in reasoning and in advanced mathematics. *Legendre's Geometry* came within the scope of this new orientation for secondary and higher education.

Legendre's Geometry borrowed the method of proof exposed in *Euclid's Elements*. The proofs relied on deductive reasoning, essentially through synthetic order. Legendre also included *reductio ad absurdum* in his demonstrations, as he did to establish the area of a circle (Legendre, 1817, pp. 102-121).

Yet, unlike Euclid, there were very few problems and constructions in his textbook. He also rearranged the order of several properties, especially in book 1, to make it more understandable. He tried to prove some propositions that stood as axioms or postulates in *Euclid's Elements*, which actually could not be proven. This is the case of Euclid's last and fifth axiom concerning the unicity of a line parallel to another drawn from any point. Legendre wrote many proofs of that postulate which all appeared to be erroneous [5]. The demonstration, produced in the 1817 edition, relied on the observation of a particular diagram, whose evidence gave the conviction of the truth of the proposition, according to the French man. Legendre wrote a same kind of proof, which Euclid had always discarded, in order to establish the equality of two right angles in proposition 1 of book 1[6]. Another difference with Euclid was the role of arithmetic and algebra in *Legendre's Geometry*. The French man assimilated magnitudes to numbers in order to perform operations on lines, surfaces and volumes - provided a length unit had been chosen. Thus, unlike Euclid, he gave formula for the area of polygons and the volume of solids. Finally, Legendre removed all the *protasis,* keeping only the particular statement for each proposition.

The soft breaking of John Farrar and the reform of Harvard curriculum

In 1819, Legendre was first translated in the United States by John Farrar (1779-1853), professor of mathematics at Harvard College. John Farrar graduated from Harvard in 1803. He took the chair of mathematics and natural philosophy in 1807. In the 1810s, Harvard's president, John T. Kirkland (1770-1840), a liberal and a reformist, asked Farrar to produce a new series of mathematics textbooks for Harvard curriculum. At that time, Harvard's mathematical studies were pursued with the help of *Webber's Mathematics* and *Playfair's Geometry*. In 1818, probably influenced by the Bostonian open-minded literary, cultural and scientific activity [7], Farrar started writing the translations of eight French textbooks, five of which were dedicated to mathematics, borrowing ideas from Legendre, Lacroix and Bézout [8]. The *Farrar's Legendre's geometry* was the first English language translation of the French textbook ever published in the world.

Legendre was praised by the Harvard's professor. His geometry united "the advantages of modern discoveries and improvements with strictness of the ancient method" (Farrar 1819, p. iii) and its "celebrity" across France and Europe was very well known in America. Introducing Legendre's book for an American audience, Farrar wanted to take his distance from the rigidness of Euclid as he explained to Kirkland:

> There is scarcely anything in which our superiority over the ancients is more manifest and palpable than in mathematics and yet this is almost the only branch of knowledge in which we continued to acknowledge them as our teachers (Farrar, 1817).

Its presentation of Euclidian geometry, using algebraic symbolism and a new arrangement of properties, was perceived by the Harvard's scholar as a good compromise for his teaching between classicism and modernity. Thus, John Farrar's translation of *Legendre's Geometry* was very faithful and introduced a breaking in the teaching of geometry in United States. The author did not introduce major changes, and the small alterations were only concerned with the removal of propositions (Preveraud, 2013a).

DAVIES AND LOOMIS. THE PUBLISHING CONTEXT AND THE COMEBACK TO EUCLID (1828-1849)

With Charles Davies's and Elias Loomis's versions of *Elements of Geometry,* very important changes came to light within the French original.

Charles Davies (1798-1876) was a professor of mathematics at West Point Military Academy and had started publishing a series of textbooks for cadets and more generally higher education students. In 1828, he made use of a previous adaptation of Legendre, published in 1822 by Scottish scientist David Brewster (1781-1868) [9] to publish *Elements of Geometry* [10]. He produced many reprints of his textbooks (Ackerberg-Hastings, 2000, pp. 238-248), notably in 1834. Another American adaptation of Legendre's textbook was intended for college students. It was the publication of *Elements of Geometry* in 1849 by New York University's professor Elias Loomis (1811-1889). The author had spent a few years in Paris and studied natural philosophy and medicine (Newton, 1889-1890, p. 326). Back to the United States, he became a professor of mathematics at New York University at the end of the 1840s. During his position in New York, he started a successful career as an author of mathematics textbooks designed for universities. Both Davies's and Loomis's textbooks were circulated in American higher education, as they each entered at least a dozen colleges curricula as shown in (Cajori, 1890).

Both authors reintroduced many characteristics of Euclid's book that Legendre had removed from his own. First, they added eight original Euclidean axioms to Legendre's first five. Also found in Playfair's and Simson's textbooks, Davies and Loomis added axioms regarding the addition and subtraction of the same magnitude to equal magnitudes. They removed Legendre's proposition I in book I that proved the equality of two right angles, and they changed it into an axiom. They also completely discarded all of Legendre's proofs of the fifth postulate (Davies, 1834, p. 13 & Loomis, 1849, pp. 12-13). Davies judged that the demonstration given by Legendre, in the 1817 edition, was not rigorous enough for a geometry textbook:

> The preceding investigation, being founded on a property which is not deduced from reasoning alone, but discovered by measurements made on a figure constructed accurately, has not the same character of rigorousness with the other demonstrations of elementary geometry. It is given here merely as a simple method of arriving at a conviction of the truth of the proposition. (Davies, 1834, p. 17)

They rewrote many proofs to make them "more Euclidean", including the statements of some propositions that appeared to be closer to Euclid's (or Simson's and Playfair's) than to Legendre's. Thus, Legendre's proposition "Deux triangles sont égaux lorsqu'ils ont un angle égal compris entre deux côtés égaux chacun à chacun" (Legendre, 1817, p. 20) became for Loomis:

> If two triangles have two sides, and the included angle of the one, equal to two sides and the included angle of the other, each to each, the two triangles will be equal, their third side will be equal, and their other angles will be equal, each to each. (Loomis, 1849, p. 17).

A last significant change occurred in Loomis's version with the almost complete abandon of magnitudes arithemization. Symbols +, -, =, >, etc. were removed by the New York University's professor as shown in the following example (Fig. 3).

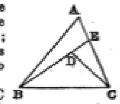
fig. 24. *Si d'un point* O *pris au-dedans du triangle* ABC, *on mène aux extrémités d'un côté* BC *les droites* OB, OC, *la somme de ces droites sera moindre que celle des deux autres côtés* AB, AC.

Soit prolongé BO jusqu'à la rencontre du côté AC en D; la ligne droite OC est plus courte que OD + DC*: ajoutant de part et d'autre BO, on aura BO + OC < BO + OD + DC, ou BO + OC < BD + DC.

*pr. 8. On a pareillement BD < BA + AD; ajoutant de part et d'autre DC, on aura BD + DC < BA + AC. Mais on vient de trouver BO + OC < BD + DC; donc à plus forte raison, BO + OC < BA + AC.

If, from a point within a triangle, two straight lines are drawn to the extremities of either side, their sum will be less than the sum of the other two sides of the triangle.

Let the two straight lines BD, CD be drawn from D, a point within the triangle ABC, to the extremities of the side BC; then will the sum of BD and DC be less than the sum of BA, AC, the other two sides of the triangle.



Produce BD until it meets the side AC in E; and, because two sides of a triangle are greater than the third side (Prop. VIII.), the two sides BA, AE of the triangle BAE are greater than BE. To each of these add EC; then will the sum of BA, AC be greater than the sum of BE, EC. Again, because the two sides CE, ED of the triangle CED are greater than CD, if DB be added to each, the sum of CE, EB will be greater than the sum of CD, DB. But it has been proved that the sum of BA, AC is greater than BE, EC; much more, then, are BA, AC greater than BD, DC. Therefore, if from a point, &c.

Fig. 3. Proposition IX, Book 1 in Legendre's *Éléments de géométrie* (left) (Legendre, 1817, p. 12) and in Loomis's (right) (Loomis, 1849, p. 19)

The reason why Davies and Loomis came back to Euclid in their adaptations is to be found in the publishing market context and the local teaching uses. During the first half the nineteenth century, the teaching of geometry in American colleges and schools borrowed mostly from uses and methods that came from England, based on rather strict Euclidean geometry (and also practical geometry textbooks). As a consequence, a large range of textbooks in use and published in America were closer to Playfair's work than to Legendre's. Both authors noticed the gap between the geometry offered in Legendre's and the geometry Americans used to learn and practise, and they decided to fill it. For Loomis and Davies, French original textbooks presentation of mathematics was unsustainable for the horizon of expectation, as defined in (Jauss, 2010), of American readers. Indeed, both authors intended to widely sell their textbooks within the publishing market. In association with Alfred Barnes (1817-1888), an editor from Hartford, Connecticut, Davies published several national series of textbooks, designed for elementary, high school and higher education, based on West Point Davies's first publications, and became a businessman in mathematics publishing (Ackerberg-Hastings, 2000, p. 215).

THOMSON AND SMITH. BACK TO EUCLID VERSUS MODERNIZATION: PEDAGOGIC NEEDS LED TO A COMPROMISE (1844-1867)

In the second half of the century, two other adaptations of *Legendre's Geometry* were published in the United States, but they were not as successful as their predecessors. The first one was intended for a growing but a new audience, and the second for a very specific and small group of readers.

Thomson's (1844) and Smith's (1867) adaptations intended to specific audiences

The 1840s marked the time when geometry started to be taught in high schools (Sinclair, 2008, p. 19). In the mid-century, high schools mainly trained young students for college admission where arithmetic and some algebra were required. From the 1870s, geometry became a requirement in most college admissions. Nevertheless, it had been taught in high schools before then. In 1844, the schoolteacher James Bates

Thomson (1808-1883) wrote another adaptation of *Legendre's Geometry* using Brewster's adaptation, but wrote it for high school readers. He was the co-author of a series of textbooks abridged from Jeremiah Day's series initially published in the 1810s [11].

In 1839, a new Military Academy opened in Lexington, Virginia. The Virginia Military Institute was designed to train military engineers of the South of the United States, as West Point did for the North. As an ex-cadet of West Point, the first superintendent and professor of mathematics, named Francis Henry Smith (1812-1890), organized the Academy explicitly referring to West Point structure, methods and curricula (Wineman, 2006, p. 40) largely relying on French pedagogical methods and textbooks translations (Preveraud, 2013b). He translated Louis Lefébure de Fourcy's *Elements of Trigonometry* (1868) and a new French edition of *Legendre's Geometry*, published in Paris around 1850 by Alphonse Blanchet (1867). When Legendre died in 1833, Blanchet, a mathematics teacher at College Sainte Barbe in Paris, shouldered the publishing of the following editions of the textbook. Starting in 1848, he introduced several changes in the text, and from that moment on he considered himself as a co-author. The main transformation was the introduction of the concept of limits. Blanchet used limits for the writing of the proofs for the measurement of circles and round bodies, whereas Legendre had applied *reductio ad absurdum.*

In the writing of their adaptations, how did Thomson and Smith take into consideration the previous adaptations and the specific needs of their readership? Did they introduce changes to the original, as Davies and Loomis did? Did the pedagogical and publishing context, in which the adaptations were produced, and the type of students they intended to target, also have as consequence a comeback to Euclid?

Thomson and Smith both came back to a Euclidean presentation of geometry, but they did search for a compromise between the virtues of Euclidean textbooks and the advantages of more modern methods - highlighted in this article by the following examples: the status and the role of abstraction in the teaching of geometry and the introduction of analytical tools to facilitate the understanding of some proofs.

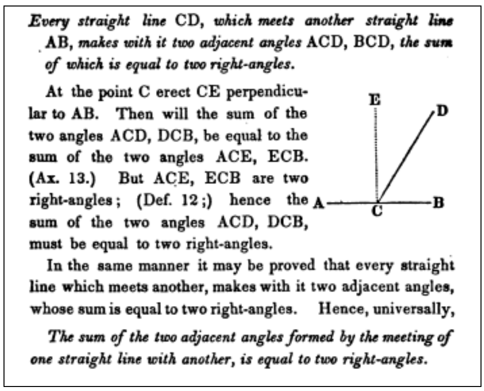
Geometric truths, abstraction and the yardstick of teaching contexts

Unlike Legendre who got rid of the Euclidean *protasis*, Davies re-introduced the general statement of the propositions as he clearly explained in his preface:

> In the original work […], the propositions are not enunciated in general terms, but with reference to, and by the aid, of the particular diagrams used for the demonstrations […]. This method seems to have been adopted to avoid the difficulty which beginners experience in comprehending abstract proposition. But in avoiding this difficulty, and thus lessening, at first, the intellectual labour, the faculty of abstraction, which it is one of the peculiar objects of the study of Geometry to strengthen, remains, to a certain extent, unimproved. (Davies, 1828, p. iii).

According to Davies, Legendre, who had removed the *protasis,* took his distance from one of the virtues of Euclidean geometry, that is to say the work the mind had to achieve in order to lead the learner "to the temple of the truth" (Davies, 1828, p. iv). As a consequence, and because Davies' *Elements of Geometry* were meant to be taught in colleges and academies, it was necessary to change Legendre's way of enunciating the propositions.

In his *Elements of Geometry* designed for high school, Thomson found a compromise between Legendre's and Davies's approaches. He chose to place the *protasis* at the end of the proof as shown in proposition II, book 1 (Fig. 4).



Every straight line CD, which meets another straight line AB, makes with it two adjacent angles ACD, BCD, the sum of which is equal to two right-angles.

At the point C erect CE perpendicular to AB. Then will the sum of the two angles ACD, DCB, be equal to the sum of the two angles ACE, ECB. (Ax. 13.) But ACE, ECB are two right-angles; (Def. 12;) hence the sum of the two angles ACD, DCB, must be equal to two right-angles.

In the same manner it may be proved that every straight line which meets another, makes with it two adjacent angles, whose sum is equal to two right-angles. Hence, universally,

The sum of the two adjacent angles formed by the meeting of one straight line with another, is equal to two right-angles.

Fig. 4. Proposition II, Book 1 in Thomson's *Elements of Geometry* (1844) (Thomson, 1844, p. 21)

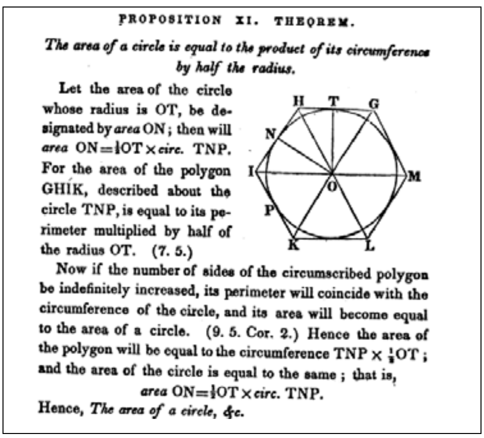
The reason is that he intended his book for high school students, who could experience stronger difficulties to face abstraction than college students might have:

> The principal embarrassment which young minds experience in the study of geometry, arises from the difficulty of comprehending abstract propositions. Legendre has essentially removed this difficulty by enunciating the propositions by the aid of particular diagram which he uses in the demonstration […]. It is found, however, to be inconvenient for scholars to quote a proposition enunciated with reference to a particular diagram […]. To obviate this inconvenience, after the truth of the proposition has been established with respect to the particular diagram in question, the general principle is then deduced, and for the sake of more convenient reference is printed in italics. Thus we begin with a particular case, and arrive at a general conclusion. (Thomson, 1844, p. 6).

In Thomson's approach, the example led the student to the general statement of the proposition, but this statement could not be omitted as conducting the mind to a universal truth, one of the objects of the teaching of geometry.

Introduction of analytical tools for simplification of proofs

Even if Thomson and Smith made Legendre look like Euclid, they openly took their distance from the proofs that relied on *reductio ad absurdum*. Thomson judged them "less satisfying for the mind" than the direct method (Thomson, 1844, p. 222). Indeed, the indirect method was only conclusive provided the conclusion was known before starting the demonstration, and it never gave the path that conducted the mind to the invention of the solution. It also mostly produced long and abstruse proofs for general-interest college or high school students. Consequently, in Thomson and Smith's books, most of the *reductio ad absurdum* proofs were replaced by more modern tools. For example, the proof of the area of a circle was based on the double assertion that the search for area could not be equal to the area of a larger circle nor a smaller one. Thomson changed the nature of the proof, using the limit – without saying the word – of the area of an inscribed polygon whose apothem became closer to the radius of a circle, and whose number of sides "indefinitely increased" (Thomson, 1844, p. 144) (Fig. 5). Noting that those "demonstrations, if not so rigorous as some, had the advantages of being more easily understood than the others" (Thomson, 1844, p. 233), Thomson took into account the needs of his readership in terms of pedagogical methods.



PROPOSITION XI. THEOREM.

The area of a circle is equal to the product of its circumference by half the radius.

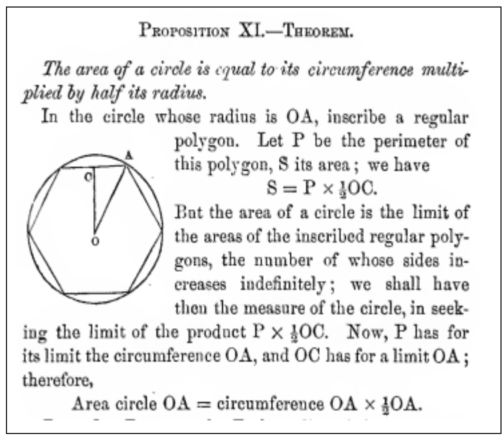
Let the area of the circle whose radius is OT, be designated by *area* ON; then will *area* ON$=\frac{1}{2}$OT$\times$*circ.* TNP. For the area of the polygon GHIK, described about the circle TNP, is equal to its perimeter multiplied by half of the radius OT. (7. 5.)

Now if the number of sides of the circumscribed polygon be indefinitely increased, its perimeter will coincide with the circumference of the circle, and its area will become equal to the area of a circle. (9. 5. Cor. 2.) Hence the area of the polygon will be equal to the circumference TNP $\times$ $\frac{1}{2}$OT; and the area of the circle is equal to the same; that is,

area ON$=\frac{1}{2}$OT$\times$*circ.* TNP.

Hence, *The area of a circle, &c.*

Fig. 5. Area of a circle proof in Thomson's adaptation (Thomson, 1844, p. 144)



PROPOSITION XI.—THEOREM.

The area of a circle is equal to its circumference multiplied by half its radius.

In the circle whose radius is OA, inscribe a regular polygon. Let P be the perimeter of this polygon, S its area; we have

$$S = P \times \tfrac{1}{2}OC.$$

But the area of a circle is the limit of the areas of the inscribed regular polygons, the number of whose sides increases indefinitely; we shall have then the measure of the circle, in seeking the limit of the product P $\times$ $\frac{1}{2}$OC. Now, P has for its limit the circumference OA, and OC has for a limit OA; therefore,

$$\text{Area circle OA} = \text{circumference OA} \times \tfrac{1}{2}\text{OA}.$$

Fig. 6. Area of a circle proof in Smith's adaptation (Smith, 1867, p. 131)

Years later, Smith used the exact same kind of proof but, then, with the concept of limit (Smith, 1867, p. 131) (Fig. 6). He intended his textbook for future engineers, whose education included differential calculus. The introduction of analytical tools and analytical methods in the demonstration came with the scope of simplifying the reading, the teaching and the learning, considering the needs and the existing knowledge of the readers.

CONCLUSION

One can be surprised by the way American scholars, after Farrar's first translation publication (1819), transformed Legendre's in such a manner that the French original almost disappeared, as in late Davies's or Loomis's versions, while authors still claimed to Legendre's influence. One could argue that this inconsistency can be explained, at least in part, by the international recognition attributed to the French textbook; for an American author, mentioning the name of Legendre on his front cover unquestionably promoted his book sales. Furthermore, it should be correlated to the nineteenth century long-term period of the history of mathematics education in America. Since 1819, decades passed before Davies, Loomis, Thomson and Smiths published their works. If Farrar was searching for a new geometry textbook, matching the reform of Harvard curricula, his successors were rather concerned by pedagogical tools, presentation and contents adapted to the audience they intended to catch and wrote textbooks as close as possible to the standards of publishing to make their diffusion possible. *Legendre's Geometry* adaptations offered mixed mathematics, borrowing from both French and local uses. *Legendre's Geometry* fate was similar to other French textbooks' in the United States (Bourdon's *Eléments d'algèbre* for example), as shown in (Preveraud, 2014, Chapter 5). Therefore, the study of the adaptations of *Legendre's Geometry* provided a relevant and significant example of the way in which French mathematics were transferred and received in America in the 19th century, as well as how teachers and textbooks authors adapted it to reach different audiences. Also involved in the sales of their work, Davies, Thomson and Loomis integrated transformations to adapt to the publishing market. In their textbooks, Legendre's original was the frame of the writing; in its name, its inner structure and organization, its arithmetization of geometry, these features were retained by each of the American authors. In the second half of the 19th century, many other geometry textbooks were also shaped using also *Legendre's Geometry*, and integrated the transformations previously made by Davies and Loomis, as shown in (Preveraud, 2014, p. 282-289). This prolonged and fruitful sedimentation of the French book, in the teaching of geometry, furnishes evidences of the standardization of mathematics practice, teaching and diffusion that would be achieved in that country by the end of the century (Parshall & Rowe, 1994).

NOTES

[1] The teaching of geometry was generalized in high schools not before the 1850s. See (Sinclair, 2008).

[2] See (Preveraud, 2012) for a complete analysis of the textbook and the adaptations Adrain introduced in his American version.

[3] Examples were given in (Preveraud, 2014, p. 94-97).

[4] For an extensive study of Playfair's *Elements of Geometry*, see (Ackerberg-Hastings, 2002). The author analyzed the « styles » Playfair wrote his proofs with.

[5] The most striking proof was published in the 1823 edition. Legendre used an analytical proof relying on the sum of the angles of a triangle. This new demonstration implied a complete reorganization of Book 1.

[6] This was an axiom in *Euclid's Elements.*

[7] In the 1810s, Farrar became a member of the Anthology Club, a Boston literary Society that founded the Bostonian Athenaeum, an independent library where many scientific books were gathered. One of the members, William Tudor (1779-1830), a businessman who had travelled in Europe, had brought back from France mathematics books Farrar had been able to read.

[8] Specific studies were produced, such as for *Lacroix's Algebra* translation in (Pycior, 1989). For a general analysis of Farrar's corpus of French translations, included the upgraded reprints Farrar wrote until 1840, see (Preveraud, 2014, Chapter 2).

[9] Brewster edited the work but did not write the adaptation of Legendre's *Elements.* This task was completed by Scottish scholar and historian Thomas Carlyle (1795-1881). See (Preveraud, 2014, p. 232).

[10] Davies published other translations and adaptations of French textbooks as *Jean-Baptiste Biot's Analytical Geometry* or *Louis P.M. Bourdon's Algebra* (Preveraud, 2014, pp. 253-261 & pp. 269-273).

[11] Day (1773-1867) was the professor of mathematics at Yale College before becoming its president.

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