Workshop

MATHEMATICAL ACTIVITIES IN THE CLASSROOM ON THE OCCASION OF THE EXHIBITION « ART ET SAVOIR DE L'INDE » DURING THE FESTIVAL EUROPALIA-INDIA (BRUSSELS, OCTOBER 2013 – JANUARY 2014)

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In the context of the Europalia-India Festival, an exhibition « Art et Savoir de l'Inde » took place, of which mathematics was an essential part. Teachers and future teachers were thus involved in the preparation of math lessons inspired by this Indian context. The construction of the fire altar in the shape of a bird leads to fractional calculations through the manipulation of wooden puzzles, while the construction of a square with poles and a cord asks interesting questions about exactness and proofs. After the description of the implementation of these activities in the classroom, we add some remarks about the advantages teachers and pupils could take from such historical approaches to mathematics, remarks corroborated by the observations already made by our students during their teaching practice.

INTRODUCTION

Every second year, Europalia presents, In Brussels and in Belgium at large, a grand festival about every cultural aspect of an invited country, for instance China in 2010 and Turkey in 2015.

In 2013, India was the invited country and, as specialists of Indian mathematics and astronomy, we took this opportunity to conceive an exhibition on Indian science. With the help of some partners, we decided to develop six aspects of Indian rational thinking : ayurvedic medicine (Pierre-Sylvain Filliozat and Sandra Smets), architecture (Kiran Katara and her students at the University of Brussels : Serge Delire, Nikita Itenberg, Manuel Leon Fanjul, Milosz Martyniak, Lucas Brusco, Jorge Serra, Vincent Guichot), ritual geometry (Jean Michel Delire), astronomy at Jai Singh's time (J.M.Delire), counting systems and operations (François Patte), games of India (Michel Van Langendonckt).

The exhibition, entitled « Art et Savoir de l'Inde », took place at the University of Brussels, accompanied by a cycle of lectures on the different subjects and a two-day conference on Indian games organized by M.Van Langendonckt and J.M.Delire at the Haute Ecole de Bruxelles (HEB) and the University of Brussels.

My wife Béatrice and I, as mathematics teachers, could not conceive of this exhibition without proposing a guided visit to secondary and (end of) primary classes. Naturally, the idea of building classroom activities based on the mathematical part of the exhibition arose from discussions with our colleague and students at the HEB. Robert

Sadin who is teaching at the primary school L'école du petit chemin (Drogenbos, a suburb of Brussels) and a maître de formation pratique at the HEB; and Chaimaa Haddar, Charlotte François, Mariam Ben Messaoud, Jessica Sassatelli, Sara El Yacoubi, Stéphanie Petit, Karim Abboud, Tariq El Boujamai and Rabi Nejjari who are future math teachers in the secundary school.

We built two activities and presented them in different schools: the calculation of the size of the bricks for the fire altar in the shape of a bird of prey and the exact construction of a square with cords and poles.

THE FIRE ALTAR IN THE SHAPE OF A BIRD OF PREY (ŚYENA-CIT)

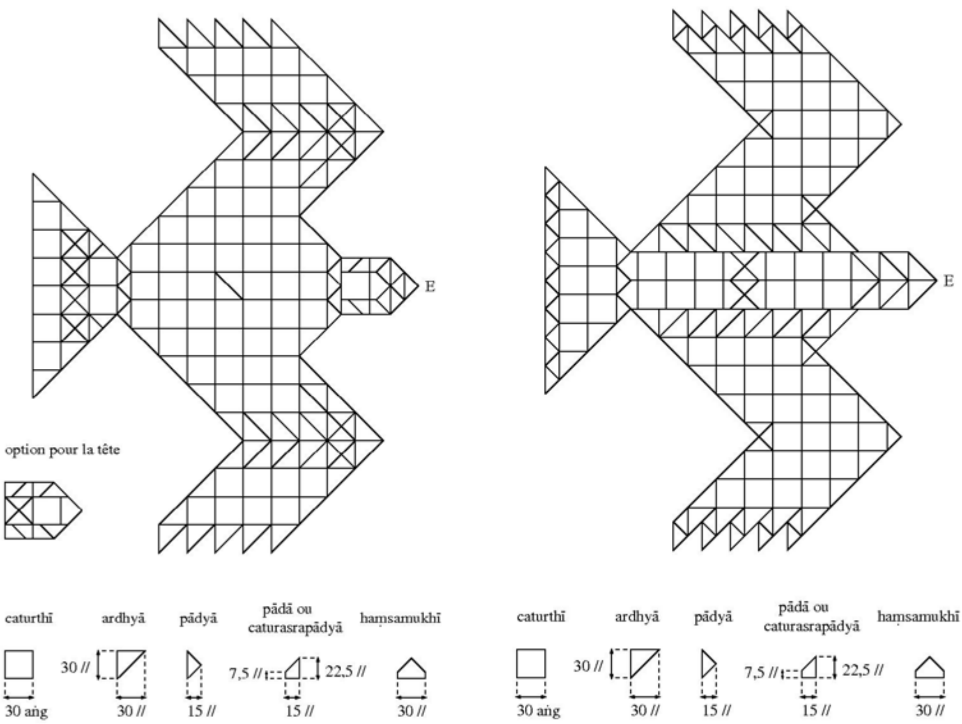


Figure 1 : odd layer **even layer**

Some Indian texts, the *Śulbasūtras*, dating from the last millenium BCE, precisely describe the shapes and sizes altars must have. These altars were prepared to receive the fire that makes the junction between men and gods in the sacrifice. Some were circular, like the *Gārhapatya* representing the world of men. Some were squares, like the *Āhavaniya* representing the world of gods. Others had much more complex shapes, like birds, turtle, a spoked wheel. But all of them had to cover the same area of 7.5 puruṣas², where the puruṣa is the height of the sacrificer with his hands pointing upwards. An altar was made of five layers of 200 bricks each. In the case of the śyena-cit (Figure 1), there are two types of layers (even and odd) and five types of bricks, of which the simplest is a square called caturthī because its side is one fourth (caturtha[1]) of a puruṣa.

Classroom exploitation of the fire altar śyena-cit

For our first mathematical activity, we asked the pupils to calculate the side of this brick with respect to their own puruṣa. Since they do not know the meaning of caturthī, they can not do better than transform 200 bricks of different shapes into an equivalent number of caturthī bricks. For that purpose, they had to count the number of bricks of each type and transform their total area into a number of caturthīs by comparing their respective areas with the caturthī's. To facilitate the procedure, we suggested them to fill the following table :

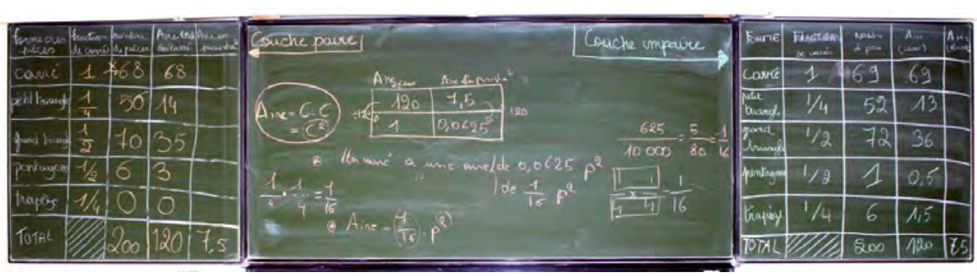
Shapes of the bricks	Fraction of the square	Number of bricks	Total area of the bricks (square = unit)	Total area of the bricks in puruṣa²
Total area				7.5

When the table is completed, we ask the pupils to fill the following table:

Number of squares	Area in puruṣa²
1	

Which necessitates dividing the area of 7.5 puruṣas² by the number of caturthī bricks of which total area is identical to the 200 initial bricks.

The activity ends with the calculation of the side length of the caturthī brick, by extracting the square root of the last result. This step could be tricky if the pupils do not like to work with fractions and prefer to work with a calculator.



Organizing and observing the activities

The activities were initiated by a short – 15 minutes – introduction with slides to the Indian context of the sacrifice, the different shapes of the altars and their dimensions. After this was completed with all the pupils together, we placed them in groups of four to six and gave to each group a plastified map of one layer (odd or even) and a puzzle made of 200 wooden bricks. To each pupil we gave a sheet of paper with the tables.

The puzzle was useful for comparing the areas of the different bricks. The pupils usually began by covering a larger brick by two smaller ones, for instance one caturthī by two ardhyās or one ardhyā by two pādyās[2].

Finding that four pādās exactly cover a caturthī, without knowing that pāda means 'fourth', was more difficult. One has to unite two pādās so that they form a rectangle half of a caturthī (Figure 2).

Figure 2 : comparing the bricks' areas

The most difficult comparison was that of the haṃsamukhī ('goose beak') brick with the caturthī. One cannot cover a caturthī by two haṃsamukhī to show that its area is half of the square. The only possibility, which is also mathematically interesting, is to notice that two pādās cover a haṃsamukhī, so that its area is equal to $2.\frac{1}{4} = \frac{1}{2}$ of a caturthī. One can also imagine halving one haṃsamukhī along its axis and rearrange it around another haṃsamukhī (Figure 2).

When counting the number of bricks of each type, the pupils used two different strategies, either they pile up the wooden bricks by tens or they distinguish the bricks on the map with different colors. To control their work, they can check that the total number of bricks is 200.

Computing the total areas for every kind of bricks in terms of caturthīs did not present any difficulty. Adding them yields 120.

But the division of the total area, 7.5 puruṣas², by 120 leads to problems when it is made with the help of a calculator. In that case, the result would be 0.0625 puruṣas² of which square root, 0.25 puruṣas could be obtained by using $625 = 25^2$. In the case of fractions, one can replace $\frac{7.5}{120}$ by $\frac{15}{240} = \frac{1}{16}$ whose square root $\frac{1}{4}$ (one caturtha) can be found if one knows how to multiply fractions. This root extraction was the only task the pupils could not complete without our help.

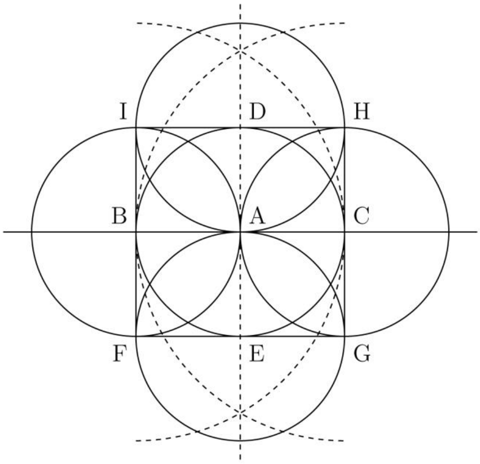
All these tasks were accomplished by the different groups of pupils, before completing the recreational activity of building the puzzle. This necessitates organization due to the number of bricks to be placed and the huge size of the puzzle. With a scale of one-tenth the actual size of the altar, the measure of the wingspan was about 90 centimetres. This forced the pupils to put two tables together and to distribute their roles, one for the head, one for each wing, one for the tail and one for the body. Since they had to follow a single map and to find the bricks they needed in a single box, they often helped each other.

THE CONSTRUCTION OF A SQUARE WITH POLES AND A CORD

In the introduction to the Indian context of the sacrifice, some steps of the actual construction of the altar are visible[3] and one of them shows that, before putting any brick in place, the shape of the bird was outlined with the help of a cord attached to poles driven in the ground.

The role played by the caturthī brick and the fact that some square altars, like the *Āhavaniya*, had to be built attest to the importance the square had for the Indian rituals. The *Śulbasūtras* give a construction of the square with poles and a cord following a procedure we have written in eight steps for pedagogical reasons:

1. Make loops at the two ends of a cord of length equal to the side *of the square.*
2. Drive a pole (A) on the West-East line[4].
3. Hook the two loops of the cord (on A) and draw a circle around.
4. Draw two poles at the West (B) and East (C) extremities of the diameter. At each of them fix a loop and draw a circle.
5. At the points where those two circles meet, draw a straight line which contains a second diameter that is perpendicular to the West-East line.
6. Fix two poles at the North (D) and South (E) extremities of this new diameter.
7. Fix the two loops respectively at the four poles (B, C, D, E) and draw four circles.



8. The vertices *of the square* are to be found at the intersections (F, G, H, I) of these circles.

Classroom exploitation of the construction of a square with poles and a cord

We did not give the entire procedure to the pupils. We did not mention that we wanted to build a square. In the first and last steps, the italicised words were missing. The reason of this omission was pedagogical. We wanted the pupils to guess that the shape they built is a square, but they also had to realize that this square was not perfect due to their imperfect way of tightening the cord. We asked them to prove that the figure aimed by the text is a square by retracing their building steps and find in (some of) them what properties of the square are exhibited.

Organizing and observing the activities

In this activity, we wanted the pupils to work as much as possible exactly as the Indians did. Since it was not possible to drive poles in the ground, we replaced them by drawing pins to be driven into a large cardboard placed on a table or on the ground. We had a cord and pens to draw the circles. Depending on time, we have either asked each of the pupils to complete collectively only one step, or to do the entire construction beforehand and then with the rest of the class complete a common drawing step by step. Generally, the pupils were asked to work in pairs. The use of pens placed in the loops for drawing circles was a source of errors, but this ensured that the final square would not be perfect and serves our purpose of requiring a proof.

The collective way of drawing was less satisfactory because the "step consisting of letting the pupils dive into the problem and try to find by themselves the intented shape was missing"[5]. In the other case, the pupils "were in a scientific process where, after finding the intented shape (induction), they had to recollect all the information from the steps. They then selected the most useful of them for the proof. Finally they wrote the argument in a logical sequence (deduction)".[6] In both cases, we encourage the pupils to express their arguments orally, in order to raise the level of the debate and make it more lively.

Another exploitation of the construction of a square with poles and a cord

We also organized this activity in the primary school of our colleague Robert Sadin. In that case, Robert Sadin had prepared a large box containing leveled clay and asked the pupils to draw the figure with the help of cords and little wooden pegs and markers carefully carved so that the cord can be kept in the same position while drawing. This equipment ensures a correct square shape if the pupils tighten the cords and do not prompt them to prove this fact. This was not our objective in primary school. We rather encouraged the pupils to discover the properties of the circle.

THE OPTIONAL HEAD IN THE ODD LAYER OF THE FIRE ALTAR

One can wonder why another disposition of bricks is given for the odd layer of the fire altar. This is related to the requirement, expressed in the *Śulbasūtras* that no crack between two bricks stands on the crack between the lower bricks. That is the reason why the odd and even layers are different. We seldom had the opportunity to build one layer above the other with the pupils, but Robert Sadin imagined a way to do it easily, by asking a team to build one layer on a plexiglass sheet and move it afterwards above a layer made by another team. Thanks to this experiment we discovered the reason why the *Śulbasūtras* propose an optional disposition for the head. This is left as an exercise for the reader.

HOW CAN SUCH CONTEXTUALIZATIONS OF MATHEMATICS BE OF GREAT BENEFIT TO TEACHERS AND PUPILS?

Before letting Mariam Ben Messaoud and Chaimaa Haddar, who wrote their memoirs about slightly different aspects of this contextualization in the case of Indian mathematics, I want to emphasize three of the many advantages offered by teaching mathematics with the help of history:

1. Mathematics is a human activity.
   Contextualization of mathematics in its history helps pupils to understand that they were invented by people like them over a long period of time and that they are still evolving nowadays. It also shows that the concepts were built to solve problems and not, as in too many mathematics lessons, the problems to illustrate concepts. Since problems vary according to places and times, the concepts are also different.[7] Moreover, taking the history into account motivates pupils who are more interested in history and language, especially if they have the opportunity to read ancient mathematical texts.

2. The origins of mathematics are varied.
   Mathematics are universal but history shows that all civilizations, even remote, made their own contribution. The teacher can bring the pupils' attention to that point by recalling the name and a concise biography of the mathematician who

discovered the application they are studying. For instance, the « generalized Pythagoras' theorem » is now called « al-Kāšī's theorem », because this Arab mathematician of Iranian origin, born at the end of XVth century, stated this theorem with the help of trigonometric functions[8]. To study the ill-named « Pythagoras' theorem », discovered by many civilizations, the teacher can show an application or a proof provided by each of them, and decide to give it a more neutral name as « theorem of the diagonal of the rectangle ».The teacher can also ask the pupils to prepare a short presentation of a mathematician. This has the advantage of involving them and making the pupils of foreign origin, numerous in Brussels and other large European cities, more aware of mathematics, especially if they choose a mathematician from their own country.

3. Mathematics are sensible
   History of mathematics usually leads to a shifting away from the centre and often helps us to give more meaning to the automatisms learned in primary school. For instance, the explanation of the binary couting system used in computers or of the sexagesimal one still present in our watches and necessary for trigonometry throws light on the decimal one.[9] Sara Ozdemir, one of my former students, who prepared an exhibition on the history of ancient numerations with her own pupils, wrote[10] « (…) the pupils less motivated by mathematics were the most interested by the activities and discoveries about the history of counting systems. They have accepted and loved to learn new methods of calculation ». Moreover, the reaction of one of Sara's pupils was[11]: « Mrs Ozdemir helped us to understand everything, so to better assimilate the subject, so to love it and she gave us the desire to learn it. That reminded me a lot of things in mathematics and to explain them myself was for me a challenge and a real fun. »

CONCLUSIONS

Mariam Ben Messaoud[12]: « The positive point was the pupils' motivation to learn differently, to make use of other instruments and to discover other aspects of mathematics. These activities allowed me to remind them of notions like area, rule of three, fractions and geometrical figures. They also induced the pupils to make use of their knowledge to take up a challenge. They had to work together to reach it and this showed them that we always succeed by mutual aid. »

Chaimaa Haddar[13]: « Erasing the name of the quadrilateral [in the exact construction of a square] added to the activity a challenge. Some pupils even considered this exercize as an enigma, which renders the atmosphere of the class very pleasant. About the motivation, one must add that introducing this activity by history gave the opportunity to the pupils who were not especially interested by mathematics to participate as well as the others. So they found a kind of entrance (…) different from those they already knew. »

Using history in a mathematical course offers those who have taken time to keep informed many advantages and few drawbacks except, perhaps, a slight loss of time in the classroom. The question is: would the pupils and the teacher really loose time if they allow history to make their mathematical activity more meaningful?

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[1] One can guess catur is akin to Latin quatuor ‘four’, -tha being a suffix for fractionnal or ordinal numbers, as -th in four-th. These similarities are due to the appartenance of English, Latin and Sanskrit, the language of the *Śulbasūtras*, to the same linguistic family.

[2] It is linguistically interesting to note that pādyā derives from pāda, which means ‘foot’ or ‘fourth’ (animals have four feet).

[3] As in the film *Altar of Fire*, by R.Gardner and J.F.Staal for the Film Study Center at Harvard University, color, 58 min, 1976 (http://www.der.org/films/altar-of-fire.html).

[4] This line is considered as the spinal column of the sacrifice. Its position is usually fixed by a method based on astronomy.

[5] Chaimaa Haddar, *Comment sensibiliser les élèves à la notion de démonstration (en mathématiques et ailleurs) en s’inspirant de son évolution historique*, Memoir presented for the diploma of BA – teacher in mathematics, Haute Ecole de Bruxelles, 2014, p.32.

[6] Chaimaa Haddar, *ibidem.*

[7] M.Wolff, *La bosse des maths est une maladie mentale ?*, La découverte, Paris, 1984, p.32, and J.P.Friedelmeyer, « *L’indispensable histoire des mathématiques* », *Repères IREM*, n°5 (oct.1991), p.25 : « Les constructions mathématiques ont répondu aux problèmes d’alors, dans un contexte de civilisation donné. »

[8] Following Youschkevitch, *Les mathématiques arabes*, Paris, 1976, pp.71 and 136. It was already in Euclid, *Eléments*, II.12 and 13, but without trigonometry, of course.

[9] About an interdisciplinary work on the Cassini’s map of France (1666-1715), Xavier Lefort wrote (« *L’histoire de la carte de France de Cassini* », *Repères IREM*, 14 (January 1994), p.25) : « In mathematics, for instance, one must endeavour to find again the ancient techniques ; then the pupils look at many notions in a new light. »

[10] Sara Ozdemir, *Utiliser l’histoire dans l’enseignement des mathématiques les rend plus humaines*, Memoir presented for the diploma of BA – teacher in mathematics, Haute Ecole de Bruxelles, 2009, p.45.

[11] Sara Ozdemir, *ibidem.*

[12] Mariam Ben Messaoud, *Amener une autre vision des mathématiques via l’exploitation de* l’exposition « Art et Savoir de l’Inde », Memoir presented for the diploma of BA – teacher in mathematics, Haute Ecole de Bruxelles, 2014, p.36.

[13] Chaimaa Haddar, *op.cit.*, p.32.