Plenary Lecture

PROMOTING AN INTERDISCIPLINARY TEACHING THROUGH THE USE OF ELEMENTS OF GREEK AND CHINESE EARLY COSMOLOGIES

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Abstract: Most of the curricula, at an international level, encourage an interdisciplinary approach for the teaching of both mathematics and sciences. In this context, interdisciplinarity is often promoted as a fruitful way of making students aware of the links existing between mathematics and the sciences. History of science can be considered as an inspiring ground for the elaboration of teaching sequences where mathematical and scientific knowledge and skills are integrated. In this paper, examples of such integration are presented through the use of two distinct historical episodes dealing with Greek and Chinese early cosmologies. From these cosmologies teaching sequences (involving historical elements mixed with non-historical ones) have been elaborated in order to provide students with elementary astronomical knowledge dealing with scientific and mathematical knowledge and skills.

INTRODUCTION

Most of the curricula, at an international level, encourage an interdisciplinary approach for the teaching of both mathematics and sciences (see for example AAAS 1989, Rocard 2007). In this context, interdisciplinarity is often promoted as a fruitful way of making students aware of the links existing between mathematics and the sciences. As an example, the third pillar of the French *common base of the knowledge and skills* for primary and lower secondary school claims for "concrete and practical approaches to mathematics and sciences" that should allow students to acquire the "scientific culture needed to develop a coherent representation of the world and an understanding of their daily environment" and help them grasp that "complexity can be expressed in fundamental laws" (French MEN 2006 – my trans.). Here, mathematics and experimental sciences are considered altogether in a global enhancement project of scientific literacy.

Nevertheless, nothing is easy about effectively integrating mathematics and science in the classroom since the disciplinary isolation of the two disciplines in the traditional teaching organizations has to be overcome (Czerniak et al. 1999). Indeed, in most cases, the separation between science and mathematics is rigorously maintained, even in primary school where both mathematics and science are taught by a unique teacher. Moreover, few teaching materials involving both mathematics and science have been developed (Davison & al. 1995). Nevertheless, research addressing interdisciplinarity issues show that even young students are able to acquire skills in the domains of mathematics, science, and scientific processes such as measuring, modeling, etc. (Munier & Merle 2009). The lack of teaching resources of that kind may be puzzling if one considers the interrelations between science and mathematics in their historical developments. In this regard, history of science can be considered as an inspiring ground for the elaboration of teaching sequences where mathematical and scientific knowledge and skills are integrated.

In this paper an example of such integration is presented through the use of three distinct historical episodes dealing with early Greek and Chinese cosmologies. From these cosmologies teaching sequences (involving historical elements mixed with non-historical ones) are elaborated in order to provide students with elementary cosmological knowledge dealing with scientific and mathematical knowledge and skills (quasi-parallelism of Sunrays, shape and size of the Earth, Sun-Earth distance, measuring and computing, etc.).

MATHEMATICS AND PHYSICS AS INTERRELATED AREAS OF KNOWLEDGE

Claims for bridging mathematics and physics in science teaching often refer to Galileo Galilei who wrote:

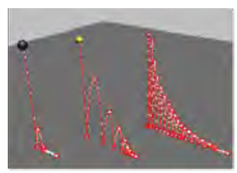
> Philosophy is written in that great book which ever is before our eyes - I mean the universe - but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is **written in mathematical language, and the symbols are triangles, circles and other geometrical figures**, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth (Galileo, *Il Staggiere*, 1623).

Actually, physics embraces much more mathematics than Euclidian geometry and the intricate connection between mathematics and physics have been valued by lots of scientists before and after Galileo (see Siu, 2009). In a few words, physics can be defined as a domain of knowledge which explores "inanimate nature" (Wigner 1960, p. 3) from infinitely large to infinitely small. This exploration can concern structure, organization, movement of matter; it can involve elementary objects or interactions between objects, etc. The generic process of the discovery of laws of nature is to translate natural phenomena - observable or not, combining measurable quantities in order to establish laws expressed mathematically; these quantities refer to formal concepts (ie: concepts that don't have empirical correspondences and which are defined by their attributes – such as Force, Energy, Field, etc.). All the laws of nature are conditional statements which permit a prediction of some future events on the basis of the knowledge of the present. As a consequence, the validation of knowledge in physics is absolutely based on both "reproducibility" and "predictability":

Historically, physics progressively passed from a construction activity interrelated to (Euclidian) geometry to an activity that describes the variation of matter and radiation in space and time. Consequently, most of phenomena the physicist is interested in are described by the second derivatives of positional or temporal coordinates.

To sum up, the language of the physics applies to systems extracted from the real-world. It is structured by figures, graphs, mathematical symbols, or proposals formed by words. It allows predictions and relies on causal relationships established through measurements. In this context, problems under physics are diverse (explanation, creation of phenomena, of objects, predictions of behaviour, etc.) but globally, their solutions take the form of laws which are assumed to govern the reason for the in-animate nature (why nature - matter, radiation - is as it is?), the how of its past (how did it get there?) and its future (what would happen if…).

Let's consider an example: the bouncing of balls. Here one can focus on "why" balls bounce or on "how" they bounce. If one focuses on "how" balls bounce the physics enterprise consists in looking for a relationship between different quantities on the basis of conservation laws. We find that a constant quantity exists which connects the bounce height and is the drop height. And what is very interesting is that this quantity - the coefficient of restitution (*k* in fig. 1), allows predicting the total distance *D* travelled by the ball according to the number n of bounces and the associate length of time *T*.



$$D = d_0 + 2d_1 \frac{1-k^{2n}}{1-k^2}$$

$$T = t_0 + \sum_{t=1}^{n} 2t_n = t_0 + 2t_1 \frac{1-k^n}{1-k}$$

$$k = \frac{v_2}{v_1} = \frac{t_2}{t_1} = \sqrt{\frac{h_2}{h_1}}$$

Fig. 1: Mathematics modelling of the bouncing ball. *D* represents the total distance covered by the ball and *T* the length of time of the movement of the ball.

Looking at the formula (fig. 1) one can mathematically admit an infinite number of bounces and find a finite distance *D*, but this infinite limit does not make sense from a physics viewpoint, since the ball stops its movement after a while. This aspect can be puzzling for students.

MATHEMATICS IN PHYSICS LEARNING

Several researches in physics education have been carried out where interplays between mathematics and physics have been questioned (Artigue & al. 1990, Gill, 1999, Albe & al. 2001, Melzer, 2002, Hestenes, 2003). Some researchers have focused on the difficulties encountered by students when using/facing mathematics in physics (manipulation of vector quantities); others have shown that the process of conceptualizing in physics strongly takes advantage of a good mastery of mathematics: the meaning of the constant of integration allows to understand the importance of the initial conditions of velocity for example in mechanics, or, a single point in a space-time diagram allows to better grasp the deep signification of what an event is in the framework of Special Relativity:

> There is a significant correlation between learning gain in physics and students' pre-instruction mathematics skill (Meltzer, 2002)

As a conclusion, considering mathematics as a part of the teaching of physics is an epistemological reality and a cognitive opportunity since the mathematical abstraction favours the conceptualization process in physics (and *vice versa* ?). Thus, it appears as a real didactic necessity[i]. In the following the potential of history of science in the promotion of teaching sequences based on effective mathematics-physics interplays is highlighted.

HISTORY OF PHYSICS AND PHYSICS LEARNING

Today researches involving science education and history of science follow two different orientations – which are not mutually exclusive. The first one aims to provide students with element associated with Nature of Science (science activity, elements of epistemology, etc. see Abd el Khalick & Lederman, 2000); the second one searches for elements that may favour a better appropriation of concepts and laws. In the way I work, the spontaneous reasoning of students plays a determining part. Actually, the common sense is very powerful in providing operational and coherent explanation while facing empirical phenomena. Today, the whole community of researchers in physics education agrees that learning physics is based on a negotiation process between the rationality of the common sense[ii] and the rationality of physics. In this regard, using history of science for physics learning in a conceptual perspective should take into account the type of reasoning a student can hold concerning a phenomena to be studied. And a way of managing this is to search within history of science ideas that could, to some extent, echo with common students' ideas or conceptions in order to create a problem directly inspired by an historical episode that could meet student's interest and thus, be accepted by them.

First example: Earth is spherical

My first example leans on a teaching sequence for 10 year-old children elaborated by the French science education researcher Hélène Merle (2002).

The context is Greek astronomy, also qualified by historians of science as "mathematical astronomy" (Neugebauer, 1957) or "geometrical astronomy" (Coveing, 1982). In Ancient Greece, the prevalent assumption is that the movements of celestial bodies are circular and uniform:

> Pythagorism turned geometry into the instrument for astronomy as a contemplative science of the natural being. (Coveing, 1982, p. 146).

The historical text that inspired the teaching sequence is an excerpt of Aristotle's *Treaty of the Sky* in chapter 14. In this text, Aristotle argues for a spherical Earth on the basis that:

> According to the way celestial bodies show themselves to us, it is proved that not only the Earth is round but what is more it is not very big; because we just have to make a

> small travel either at the South or at the North, so that the circle of the horizon becomes obviously quite different. So the celestial bodies which are over our heads undergo a considerable change, and they do not seem to us any more the same, as we go to the South or to the North. **There are certain celestial bodies that we see in Egypt and in Cyprus, and that we do not see any more in the northern parts of the country**. On the contrary, some celestial bodies that we see constantly in the northern countries lie down when we consider them in the parts of the country which I have appointed. This proves not only that the shape of the Earth is spherical, but still that its sphere is not big (Aristotle, *Treaty of the sky,* chap. 14).

The sequence addresses children's ideas about "horizon" and takes into account the idea that vertical and horizontal notions are only considered by 10 year-old children locally. As an example, children represent the level of a given quantity of water contained in a bottle, as a straight line, and its direction is perpendicular to the boundaries of the bottle itself (Ackermann, 1991) – whatever the orientation of the bottle.

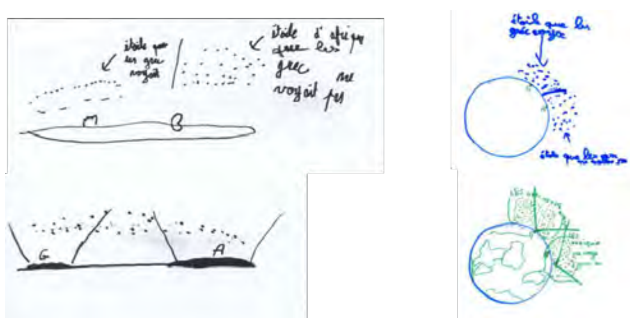


Fig. 2: Drawings provided by children involved in Merle's research (2002) who are asked to explain the reason why some stars disappear when travelling to the South. ***Translation of children comments: "stars seen by Greeks" / "stars Greeks cannot see".***

First, the problem is transformed and adapted so that children are asked to explain the reason why some stars disappear when travelling to the South. They provide some relevant drawings, relevant in the sense that they fit with Aristotle observation (fig. 2). But some drawings involve a flat Earth, while others involve a spherical Earth because the way children represent the "field of vision" (vertical or oblique lines) fit with both shapes (fig. 3). In other words, Associated with the way children represent the "field of vision", Aristotle observation is not sufficient to discriminate a spherical Earth from a flat one.

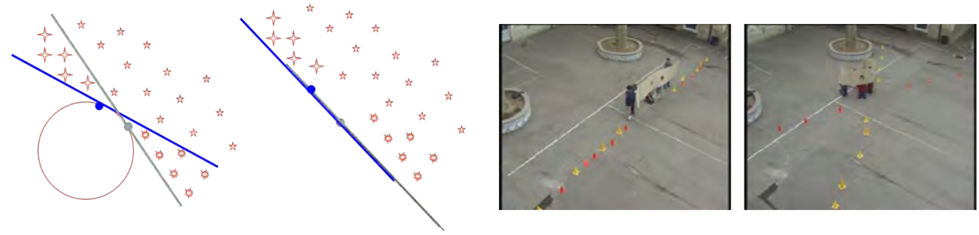


Fig. 3a: Horizon seen as a tangent line to the terrestrial sphere allows Aristotle to explain why some stars disappear when travelling to the South on a curve ground. In figure 3b, two children are placed behind a cardboard where two windows have been opened so that they can see other children transporting coloured cones (red and yellow). Each child is responsible for a given coloured cone and is piloted by one of the children placed behind the cardboard. Children transporting a cone are asked to drop it when it cannot be seen through the window. In the pictures two geometries of Earth's ground are modelled: a flat one and a curve one; to each geometrical model corresponds two lines of cones (ie: two horizon lines) differently arranged.

In other words, for Aristotle, the differences, in visible stars from two different places is a hint of the spherical shape of the Earth, but not for children.

The interesting thing here is that history of science provides a fruitful problem-to-be-solved: the problem of the stars can be solved by children and make their conception of the field of vision (as a delimitation for the visible space) be expressed. Because children know that Earth is round, they can use this knowledge in order to initiate a conceptual change and pass from an inappropriate modelling of the visible space to a correct one. Note that there is no parallelism between a (supposed) historical path and children's conceptual development, since horizon is a geometric tool that allows Aristotle to argue in favor of a spherical Earth, while children use a spherical Earth hypothesis to build the concept of horizon. In this regard, the way the sequence is conducted in order to make children conceptualize the horizon line is totally a-historical (see fig. 3b).

Second example: cosmological distances are measurable (see de Hosson & Décamp, 2014).

The second example leans on two different cosmologies: the Chinese one and the Greek one seen in the light of various historical sources: *the Zhou Bi* in the Cullen (1996) and Kalinowski (1990) translations, Ho Peng Yoke's *Astronomical chapters of the Chin shu* (1966), and Clemoledes' *De Motu circulari corporum caelestium* in the Weir translation (1931).

A current astronomical activity in primary school in France (carried out in both mathematics and science courses (see for example, di Folco and Jasmin 2003; Kuntz 2006) consists in exploiting the procedure supposedly used by Eratosthenes in the 3rd century BC in order to measure the perimeter of the Earth. This procedure leans on two observations: (1) A gnomon located in Alexandria (northern Egypt) at noon the day of the summer solstice casts a shadow of a certain length, (2) At the same time a gnomon located in Syene (middle Egypt) casts no shadow since the Sun appears at the zenith. From the pedagogical use of Eratosthenes procedure, some researchers questioned children's difficulties while modeling the Sunrays (Feigenberg et al. 2002, see fig. 4). Children are asked first to explain why during summer solstice at noon, a gnomon located in Alexandria (northern Egypt) casts a shadow while another (identical to the previous one) located in Syene (middle Egypt) casts no shadow.

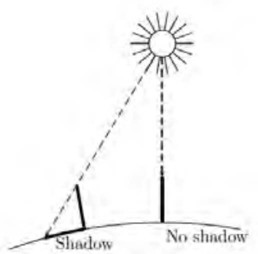


Fig. 4: Drawing provided by students who are asked to model the reason why gnomon placed at Alexandria casts a shadow, whereas at the time the day of summer solstice (at noon) a gnomon placed at Syene casts no shadow.

As an explanation (of what we will call the 'shadows observations'), some children draw non-parallel rays coming from a sketched Sun down onto a curve (or a plane) surface of the Earth. This drawing is also typical of those proposed by most of the primary teachers (target of the following sequence) explaining the same observation (Merle 2000).

The Chinese text presented hereafter (Doc. 1) is taken from the Chin Shu, a book written around 635 A.D.

> According to the Chu Li (Rites of Zhou), the shadow of the Sun at midday during the summer solstice was 1 chi 5 tsun. The place where this particular observation was made was known as the 'Earth centre'. Cheng Chung said that the length of the gnomon shadow template was 1chi 5 tsun and that the place where a vertical pole 8chi in length at midday of the summer solstice cast a shadow the same as that of the shadow template, was called the 'Earth centre'. The place corresponds to the present location of Yangchen, in Yingchuan. Cheng Huan said that the shadow cast by the Sun on the Earth surface changed by a length of 1 tsun for every change of 1000 li in the horizontal distance (north or south). Since the length of the shadow is 1chi 5 tsun, the Sun is 15000 Li away and to the south of the observer. From this it can be deduced that the vertical distance of the Sun is 80000 Li from the Earth's surface"

Doc. 1: *The Chin Shu*, Ho Peng Yoke (1966), p.65 - Units of length: 1chi = 10 tsun = 35.8 cm; 1 tsun = 3.58 cm, 8chi = 2.86 m and 1li = 560 m.

The astronomical part of this book has been written by Li Shun-fêng. The proposed excerpt refers to the astronomical knowledge under the Zhou dynasty that began about a thousand year B.C. Another historical text, *the Zhou bi* (namely, the gnomon of the Zhou) gives similar elements to those found in this Chin Shu. The proposed excerpt presupposes children and teachers main type of explanation of the 'shadows observations' and based on it computes some measurements: the shadow of a vertical eight chi long gnomon (2.86 m) located in Yangchen is 15 tsun (53.7 cm) long, at noon, on the day of the summer solstice. The same day at the same time, an identical gnomon located 1000 li (560 km) south of Yangchen will cast a 14 tsun (50.1 cm) long shadow, and if it is located 1000 li (560 km) north of Yangchen, this gnomon cast a 16 tsun (57.3 cm) long shadow. The text presented in doc. 1 deduces from these measurements that the Earth-Sun distance is 80,000 li (44,800 km). Figure 5 helps us to understand this result.

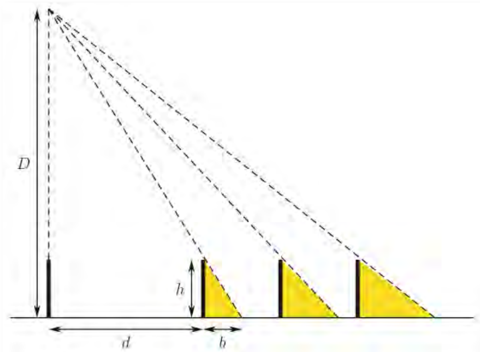


Fig. 5: Distance *d* is computed knowing that each 1000 li, a *h* height gnomon casts a shadow whose length decreases of 1 tsun. Since *b*=15 tsun, *d*=15 000 li. Today, one can compute Distance *D* by using similar triangles property: (*D*-*b*)/*d*=*h*/*b*; since *h*=8chi, *D*≈80 000 li.

A similar observation supports a spherical geometry, which seems to be at the root of Eratosthenes measurement of the Earth's perimeter. Unfortunately, the original writings of Eratosthenes (2 books) were lost. We have access to his work only through authors of antiquity such as Cleomedes, Pliny, Strabo. The most detailed among these writings is a short review by Cleomedes. We are told by Cleomedes (see the translation of *On the circular motion of the celestial bodies*, book 1, Chap. 7, by Weir 1931) that Eratosthenes made measurements with a gnomon that cast a shadow onto the graduated inner surface of a hemispherical sundial named *scaphe*. Eratosthenes knew that on a certain day (summer solstice) at noon in Syene, the gnomon of a *scaphe* cast no shadow, whereas the same day at the same time in Alexandria (located at 5000 stadia -800 km- at the north of Syene) the shadow cast by the gnomon of an identical scaphe reaches an arc equal to 1/50th of a circle from the base of the gnomon (Fig. 6). Assuming the parallelism of the sunrays that reach Syene and Alexandria and the fact that both cities are on the same meridian, it is easy to deduce that the distance between Syene and Alexandria is also equal to 1/50th of Earth's circumference and then to compute this measurement. The method Eratosthenes used to compute the distance between Syene and Alexandria has been subject to debate. It seems to be based on maps of Egypt or on accurate distance estimations made by bematists. These men were trained to make regular paces when marching from one place to another and to record their numbers (Dutka, 1993).

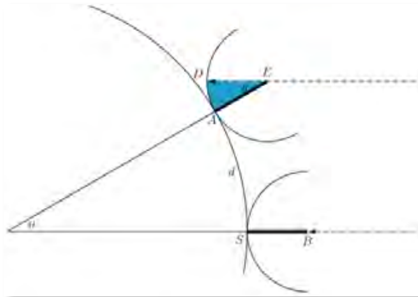


Fig. 6: Illustration of Eratosthenes' procedure as described by Cleomedes in *On the circular motion of the celestial bodies* (Weir 1931). The shadow *AD* cast by the gnomon in the hemispherical sundial reaches an arc equal to 1/50th of a circle of radius *AE*. The ratio of 1/50th of the circumference of the Earth corresponds to the distance AS between Alexandria and Syene (de Hosson & Décamp, 2014).

There are many similarities between the Chinese and Greek chosen measures. In both cases, the astronomers have chosen noon of the summer solstice to make their measurements. This is probably not by accident: midday (solar time) of the summer solstice corresponds (in the Northern hemisphere) actually to the moment at which the shadow of the gnomon is the shortest during the year. In both cases, they also computed a terrestrial surface measurement and derived from it a vertical measurement using a strategy based on proportionality. Both used a sundial but an interesting difference is the fact that Chinese and Greek instruments are not exactly the same. The Greek *scaphe* and its gnomon are an hemispherical sundial. It gives direct access to the searched portion of the circle (we would say to the angle in modern terms) and this is the useful measurement in a spherical Earth cosmology. The Chinese *bi* is on the contrary a flat sundial which gives access to the angle tangent, a more adapted measure for a flat Earth cosmology. This is an interesting illustration of Bachelard's thought:

> A measuring instrument always ends up as a theory: the microscope has to be understood as extending the mind rather than the eye. (Bachelard 2002, p. 240).

The Chinese astronomical model promotes a hypothesis very close to prospective primary teachers' ideas about the propagation of the Sunrays. This model (flat Earth/close Sun sending divergent Sunrays) was chosen as an anchoring situation that would echo students' prior knowledge. Prospective teachers were then engaged in operating this model through an experimental activity. By confronting Chinese data (e.g. the Sun-Earth distance) with the current one, they were prepared to elaborate an alternative way of modeling the shadows situations, based on parallel Sunrays. Nevertheless, the majority of these future teachers did not understand how a single point could send parallel rays as illustrated in the following piece of transcription between a prospective teacher T and the researcher R:

> [T] I still don't understand why the light sent by a single luminous point can be modeled using parallel rays
>
> [R] Actually, the Sunrays are not exactly parallel. But if I stretch out two long strings from the same mooring, their extremities can be considered as parallel lines under certain conditions? Which one?
>
> [T] Hum… if it is nearly parallel it is very different!
>
> [R] Why?
>
> [T] Because in the case of nearly parallel lines from a single point it is obvious that they can have the same origin, whereas if they are really parallel they will never cross each other; they cannot come from the same point
>
> [R] Ok. But what could allow considering these two lines having the same mooring as parallel?
>
> [T] If the extremities are very close
>
> [R] Do you think Syene and Alexandria are close enough to consider the Sunrays reaching them as parallel lines?
>
> [T] Well… oh… ok… Yes, since the Sun-Earth distance is much larger than the distance between the two towns! The distance between the extremities of the Sunrays should be very very small

Actually, the Sun is not a point source of light but an extended one and its angular diameter is about 0.5°.

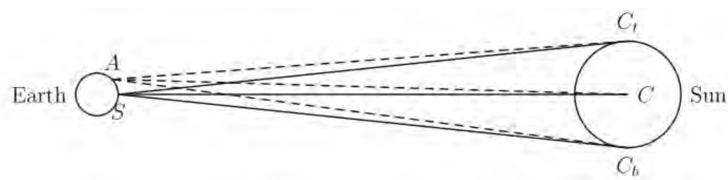


Fig. 7: Illustration of two cones of light coming from the Sun and reaching Syene S and Alexandria A. Figure not drawn to scale: $\widehat{C_tAC_b} = \widehat{C_tSC_b} \approx 0,52°$whereas $\widehat{ACS} \approx 1,1''$, (Décamp & de Hosson, 2012).

Half of the Earth's surface is struck by an infinite number of cones, each containing an infinite number of rays sent out by each point of the Sun. In Fig. 4 we have illustrated two of these cones reaching the towns of Alexandria and Syene. One must consider that the angle $\widehat{C_tAC_b} = \widehat{C_tSC_b} \approx 0{,}52$. This angle is small because the diameter of the Sun [C_tC_b] is small with respect to the distance between Sun and Earth. This is the reason why the Sun can roughly be considered as a point source. In the same picture we see that the angle $\widehat{C_tAC_b}$ is even smaller. Therefore the sun's rays can be considered parallel. This clarification illustrates the complexity of the process underlying the hypothesis of parallelism of sunrays, which at the scale of a portion of 1/50th (or 1/48th) of the Earth's circumference remains an approximation. Similarly the assimilation of the Sun to a single point stems from an identical complexity. Yet, the fact that the Sun is an extended source explains that the shadows cast by gnomons are surrounded with a partial shadow area.

Only a rigorous geometrization of the astronomical construction legitimates the approximation usually presented to students. In that perspective mathematics gives sense to physics, not only for the understanding of concepts but also for the grasping of the deep meaning of what physics is: a construction of theory and models validated through the predictions they allow within certain domain of validity and taking into account measurement uncertainties.

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[i] In France, something quite surprising happens concerning physics teaching. In order to make physics more attractive, in 2011, the curriculum designers have make mathematics almost disappear and we can read in the K-12 curriculum that "using mathematical tools is not the first target of physics learning, even though it is sometimes necessary to carry a study out". It seems that physics teaching in France is a paradigm where mathematics reduces physics attractiveness…

[ii] Common sense can be considered as a set of representations of the world shared by most of people and capable to generate operative and relevant explanation concerning natural phenoma but inappropriate according to scientific rationality (see Viennot, 2001, 7-11).