Plenary Lecture

CALENDARS AND CURRENCY – EMBEDDED IN ICELANDIC CULTURE, NATURE, SOCIETY AND LANGUAGE

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Iceland was settled from Norway in the 9^th^ century. The settlers brought with them a tradition of a seven-day week. Placed so far from other inhabitancies, they had to create their own week-based calendar. They developed their own currency, based on available commodities, such as fish, which was discarded at the turn of the 20^th^ century with the aid of arithmetic textbooks. Observations of the solar cycle soon revealed errors in the calendar which was cleverly amended. The calendar was later adjusted to the Roman calendar. It remained in common use for secular purposes until the 19^th^ century. Special occasions related to it are still celebrated. Both systems, the currency and the calendar, are embedded in the local language and serve to link generations together in their scope of time, nature and valuables.

INTRODUCTION

Iceland was settled from Norway by Viking raiders, bringing with them Celtic slaves, in the latter half of the 9th century. Icelanders became literary in the 12th century. Once literate, they began to write voluminously, initially to document the laws of the newly-founded Commonwealth (Kristjánsson, 1980, p. 29). The settlers brought Norwegian traditions of sheep and cattle farming, the seven-day week and an empirical lunar calendar. They developed their own calendar of the year, based on their heritage of counting time in weeks, and on their observation of nature. A thirteenth-century law-codex *Grágás* (1980–2000) contains a concise description of a week-based calendar, created in Iceland in the 10th century. The calendar, called *misseri*-calendar or farmers' calendar, was synchronized to the Julian calendar in the 12th century and later to the Gregorian calendar. Remnants of the *misseri*-calendar have survived in the country to present days, and given rise to mid-winter-feasts and the First Day of Summer as a public holiday.

Gradually, Icelanders also made up a system of currency, based on the commodities they had to sell to foreign merchants. Due to Iceland's geographic position, sailing was only possible during summer so the society was isolated. Their trade was mainly made by bartering where the basic trade unit was *fish*, a so-called *valid fish*. This unit became linked to the price of another basic commodity, an ell of woollen cloth, as well as the price of land and salaries of farm workers.

Both systems, the calendar and the currency system, became unique for Iceland and interwoven into the Icelandic culture and language while the Nordic languages diverged apart from each other in the early modern era. In this article, these systems will be explored and explained as an example of how systems could be developed in isolated ancient civilizations.

LITERATURE REVIEW

Ethnomathematics

Ethnomathematics is the study of the relationship between mathematics and culture. The term "ethnomathematics" was introduced by the Brazilian educator and mathematician Ubiratan D'Ambrosio. The subjects of ethnomathematics are the topics of everyday life: In different environments, ethnomathematics differs in terms of *counting time*, *measurements of land and distances*, *systems of taxation*, and arithmetic dealing with the economy such as *trade in the form of barter*, by which trading goods were allocated monetary value.

Researchers in various parts of the world, such as Bill Barton in New Zealand, Eduardo Sebastiani Ferreira in Brasil, and Paulus Gerdes in Mozambique, have made valuable contributions to research of ethnomathematics (Miarka and Viggiani, 2012). Their pioneering work has led the attention of other researchers to their own mathematical cultural heritage. In this paper, the focus is on Icelandic mathematical heritage which developed in relative isolation from the 9th century onwards.

Calendars

Mapping the time – The calendar and its history by E. G. Richards (1998) is a comprehensive overview of the calendar in theory, calendars of the world, calendar conversions and computations of dates of Easter. It mentions briefly the Icelandic Misseri-calendar where the author explains the geographical difficulties in using lunar calendars, practiced by Teutons, including the Angles, Saxons and the Vikings. Therefore, calendars operating according to different principles were developed. In Iceland, resting on the Arctic Circle, the lunar calendar was impossible to use in summer, so they took to counting the weeks instead (Richards, 1998, pp. 203–205).

Svante Janson (2011) has written a scientific analysis of the Icelandic calendar from earliest times. Janson built his work partly on research by physicist Þorkelsson (1923; 1930; 1932; 1936), physicist and science historian Vilhjálmsson (1989; 1990; 1991), as well as on Icelandic medieval sources.

The ancient handbook, Bishop Árnason's (1739; 1838; 1946) *Dactylismus Ecclesiasticus eður Fingra-Rím* is a valuable source of the farmers' calendar. Furthermore, Bjarnadóttir (2010) has published an overview of the pagan calendar.

Currency

Statistics Iceland, the official site of statistics for Iceland, has published Icelandic historical statistics (Jónsson and Magnússon, 1997) where the currency system is explained. Professor emeritus Gísli Gunnarsson (1976; 2002) has written several studies on the currency system, both scientific (1976), and for public education (2002). Arithmetic textbooks and tables, published in the 18th and 19th century, such as Stefánsson (1785), and Briem (1869; 1880; 1898) explained the relations between the various currencies.

THE ICELANDIC CURRENCY SYSTEM

The currency units

Once the Viking loot had been spent, metal for coins was not available. Exchanges with foreign merchants were made by barter. The selling commodities were fish, fat, woolen goods, measured in ells, and walrus teeth, exchanged for linen, grains, wood for boats and larger buildings, etc. All farms were recorded during 1702–1714 (Magnússon and Vídalín, 1980–1990). The records reveal rents paid in butter, cheese or hay, and feeding of livestock: sheep or cattle.

A complicated system of valuing land and goods developed gradually:

> **A hundred** equalled **120 ells** (originally a measure of woollen cloth).
>
> **A hundred** equalled also **240** (valid) **fishes**, that is processed and dried fish, each weighing about 1 kilogramme.
>
> **A hundred** was the equivalent of **a cow**, i.e. a middle-aged, faultless cow in spring, or **six sheep**, woolly and carrying lambs, in spring.
>
> **Farmlands** were also measured by hundreds: An average farm was valued at **20 hundreds**, and it was supposed to support livestock of **20 cows** or **120 sheep** (Jónsson and Magnússon, 1997).

This Icelandic currency system existed from medieval times up to the 20th century. Evaluation of farmlands in hundreds was established in 1096 along with the tithe, and was finally abandoned in 1922. The tithe was a property tax in Iceland and was based on the evaluation of farmlands, while in other countries it was an income tax. Farmlands may have been originally measured by how much livestock they could carry. They were only re-evaluated in case of damage due to floods, erosion etc., or changes in national industries, such as advantages due to access to fishing when fish became the main export commodity after 1200/1300 (Gunnarsson, 2002).

In 1880, the currency of the Danish *króna*, crown, was adopted and arithmetic textbooks emphasized teaching computations by exchange rates. This was done simultaneously with the establishment of the first bank, the National Bank of Iceland, NBI. Its guardian was the reverend and arithmetic textbook author Eiríkur Briem (1846–1929), who was professor of theology by profession.

Two examples from Briem’s Arithmetic Textbook

Briem’s arithmetic textbook was first published in 1869 and in an expanded version in 1880 with increased emphasis on decimal fractions and the new decimal currency system. I will take examples from Briem’s textbook seventh edition of 1898 that my grandfather, born in 1877 and whom I well remember, may have studied. One notices three different units: fishes, pounds of fat, and ells in addition to the hundreds.

> The tithe of 5 [estate] hundreds is 3 **fishes** and proportionally of larger estates. The light-tax is 4 **pound** [sheep] **fat** and each graveyard-tax is 6 **ells**. How high is the income of a church which had tithe from 131.55 **estate hundreds**, and 87 ½ **cash hundreds**, 17 ½ light taxes and 5 ½ graveyard-taxes when each pound of **fat** was worth 35 cents, each fish in tithe 29 ½ pennies, but in graveyard-taxes 29 cents? Answer: 82 crowns, 41 cents. (Briem, 1898, pp. 76–77)

One notices that the price of ells is not given, as everyone should know that each ell was worth 2 fishes. Furthermore one may wonder about the different exchange value of fishes in tithe and in graveyard taxes, and the difference between estate hundreds and cash hundreds. Both point to economic difficulties in using the old currency, and that the time for a new currency had arrived.

Another example concerns salaries; how workers were paid for their work:

> A farmer's hired hands, a man and a woman, made 50 **horses** [i.e. horse-loads] of hay in 4 weeks. The man earned 2 quarters **butter** a week but the woman 25 marks [of butter].
>
> If the food for the man was calculated as 3 **fishes** a day but for the woman as 2 fishes, and each fish was worth 30 cents; each pound of butter 65 cents, and furthermore each horse-load cost 60 cents for hiring land, horses etc., how much would the hay cost? Answer: 156.50 crowns. (Briem, 1898, p. 76)

The reader needed to know that 1 quarter equals 10 pounds and 1 pound contains 2 marks. But the example reveals also the ratio between salaries of men and women who had different tasks in the hay-making.

Both examples bear witness about the commodities that farmers produced and could use for currency. By the turn of the 20th century, the payments probably were through the local merchants or cooperative societies which were established around the country from 1882 onwards. The farmer had an account into which his income was credited. He then took out necessary goods for his family, and so could his workers do for what was left of their share if they had accepted a lamb or feeding of sheep through the winter as part of their payment. The economy was thus basically based on bartering.

THE MISSERI CALENDAR

Introduction

> The construction of calendars, i.e. the counting and recording of time, is an excellent example of ethnomathematics. (D'Ambrosio, 2001, 12)

In 930 CE, at the close of the settlement period in Iceland, a calendar was adopted, counting the year as 52 weeks. Observations of the solar cycle soon revealed errors, which were cleverly amended. The calendar was adjusted to the calendar of the Christian Church in the 12th century. It remained in common use for secular purposes until the 19th century, and special occasions related to it are still celebrated.

The week-based calendar

The settlers of Iceland came from Norway, and they brought slaves from Britain and Ireland. Their common calendar included a seven day week, the days named after their Norse gods (Björnsson, 1990, pp. 71–74; 1993, pp. 18–19, 665–660):

> *Sunday*,*sunnudagur*, the day of the sun
> *Monday*, *mánudagur*, the day of the moon
> *Tuesday* for Tyr, the god of war
> *Wednesday* for Woden, the cunning god
> *Thursday* for Thor, the thunder god
> *Friday* for Freyr / Freyja / Frigg, the god and godesses of love/marrriage
> *Saturday*, *laugardagur*, the day of washing and bathing.

The names related to the pagan gods remain in English and the Nordic languages other than Icelandic, where they were abandoned by the Church in the 12th century for *thridjudagur* (Third Day) for Tuesday, *miðvikudagur* (Mid-week Day) for Wednesday, *fimmtudagur* (Fifth Day) for Thursday and *föstudagur* (Fast Day) for Friday. *Sunnudagur*, *mánudagur* and *laugardagur* have remained intact to this day.

Probably some of the settlers counted the time according to the cycle of the moon, 29.52 days. However, in Iceland, the nights are light from April until late August so the moon can barely be seen. It is also often low at northern latitudes, and the sky is often cloudy. Counting the lunar months in summer was therefore abandoned and counting the summer weeks was adopted instead. Moreover, difficult weather conditions may mean that the moon cannot be seen regularly in wintertime, and in time winter months were standardized at 30 days each (Richards, 1998, p. 204).

A yearly parliamentary gathering was agreed upon in year 930. According to a brief history of Iceland, Í*slendingabók* [*The Book of Icelanders*, *Libellum Islandorum*], written by Ari the Learned in the period 1122–1133, an agreement was reached at the establishment of the parliament on meeting again after 52 weeks or twelve 30-day months and four extra days. The year was to be divided into two terms, *misseri*, the winter-*misseri* to last six months, the summer-*misseri* another six months, and the four extra days were added at midsummer. This system quickly revealed the need for a more reliable system of time-computing. By the 950s it had become clear that the summer "moved back towards the spring", i.e. the summer according to this calendar began earlier and earlier vis-à-vis the natural summer (Benediktsson, 1968, p. 9). This was inconvenient, as the parliamentary gathering had to assemble after the completion of certain necessary farming tasks, such as lamb-births, and before others, such as hay-making, were due to begin. This is recorded in *The Book of Icelanders*. The book exists in manuscripts from 17th century. In this context, Benediktsson's (1968) edition of *The Book of Icelanders* states:

> This was when the wisest men of the country had counted in two misseris 364 days – that is 52 weeks, but twelve thirty-night months and four extra days – then they observed from the motion of the sun that the summer moved back towards the spring; but nobody could tell them that there is one day more in two semesters than can be measured by whole

> weeks, and that was the reason. But there was a man called Þorsteinn Surtur … when they came to the Althing then he sought the remedy … that every seventh summer a week should be added and try how that would work … (Benediktsson, 1968, pp. 9–10)[1]

The error seems to have been realized by Þorsteinn Surtur from an observation of the location of the sunset, which in northern areas moves rapidly clockwise along the horizon before the summer solstice, and subsequently anti-clockwise. According to the source, cited above, Þorsteinn Surtur recommended in year 955 that every seventh year an extra week be inserted, Summer's Extra Week, making the average year 365 days. Below, we see the view to the west direction from Þorsteinn Surtur's farm at 65° N latitude. Only at summer solstice the sun sets right north of Eyrarfjall (Vilhjálmsson, 1990).

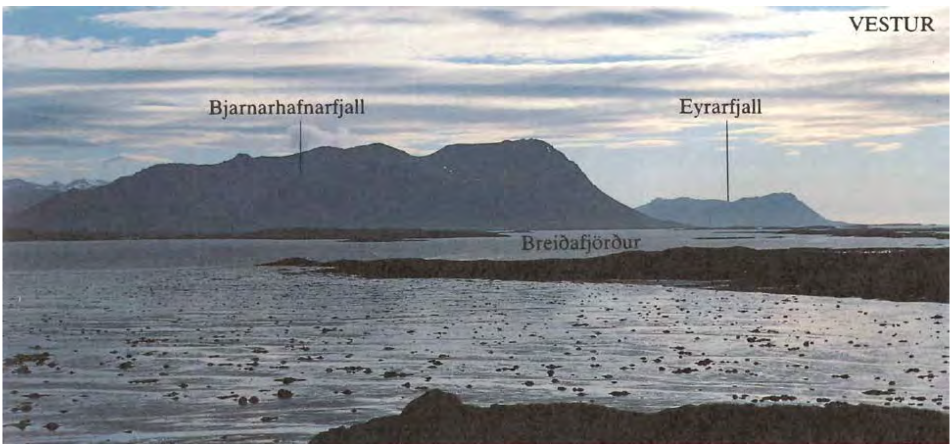

Figure 1: The view to the west direction from Þorsteinn Surtur's farm at 65° N latitude. (Photographer: Grétar Eiríksson)

By the year 1000 the parliament gathered after ten weeks of summer had passed, instead of nine weeks, which illustrates that the eleven missing leap years since year 955 had also caused "a move back to the spring" as earlier, even though slightly slower. In the *Book of Icelanders* it says: "Then it was spoken the previous summer by law, that men should arrive at *Althingi* when ten weeks of summer had passed, but until then it had been a week earlier." (Benediktsson, 1968, p. 15)

Simulation of the sun track

The **axial tilt** of the Earth is the inclination angle of the Earth's rotational axis in relation to its orbital plane around the sun. The axial tilt is currently about 23.44°. The axis remains tilted in the same direction throughout a year; however, as the Earth orbits the Sun, the hemisphere tilted away from the Sun will gradually become tilted towards the Sun, and vice versa. Whichever hemisphere is currently tilted toward the

[1] All Icelandic passages have been translated by the author, KB

Sun, experiences more hours of sunlight each day. In the northern hemisphere, the maximum tilting towards the Sun is at **summer solstice** in June and the minimum at **winter solstice** in December. At **equinoxes** in September and March, the axial tilt does not have effect on the observed sun track.

We will simulate the sun track at 65° North, the latitude of the farm of Þorsteinn Surtur, and for comparison at 42° North, the latitude of Rome. In Figure[2] 2 the altitudes of the sun at 65°N and 42°N through 24 hours are approximated by

$$\text{Iceland}(x) = -(90-65)*\cos(2\pi x/360) \text{ and}$$
$$\text{Rome}(x) = -(90-42)*\cos(2\pi x/360)$$

respectively. The 0°–360° scale on the horizontal axis denotes the direction of the sun on the 360° horizon during 24 hours, while the scale on the vertical axis denotes the altitude in degrees.

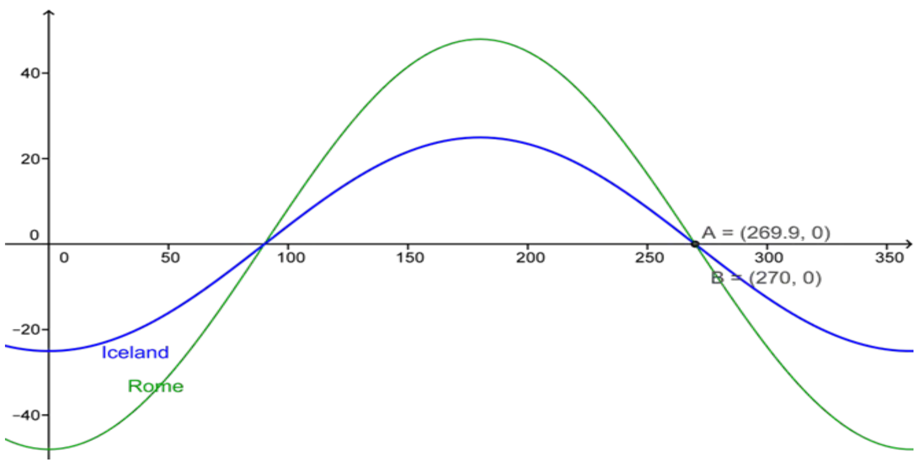

Figure 2: Simulation of the Sun track seen from 65°N and from Rome at 42°N at equinoxes

We notice that the track of the sun is flatter at northern latitudes than closer to the equator. At winter-solstice the sunset at 65°N is at 200° on the horizon. At summer-solstice the sunset is at 340°, see Figures 3 and 4. The range of the sunset is about 140°. For comparison, the approximated sunset in Rome at 42°N at winter-solstice is at 241° on the horizon, and at summer-solstice at 300°. The range in Rome is about 60°.

The altitude of the sun at 65°N at noon is 90°–65° = 25° at the equinoxes. At summer solstice the sun is therefore only 25°–23.4° = 1.6° below the horizon at its lowest position. Since the sun is so close to the horizon at that time, the night is bright enough for reading a book. The official calendar for Iceland does not record darkness in Reykjavík at 64°N from May 19 until July 23 (*Almanak fyrir Ísland*, 2014).

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[2] Figures 2, 3, 4 and 5 are made by the aid of Geogebra-software.

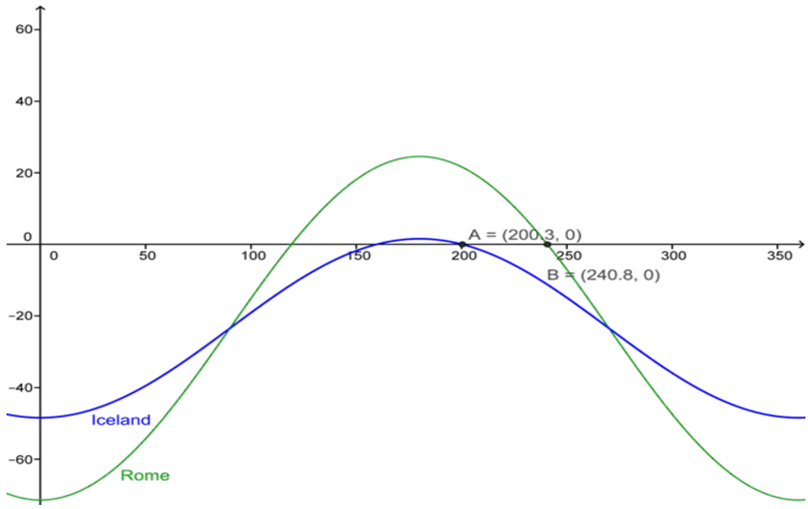


Figure 3: Track of the sun seen from 65°N and from Rome at 42°N at winter solstice when the observed tilt, –23.44°, is subtracted from the formulas for the altitudes of the Sun at equinoxes.

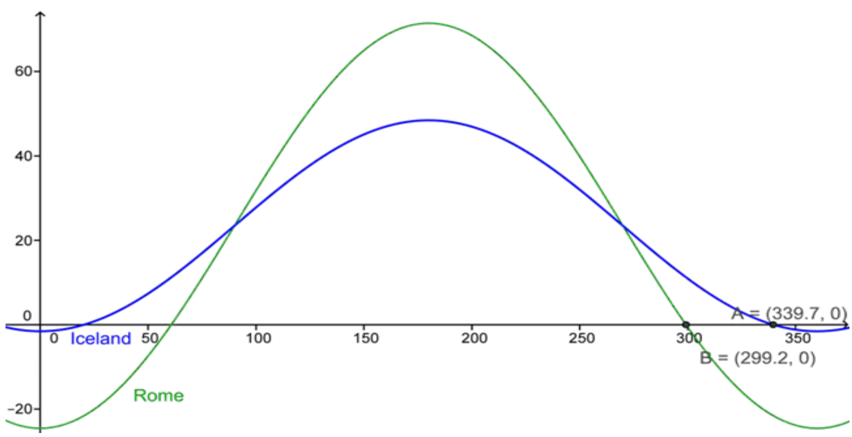


Figure 4: Track of the sun seen from 65°N and from Rome at 42°N at summer solstice when the observed tilt, +23.44°, is added to the formulas for the altitudes of the Sun at equinoxes.

As the track of the sun is flatter at northern latitudes than closer to the equator, the sunsets move more rapidly there along the horizon near the solstices than at places closer to the equator. In early June, when the observed axial tilt is one degree less than maximum, the difference then between the sunset location and the northernmost location sunset on the horizon in Rome is 1.5°, see the coordinates of points B and D on Figure 5, while at 65° latitude the difference is nearly 6°, see coordinates A and C on Figure 5.

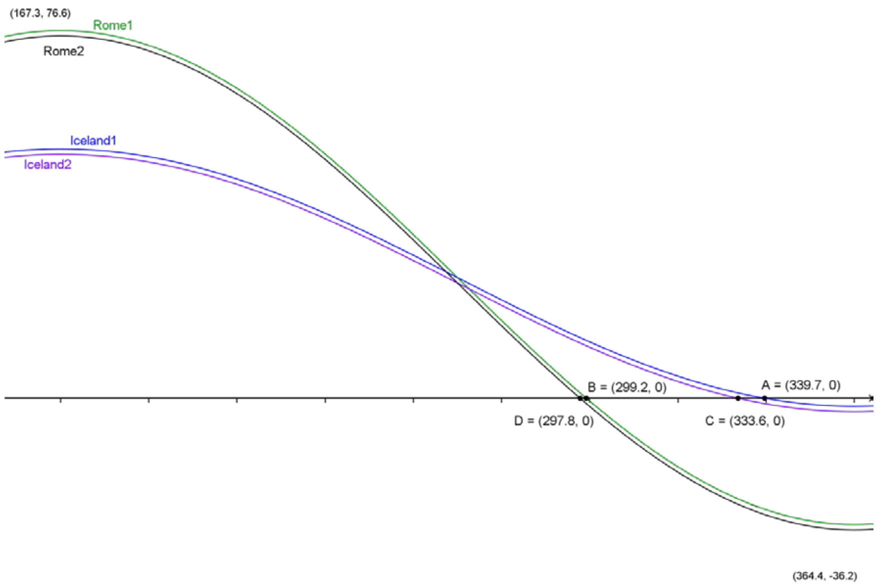


Figure 5. The placements of sunsets at 22.4° observed tilt and at maximum observed tilt, 23.4°, in Rome at 42°N and in Iceland at 65°N.

This may easily be seen from many places on the west coast of Iceland, e.g. in Öskjuhlíð hill in Reykjavík, see Figures 6 and 7.

Figures 6 and 7. The sunset on June 11 2008 at 23:55 on the left, and on June 19 2008 at 24:04 on the right. The pictures are taken from Öskjuhlíð hill in Reykjavík, Iceland, at 64°North. (Photographer: The author)

The sunset moved 2.2° clockwise along the horizon in 8 days in June 2008. The situation in nature cannot be compared computationally to the modelling drawings above, as refractions in the air have to be taken into account. However, the pictures in Figures 1, 6 and 7 may illustrate how Þorsteinn Surtur could discover in year 955 the error in the calendar, adopted in year 930.

Discovery of errors in the Christian calendar

Icelanders agreed to accept the Christian Faith in year 1000. The Christian Church as an institution was established in the late 11th and early 12th century. The Church introduced the Julian calendar with one extra day to the 365 days every fourth year, the leap year. By adding a day to 365 days every fourth year, the average length of the year became 365.25 days, while in reality it is approximately 365.2422 days. The Julian calendar assumed the summer solstice to be on June 21, decided upon in Nikea in 325 CE. In the 12th century, it fell on June 15, due to the addition of six too many leap-year days, which would have been skipped at years 500, 600, 700, 900, 1000 and 1100 according to the correction by the Gregorian calendar.

In the first half of the 12th century, Oddi Helgason (1070/80–1140/50), a farm-worker, made observations of the annual motion of the sun, of which an account is found in the ancient treatise *Odda-tala* [*Oddi's Tale*], contained as a separate treatise in the oldest part of the manuscript *GKS 1812, 4to*, written around 1192 (*A dictionary of Old Norse Prose – Indices*, 1989, p. 471). The Icelandic week-based *Misseri*-calendar was adjusted to the Julian calendar in the early 12th century. Oddi observed the summer solstice and the winter solstice to be earlier than the official date, i.e. on June 15 and December 15 instead of June 21 and December 21. Oddi also explained the curve of the height of the sun, counting the weekly increase and decrease in its height, measuring the rise as 91 sun diameters (Vilhjálmsson, 1991). In the Icelandic calendar treatise, *Rím II*, [*rím*: rhyme, meaning calendar] written in the late 13th century (Beckman and Kålund, 1914–1916, pp.81–178), it says:

> Solstice in summer is four nights before the mass of John the Baptist ... It is so in the middle of the world. Some men say that it is close to a week earlier in Iceland. (Beckman and Kålund, 1914–1916, p. 121)

The Mediterranean [Mid-Earth] Sea was considered the middle of the world. The error of the Julian calendar had thus been discovered in Iceland in the 12th century but was blamed on different latitudes. Better estimates of the year than the Julian calendar presented had already been made. Examples are shown in *Table 1*.

Researcher	Location	Year	Length of the year
?	Babylon	c. 700 BC	365.24579 days
Hippachus	Egypt	150 BC	365.2466 -
?	Mexico (Mayan)	700 AD	365.2420 -
Da Yen	China	724 AD	365.2441 -
Al-Battani	Arabia	900 AD	365.24056 -
Al-Zarqali	Arabia	1270 AD	365.24225 -

***Table 1*: Examples of early estimates of the length of the year (Richards, 1998, p. 33).**

The Julian and Gregorian Calendars

The Gregorian calendar was a reform to correct the discrepancies of the Julian calendar. By 1700, when the Gregorian calendar was accepted in the Danish Realm, eleven days were omitted, November 17–27. Evangelic Lutheran Bishop Jón Árnason (1739; 1838; 1946) published a thorough guide, *Dactylismus Ecclesiasticus eður Fingra-Rím*, to computing the ecclesiastical calendar according to the new style, both by mathematical formulas and by counting on fingers. The calendar of the "farming-year", the *Misseri*-calendar, was attached as a second part.

For both calendars the *Sunday letters*, *dominical letters*, were important. Each day of the year is assigned a letter, called *calendar letter,* A, B, C, D, E, F or G. Each year is then assigned a letter, *dominical letter*, according to the calendar letter of the Sundays that year (Richards, 302–307).

A regular 365-day year begins and ends on the same weekday, which implies that the dominical letters of succeeding years are displaced back one place – except after leap years, when they are displaced two places. February 29 and March 1 have the same calendar letter. The leap years therefore have two dominical letters. Every 4th year was a leap-year and the week counts 7 days; the lowest common multiple of 4 and 7 is 28, so that the sequence of dominical letters, called the Solar Cycle, repeated every 28 years in the Julian calendar. In the Gregorian calendar the leap years were skipped in years 1700, 1800 and 1900 so the length of the Solar Cycle including these years extended to 40 years. *Table 2* shows the reverse order of dominical letters in a sequence of 28 years. The same information on the dominical-letter sequence, arranged on the fingers and palms in Bishop Árnason's *Dactylismus*, is shown on Figure 6. This was convenient for the general public for whom paper was expensive and scarce.

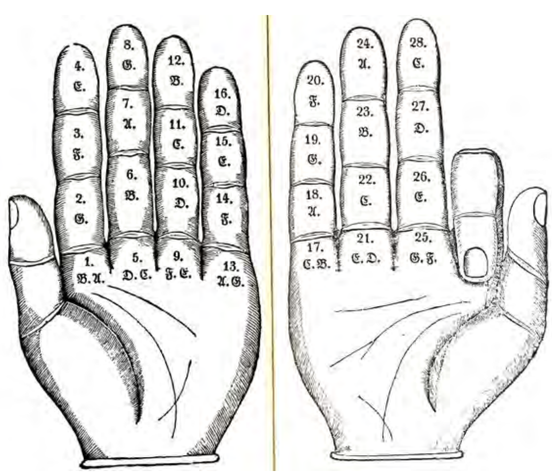


Figure 6. Dominical letters, arranged on the palms and fingers of both hands (Árnason, 1838; 1946, pp. 106–107).

Place # in the Solar Cycle	**Two last digits of the date**	**1600 2000**	**1700 2100**	**1800 2200**	**1900 2300**
1	00	BA	C	E	**G**
2	01 29 57 85	**G**	B	D	F
3	02 30 58 86	F	A	C	E
4	03 31 59 87	E	**G**	B	D
5	04 32 60 88	DC	FE	**AG**	CB
6	05 33 61 89	B	D	F	A
7	06 34 62 90	A	C	E	**G**
8	07 35 63 91	**G**	B	D	F
9	08 36 64 92	FE	**AG**	CB	ED
10	09 37 65 93	D	F	A	C
11	10 38 66 94	C	E	**G**	B
12	11 39 67 95	B	D	F	**A**
13	12 40 68 96	**AG**	CB	ED	GF
14	13 41 69 97	F	A	C	E
15	14 42 70 98	E	**G**	B	D
16	15 43 71 99	D	F	**A**	C
17	16 44 72	CB	ED	GF	BA
18	17 45 73	A	C	E	**G**
19	18 46 74	**G**	B	D	F
20	19 47 75	F	**A**	C	E
21	20 48 76	ED	GF	BA	DC
22	21 49 77	C	E	**G**	B
23	22 50 78	B	D	F	A
24	23 51 79	**A**	C	E	**G**
25	24 52 80	GF	BA	DC	FE
26	25 53 81	E	**G**	B	D
27	26 54 82	D	F	A	C
28	27 55 83	C	E	**G**	B
1	28 56 84	BA	DC	FE	**AG**

***Table 2*. Dominical letters of years 1600 to 2300 according to the Gregorian calendar.**

The larger Solar Cycle is 400 years. The *Dactylismus* finger-rhyme helps to find the number of each year in the 28-year Solar Cycle (Figure 7).

Years 1600, 2000, ... # 1, dominical letters B and A.

Years 1700, 2100, ... # 5, dominical letter C. etc.

The years in-between are counted onwards with modulus 28.

The numbers 1, 5, 9, 13 correct the cycles due to missing leap years at the turns of centuries.

The numbers 4, 1, 2, 3 at the top denote the classes of the centuries within the 400-year cycle.

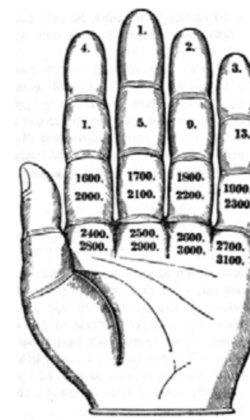


Figure 7. Numbers, denoting classes of centuries within the 400-year cycle (Árnason, 1838; 1946, p. 102).

Adjustments of the *Misseri* calendar to the Roman calendar

By the adjustments to the Julian calendar, the Summer's Extra Week was inserted every sixth year, or every fifth year if two leap years were in-between. The First Summer-Day was the beginning of the secular year. It was to fall on Thursday in the week April 9 to 15. In the late middle ages, April 9 was the beginning of the light-night period in Northern-Iceland, while the date April 15 was the Norwegian official beginning of summer (Þorkelsson, 1930). The First Summer-Day transferred later according to the Gregorian calendar to Thursday in the week April 19–25, and other dates, mid-summer, beginning of winter etc., accordingly (Sæmundsson, 1972).

The Summer's Extra Week is inserted in mid-summer, beginning on Sunday after 13 weeks. The years of Summer's Extra Week are those which begin on Monday, that is when the dominical letter of the year is G, and those which begin on a Sunday, i.e. have the dominical letter A, if it is the year before a leap year. The year that begins on a Sunday and is the year before a leap year is called Rhyme-Spoiler. This happens once in the 28-year Solar Cycle, but is disturbed at 1700, 1800 and 1900 when there are no leap years and the Solar Cycle is 40 years. The Rhyme-Spoiler moves all dates forward one day from Summer's Extra Week until leap-year's day. This means that Summer's Extra Week is always inserted in years of Sunday July 22, or Sunday July 23 when next year is leap-year. In *Table 2*, the dominical letters for years of Summer's Extra Week are marked by bold letters. To take examples of years of Summer's Extra Week, there are 2001, 2007, 2012 and 2018. Year 2023 with dominical letter A brings Rhyme-Spoiler and so does year 2051.

The present pagan-calendar 30-day months

The Icelandic *Misseri*-calendar was based on weeks and they were counted during summer months when the moon could not be seen. There were also thirty-day long months, more in use during winter. The names have been different during the centuries while the high winter months, *Þorri* and *Góa*, have remained since earliest times. The farm-year began in spring. The present system and names are as follows:

> *Harpa* begins on Thursday in April and marks the beginning of summer-*misseri*
> *Skerpla* begins on Saturday in May
> *Sólmánuður* [Sun-month] begins on Monday in June
>   *Aukanætur*, four extra nights, begin on Wednesday in July
> *Heyannir* begins on Sunday in July, at mid-summer
> *Tvímánuður* begins on Tuesday in August
> *Haustmánuður* [Autumn-month] begins on Thursday in September
>   *Summer's Extra Week* every sixth or fifth year
>   *Veturnætur*, two extra [winter]-nights
> *Gormánuður* begins on Saturday in October and marks the beginning of winter-*misseri*
> *Ýlir* begins on Monday in November, ending at *yule*
> *Mörsugur* begins on Wednesday in December
> *Þorri* begins on Friday in January
> *Góa* begins on Sunday in February
> *Einmánuður* begins on Tuesday in March

All weeks of summer-*misseri* begin on Thursdays but weeks of winter-*misseri* begin on Saturdays. Therefore, the two last days of *Haustmánuður* [Autumn-month] are called *veturnætur* [winter-nights] and are outside weeks. The last week of *Einmánuður* lasts only five days.

The names of the high winter months, *Þorri* and *Góa*, were also names of pagan gods. When *Einmánudur* [One-Month or Lone-Month] commences there is one month left until summer begins (Þorkelsson, 1928). The beginning of *Þorri* marks mid-winter and has been an occasion for mid-winter festivities.

> ***Þorri*** (masculine) begins on Friday in the 13th week of winter (in late January); this was Husbands' Day.
>
> ***Góa*** (feminine) begins on Sunday in the 18th week of winter (in late February); this was Wives' or Women's Day.
>
> ***Einmánudur*** (masculine) begins on Tuesday in the 22nd week of winter (in late March); this was the Young Men's Day.
>
> ***Harpa*** (feminine), the first month of summer, begins on Thursday in April 19–25, First Day of Summer; this was the Young Girls' Day (Björnsson, 1993, pp. 766-783).

The First Day of Summer has been a public holiday in Iceland for centuries. Youth and child-care organizations organize festivities in cooperation with local authorities. Furthermore, international Mother's and Father's Days are not much celebrated in Iceland: rather the first days of *Þorri* and *Góa* (Björnsson, 1993).

Figure 7. Celebration of First Day of Summer. (Photographer unknown)

The First Day of Summer also depends on the dominical letter of the year, see *Table 3*:

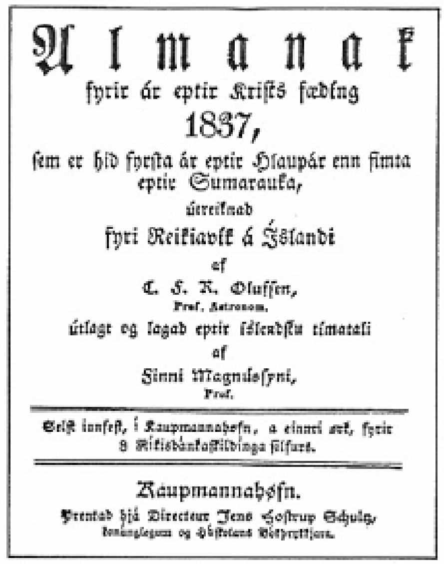
Dominical letter	First Sunday in April	First Day of Summer
A	April 2	April 20
B	April 3	April 21
C	April 4	April 22
D	April 5	April 23
E	April 6	April 24
F	April 7	April 25
G	April 1	April 19

***Table 3*. The dates of First Day of Summer.**

The Icelandic *Almanak*

Before Bishop Árnason's *Dactylismus*, Danish calendars were in use for a few centuries, but these did not meet the needs of Icelanders, most of whom were more familiar with the *misseri*-calendar. The *Dactylismus* therefore must have been the main handbook for Icelanders during the 18th century. Icelandic calendars of the years 1800 to 1836 exist in manuscripts, adjusted to the environment in Northern Iceland, but they were not continuous. In response to these, which were deemed to violate the University of Copenhagen's monopoly on publication of calendars, a calendar in Icelandic, the *Almanac* for Iceland, was first published in 1837 by the University of Copenhagen. The calendars were computed by professors at the University of Copenhagen until 1923, and translated into Icelandic by prominent Icelandic scholars. They added the *misseri* calendar with all its features, to the regular ecclesiastical calendar of the Evangelical Lutheran Church, in addition to local geographical information, such as time of the tide for various places and time of sunrise and sunset in Reykjavík. (Sigurgeirsdóttir, 1969). Figure 8 below shows the cover of the first issue of the *Iceland Almanac*. In translation it says:

Almanac for year after Christ's birth 1837, which is the first year after leap year but the fifth after Summer´s Extra Week, calculated for Reykjavík on Iceland, by C.F.R. Olofsen, Prof. Astronom, translated and adjusted to the Icelandic calendar by Finnur Magnússon Prof. (Sigurgeirsdóttir, 1969).



Almanak
fyrir ár eptir Krists fæðíng
1837,
sem er hið fyrsta ár eptir Hlaupár enn fimta
eptir Sumarauka,
útreiknað
fyri Reikiavík á Íslandi
af
C. F. R. Olufsen,
Prof. Astronom,
útlagt og lagað eptir íslendsku tímatali
af
Finni Magnússyni,
Prof.

Selst innfest, í Kaupmannahöfn, á einni mrk, fyrir
8 Ríkisbánkaskildínga silfurs.

Kaupmannahöfn.
Prentað hjá Directeur Jens Hostrup Schultz,
konúnglegum og Háskólans Bókprentara.

Figure 6: The cover page of the first issue of the *Iceland Almanac*.

The publication of the *Almanac* was transferred to the University of Iceland in 1917, and from 1923 the computations have been made by Icelandic mathematicians or astronomers (Sigurgeirsdóttir, 1969). The *Almanac* has been sold in 10,000 copies a year to a population of 320,000.

DISCUSSION

One may ask why the *Misseri*-calendar survived. There are several possible answers to that:

- It had been in use for two centuries before the introduction of the Christian calendar.
- It was maintained as a secular calendar by the law, contained in the oldest collection of laws, *Grágás*, registered in a mid-13th century manuscript *Konungsbók.* It is thus rooted in the medieval literary heritage that was studied in Iceland through the centuries.
- Bishop Árnason respected it and made it compatible to the calendar of the church in his 1739 *Dactylismus* as did 19th century scholars, at the establishment of the *Iceland Almanac,* and its later calculators.

- Registrations of births and deaths, done by the Church, were only prescribed from 1746, after the publication of bishop Árnason´s *Dactylismus.* Some birthdates were recorded according to the Misseri-calendar, a certain weekday in a certain week of summer or winter until the 20th century. The following data are from the 1920 national census:
    - Sigurður Jónsson born on Sunday in 12th week of winter 1859
    - Guðlaug Einarsdóttir on the 16th Saturday of summer in 1850

The conversion to the regular calendar sometimes caused confusion and people claimed to have another birthday than had been registered.

We may repeat the question why the pagan Misseri-calendar has survived in a Society of European Christian culture and propose an answer:

In northern latitudes such as in Iceland, the difference between darkness in winter and brightness in summer is extreme. Celebrating mid-winter *Þorri* and First Day of Summer and counting the weeks in-between is a tribute to the light and is intimately related to life in northern nature.

IMPACT ON LANGUAGE

The domestic tradition in currency and calendar has influenced the language in a variety of ways. It for example survives in poetry which has become national property. Until the mid-20th century, children of poor families might be brought up by people outside the family for as small an amount paid from public funds as possible. A well-known verse written by a poet brought up in such circumstances is:

Líf hans var til fárra fiska metið,.	His life was evaluated to only a few fishes.
Furðanlegt hvað strákurinn gat étið.	Amazing how much that lad could eat.
(Arnarson, 1942, p. iii)	

Children in the latter half of the 20th century probably did not realize that “fishes” is here a real currency unit, but all the same they may have felt the resentment and pain of these childhood memories. Official registration reveals that the poet had been placed in a farm at the age of three for 30 fishes from community funds after his father drowned, even if his mother worked at the same farm. Normal payment for a child who could not work was 240 fishes (Þráinsdóttir, 2011).

There are numerous examples of the various currency units referred to in proverbs, phrases and expressions. An example is *komast í álnir*, lit. “reach to ells”, means becoming prosperous. The Icelandic word *fé,* which is of the same origin as the English word “fee”, means both “sheep”, “livestock” in general, and “money”. *Fé,* meaning money, has remained as a natural expression in Icelandic, while its etymological roots are the Latin word *pecūnia*, “money”, and *pecū* in Latin means “a flock of sheep” (Magnússon, 1989).

The calendar was also a source of idiomatic phrases and proverbs. *Að þreyja Þorrann og Góuna*, to endure *Þorri* and *Góa*, the high winter months, means to wait patiently for better times. There are also meteorological sayings, such as "*þurr skyldi Þorri, þeysin Góa, votur Einmánuður, þá mun vel vora*", saying that *Þorri* should be dry, *Góa* windy and *Einmánuður* wet, to expect a good spring. The name of the pagan winter-solstice month, *Ýlir*, including the heathen festival "yule", has survived in the Nordic words *jól*, *jul*, for the Christian feast of Christmas.

CONCLUSIONS

In the earlier days of civilization, the mathematical elements varied greatly from one culture to another, so much that what was called mathematics in one culture would hardly be recognized as such in certain others (Wilder, 1950). D'Ambrosio has suggested an answer to why we should study ethnomathematics:

> It may compatibilize cultural forms – we should incorporate ethnomathematics in such a way that they facilitate the acquisition of knowledge, understanding, and the compatibilization of known and current popular practices. (D'Ambrosio, 1985, p. 70)

The *Misseri*-calendar was successfully compatibilized to the Julian calendar of the Christian world in the 12th century and further successfully compatibilized to the Gregorian calendar in Bishop Jón Árnason's *Dactylismus* in 1739 where the two calendars were printed parallel to each other.

Literature, phrases and proverbs preserve tradition in the language and culture, and link the present with the past by referring to human experiences, independent of time and space. The Reverend Eiríkur Briem was the right person at the right time to compatibilize the ancient currency of ells, fishes and butter to the modern currency of crowns.

At periods of rapid changes it is of great value to make the old compatible to the new in order to link generations together. This leads the attention to present times. Will the 21st century see compatibilization to the world of cyberspace, such as descendants of bit-coins?

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