Workshop

SELECTING AND PREPARING ORIGINAL SOURCES FOR PRE-SERVICE MATHEMATICS TEACHER EDUCATION IN TURKEY: THE PRELIMINARY OF A DISSERTATION

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In this paper, we aim to introduce two historical teaching modules based on the use of original sources of mathematics, namely Euclid's Elements and al-Khwarizmi's al-Jabr Wa'l-Muqabala, in order to improve junior and senior pre-service middle school mathematics teachers' mathematical knowledge for teaching, and their beliefs about mathematics and mathematics teaching. It is a preliminary study of the first author's dissertation including the introduction of the essential criteria for selecting the original sources and preparing the accompanying tasks for these sources. The study was discussed in a workshop where the participants worked on the historical modules and provided valuable inputs for their improvement and practice.

INTRODUCTION

The use of history of mathematics has been included in Turkish middle school (grades 5 to 8) mathematics curriculum with 2005 reform (Ministry of National Education [MoNE], 2005) and the following revisions (MoNE, 2009, 2013). The current curriculum (MoNE, 2013) states that history of mathematics is able to change middle grades students' attitudes towards mathematics and their learning of mathematics in a positive way. Moreover, it asserts that the history can show the discipline of mathematics as a cultural heritage. Despite the ideas on the value of history of mathematics, their integration into the relevant formal mathematics textbooks is limited to the historical information provided by secondary sources. Biographies, dates and names are some examples (MoNE, 2009, 2010). This type of integration cannot go beyond *the illumination approaches* of using history of mathematics in mathematics education (Jankvist, 2009a).

Bringing history of mathematics in mathematics classroom is a responsibility mainly on teachers' part. Mathematics teachers can be educated for this purpose in their pre-service training process or through specifically designed in-service training programs. In this study, we focus on the pre-service training because it is a relatively long formal education process before starting the teaching career. Among the undergraduate courses suggested for Turkish pre-service middle school mathematics teachers, History of Mathematics appears to be the most relevant for our focus. Alpaslan, Işıksal and Haser (2014) found that this course had a potential to increase the pre-service teachers' knowledge of history of mathematics, yet the quality of this knowledge did not seem to be promising for their future teaching. Alpaslan and Haser (2012)

investigated the content and pedagogy of such a History of Mathematics course in their case study, and they concluded that the course was ineffective for developing pre-service middle school mathematics teachers' beliefs about the use of history of mathematics. Therefore, there appears to be problems due to the nature of the existing History of Mathematics courses in pre-service middle school mathematics teacher education programs.

One of the most effective ways of using history of mathematics in teacher education is the study of original sources of mathematics (Jahnke, 2000). The original sources are defined as primary mathematical products of the studies made by great mathematicians for the purposes of (i) doing mathematics for its own sake and/or (ii) teaching mathematics (Knoebel, Laubenbacher, Lodder, & Pengelley, 2007). They show reflections of the milestones on the evolution of mathematics, which has taken place in different cultural contexts in history. Jahnke (2000) addresses that original sources can provide three distinctive benefits for both learners and teachers: (i) *replacement* of the typical perceptions about mathematics; (ii) *reorientation* of ideas about mathematical concepts through making the familiar unfamiliar; and (iii) *cultural understanding* that deals with the evolutional process of mathematics linked with technological and scientific developments in the context of various societies. The study of 'unpolished' definitions, algorithms, representations, and problems in the authentic context of the original sources can be quite valuable for pre-service mathematics teachers because it provides different mathematical and pedagogical perspectives to the learning and teaching of modern mathematics concepts (Barnett et al., 2008). In such a study, the reader is guided to understand, interpret, and discuss the mathematical reasoning of the 'masters' (van Maanen, 1997). He/she attempts to comprehend the meaning of the old mathematics through stating hypotheses. Focusing on a particular mathematics topic can yield hints about how the ideas about that topic have evolved. Original sources can also challenge the accuracy of the contemporary 'polished' forms of mathematical concepts found in the secondary sources (e.g., textbooks) which are mostly utilized in modern mathematics education today (Jahnke, 2000). Additionally, possible benefits and restrictions of contemporary mathematical forms can be noticed by students and teachers through the study of the original sources (Tzanakis & Arcavi, 2000). Herein, it should be noted that these sources require careful interpretation considering historical, mathematical, and cultural context of their own time and language. Otherwise, we may confront with the problem of Whig interpretation of history, that is, considering the past from the modern perspective only (Fried, 2001).

The Study of Original Sources in relation to Pre-Service Mathematics Teachers' Mathematical Knowledge for Teaching

In the educational context of the US, Hill, Rowan and Ball (2005) studied mathematics teachers' knowledge for teaching through a practice-based research

study, and they basically defined it as "the mathematical knowledge used to carry out the work of teaching mathematics" (p. 373). Ball, Thames and Phelps (2008) later categorized this knowledge mainly under two: *subject matter knowledge* and *pedagogical content knowledge*. *Subject matter knowledge* was formed of three components: (i) *common content knowledge* that stands for the general mathematics knowledge of the settings in and out of the classroom; (ii) *specialized content knowledge* that meets the necessary knowledge and skills particular for the mathematics classroom; and (iii) *horizon content knowledge* that involves mastering the further mathematics behind classroom mathematics. *Pedagogical content knowledge*, which collated the content of a subject and the distinguishing pedagogy for its teaching, it included three components as well: (i) *knowledge of content and students* that contained understandings about students' learning, their preconceptions, difficulties and affective dispositions; (ii) *knowledge of content and teaching* that consisted of how methods and strategies of teaching mathematics can work in the classroom to result in a meaningful understanding; and (iii) *knowledge of content and curriculum* that referred the subject within teaching programs, the related textbooks, manipulative models and the like.

Even if Ball et al.'s mathematical knowledge for teaching framework has US origin, we find it useful for our context because all of its components are regarded as necessary for Turkish mathematics teachers as well. MoNE (2008) states that a middle school mathematics teacher have to master knowledge of numbers and operations, algebra, geometry and measurement, data processing, and probability as five main learning areas (i.e., *subject matter knowledge*) in the curriculum, and to have ideas about how to use this knowledge in the teaching process (i.e., *pedagogical content knowledge*). In the relevant documents, an exemplary performance indication is that "[A mathematics teacher] knows […], representations of mathematical concepts and procedures." (MoNE, 2008, p. 13).

The study of the original sources is able to enhance pre-service mathematics teachers' *common content knowledge* on the grounds that it may broaden the meaning of the prevalent mathematics structures, concepts and algorithms through "unpolished" definitions, narratives, presentations, and approaches (Mosvold, Jacobsen, & Jankvist, 2014). Such supplementary knowledge is likely to dispute with the pre-service teachers' existing conceptions and understandings about mathematical concepts (Ball, 1990; Furinghetti, 2000). Original sources can go beyond serving *common content knowledge* by supplying symbols, proofs, representations, and illustrations which do not have widespread use today (Jankvist, 2009). This kind of knowledge addresses *specialized content knowledge* that is likely to enable pre-service teachers to connect the important ideas for their related understandings at first. Gaining such alternatives in the teaching repertoire is also acknowledged in the modern view of teaching mathematics through developmentally appropriate practices (van de Walle, Karp, & Bay-Williams, 2010). In this sense, improving *specialized content knowledge* through

history of mathematics can assist pre-service teachers in choosing more appropriate perspectives in a classroom of students with different background knowledge and various learning styles. If the pre-service teachers intend to develop *horizon content knowledge*, which is advanced knowledge of the particular topics and procedures related to teaching, they may consult the origins of those topics and procedures because the historical sources are full of further definitions, explanations, reasons, proofs, methods, and examples by the 'masters' (Knoebel et al., 2007). This process might make their mathematical work more meaningful (Ball et al., 2008). Nevertheless, such an advanced work may not be liked by the pre-service teachers because it may seem useless for teaching purposes (Mosvold et al., 2014). At this point, it is crucial to find ways to make pre-service teachers engage in the activity and opportunely using original historical sources is shown to provide such an engagement (Jahnke, 2000).

Original sources have a potential to support pre-service teachers' professional repertoire regarding their future students' ways of thinking about mathematics topics. This is related to pre-service teachers' *knowledge of content and students*. Sfard (1995) states that conceptual problems in constructing mathematical knowledge may echo in the related epistemological difficulties faced in the historical development of that knowledge. Being aware of such epistemological difficulties seems to be important for pre-service teachers because they enable pre-service teachers to better 'guard' against the common student difficulties and misconceptions, and to utilize these as opportunities to construct new knowledge (Brousseau, 1997). Additionally, the study of original sources might provide possible reasons and discussions behind such obstacles through the primary definitions and representations of concepts (Mosvold et al., 2014). Concerning *knowledge of content and teaching*, Mosvold and colleagues (2014) assert that mathematics textbooks often cover the concepts in the order of general to specific (e.g., formulas to drill). However, this is not in accordance with the nature of the development of mathematics topics in history. Abstraction gradually increased for those doing mathematics in the former times. The original sources can provide varying degrees of abstraction that can also be employed in the modern mathematics classroom. In order to serve pre-service teachers' *knowledge of content and curriculum*, original sources can be utilized to provide ways for achieving curriculum goals and objectives through enriching the available teaching materials (e.g., manipulative models, textbooks) such as developing materials considering the primary texts. Another opportunity may be studying the historical order of the subjects in the curriculum through the original sources (Jankvist, Mosvold, Fauskanger, & Jacobsen, 2012). Interdisciplinary characteristics of the concepts may also be studied with reference to these first-hand materials. In this way, pre-service teachers can give meaning to the organization of the subjects and the related objectives in the curriculum. Also, they may make connections to other subject areas such as physics. Finally, historical study of various mathematics curricula and the relevant materials

can also be helpful in understanding the different conceptions of the identical topics in these sources.

In addition to the theoretical arguments in favor of the potential benefits of using original sources for enhancing pre-service mathematics teachers' mathematical knowledge for teaching, the history and pedagogy of mathematics (HPM) literature also have some experimental studies that support these arguments. In Clark's (2012) study from the context of US, pre-service mathematics teachers stated that the study of al-Khwarizmi's *al-Jabr Wa'l-Muqabala* enhanced their understandings about quadratic equations. In particular, the geometric interpretation of the method of completing the square assisted them to comprehend the rationale behind solving quadratic equations. In Italy, Furinghetti (2007) provided pre-service teachers with cognitive roots of algebra concepts, methods and procedures by means of following their historical development through certain original sources written by the masters of algebra (e.g., Alcuin, al-Khwarizmi, Descartes, Viète). This experimental kind of studies, which have positive experiences and findings about improving pre-service teachers' mathematical knowledge for teaching through the study of original sources, has motivated the dissertation discussed in this paper.

The Study of Original Sources in relation to Pre-Service Mathematics Teachers' Beliefs about the Discipline of Mathematics and Mathematics Teaching

Pre-service mathematics teachers' beliefs can be identified as their judgments regarding the issues about mathematics and mathematics teaching (Hart, 1989), which are formed through their existing knowledge and experiences as a product of schooling and professional education (Calderhead & Robson, 1991). In this study, one of our foci is pre-service teachers' beliefs about the discipline of mathematics and mathematics teaching. Considering Jankvist (2012) and Niss and Højgaard (2011), pre-service mathematics teachers' beliefs about the discipline of mathematics can be categorized into three: (i) beliefs about application and sociologically oriented aspects of mathematics (e.g., the influence of mathematics on society), (ii) beliefs about historical and developmental aspects of mathematics (e.g., the driving forces of the historical development of mathematics), and (iii) beliefs about philosophical issues about mathematics as a discipline (e.g., whether mathematics is product of discovery or invention). The study of original sources has a potential to develop this kind of beliefs by revealing the *metaperspective issues of mathematics* within their authentic contexts (Jankvist, 2014). The other focus of the study is pre-service teachers' beliefs about mathematics teaching, particularly using history of mathematics in mathematics education as an alternative teaching strategy. These beliefs are judgements such as whether or not history of mathematics is a useful cognitive tool for teaching middle grades mathematics topics.

METHODS

Criteria for Selecting the Original Sources

In order to explain the rationale behind selecting specific original sources for the historical teaching modules intended for pre-service middle school mathematics teachers, we have set certain criteria and essential characteristics by considering the relevant workshops conducted in the previous HPM Study Group meetings, the related HPM literature, and the particular requirements of the Turkey context.

- The sources should fit into the curricula addressing the participants of the study. Herein, middle school mathematics curriculum (MoNE, 2013) and higher education policy documents regarding the pre-service teachers ([Council of Higher Education] CHE, 2007a, 2007b) constitute these curricula of interest. The use of original sources should correspond to:
  - Middle school mathematics curriculum (MoNE, 2013) by
    - serving the general aims of mathematics education (e.g., interconnecting mathematical concepts, including problems of real-world situations),
    - addressing the adopted teaching and learning approaches (e.g., teaching through problem solving, using concrete experiences to make abstraction),
    - being within the scope of the five learning areas that are Numbers and Operations, Algebra, Geometry and Measurement, Data Processing, and Probability,
    - considering the level of mathematics taught in middle grades (students aged 11 to 14), and
    - having a potential to make the curriculum objectives (e.g., to write a verbal statement in an algebraic expression) achievable.
  - CHE (2007a, 2007b) documents in relation to History of Mathematics course for pre-service middle school mathematics teachers by
    - developing the pre-service teachers' general knowledge about mathematics (e.g., mathematics as a heritage of various cultures in history), and
    - dealing with the prescribed content (e.g., analytic and modern geometry) and pedagogy (e.g., stress on mathematical aspects) of the course.
- The sources should be appropriate for the pre-service teachers' background knowledge (Barnett et al., 2008; Jankvist, 2009b). In other words, they can "show sth., that is accessible to the ordinary student and at the same time strange and different from what he has known hitherto" (Jahnke, 1991, p. 11).
- The sources should address *inner issues of mathematics* such as mathematical ideas, concepts, algorithms, methods, theories, proofs and argumentation (Jankvist, 2009a).

- The sources should able to display *metaperspective issues of mathematics*, for instance, human, social and cultural effects on the historical development of mathematics (Jankvist, 2009a).
- The sources should be milestones (i.e., initiating a field in the discipline of mathematics, being among the greatest works by 'the masters', having a great influence, exhibiting the emergence of a key concept, involving crucial ideas and fundamental problems in the development of a concept) in the historical development of mathematics (Barnett et al., 2008).
- The English translations of the sources must be reliable (Siu & Chan, 2012).

The Historical Modules

Two historical teaching modules are discussed in this preliminary study: Euclid Module and al-Khwarizmi Module. Euclid Module starts with an introduction regarding the *Elements* (3rd century BC) in which the pre-service teachers can have general ideas about where and when the book was written, what mathematical subjects it covered, and which purposes it had. The introduction is followed by open-ended questions asking the story behind the writing of the book, the topics that it contained, and the originality of the work that Euclid did. The questions here serve raising *metaperspective issues* of mathematics. After that, four selected texts from Potts' (1871) school edition of Book-I of the *Elements* are provided along with some probing questions revealing *inner* and *metaperspective issues* of mathematics and mathematical pedagogy. In particular, the four texts are Proposition-9: "Bisecting a given rectilinear angle", Proposition-18: "The greater side of every triangle is opposite to the greater angle.", Proposition-20: "Any two sides of a triangle are together greater than the third side." and Proposition-32: "In a triangle, the exterior angle is equal to the two interior and opposite angles.". The questions about *inner issues* of mathematics ask how the propositions are justified and which geometrical definitions, concepts and arguments are utilized in this justification. The pre-service teachers are also equipped with the whole Book I to be able to respond these questions. The questions regarding *metaperspective issues* of mathematics ask the nature of the axiomatic approach used in the propositions. As for the questions about mathematical pedagogy, they invite the pre-service teachers to look the propositions from a teacher's perspective (e.g., how a middle school mathematics teacher can benefit from the texts to enrich his/her teaching repertoire).

Al-Khwarizmi Module initially focuses on reading the preface written by Rosen (1831) to his translation of Mohammed Ben Musa al-Khwarizmi's *al-Kitab al-Mukhtasar fi Hisab al-Jabr Wa'l Muqabala (The Compendious Book on Calculation by Completion and Reduction)* (9th century). In this preface, it is aimed to give the related historical insights for the pre-service teachers. Accompanying open-ended questions ask the story of the writing of the book, the mathematical content of the

book, driving forces for the historical development of mathematics in general, al-Khwarizmi's particular role in the evolution of algebra, the other sciences related to mathematics and the originality of al-Khwarizmi's work when it is compared with those of Greeks and Hindus. After the translator's preface, the preface originally written by al-Khwarizmi is presented together with open-ended questions posing *metaperspective issues* of mathematics such as the appreciation of the work done by the fellow scientists, the relationship between science, religion and authority. The core of al-Khwarizmi Module follows this through excerpts about three cases of equations for completion and reduction and the demonstrations of these cases. The related questions about *inner issues* of mathematics are for understanding the algorithms that al-Khwarizmi followed in the solution of the cases, geometrical representations that he made to further clarify the cases, and the conceptual links between these algorithms and demonstrations. The questions about *metaperspective issues* of mathematics intend to enable the pre-service teachers to consider various representations of mathematics in history (e.g., verbal statements of quadratic equations) and how a mathematical concept can change and develop (for instance, quadratic equations by showing their old forms of different cases and their solution sets including only positive numbers). Lastly, pedagogy-related questions ask the pre-service teachers to evaluate the style of teaching mathematics that al-Khwarizmi adopted such as supporting algorithms with relevant demonstrations.

The Workshop

The workshop of two-hours was initiated with a brief presentation about the criteria for selecting the original sources. After that, the aim was announced as getting feedback from the workshop participants in relation to the potential of the historical modules for accomplishing their target purpose. The participants were mathematics teachers, graduate students in mathematics education master's or doctoral programs, and HPM advisory board members. The participants formed groups of two in order to work on the designed historical teaching modules. At the beginning of each module, they read the related historical contexts (e.g., social conditions of the time, mathematics of the time, why the source may be written, stories about the source) through the relevant introductions and prefaces. Then the participants read the mathematical content interested in the original sources. For each of the historical excerpts, they answered some probing questions about *inner* and *metaperspective issues* of mathematics as well as mathematical pedagogy. This was intended to be in line with the pedagogy of discovery and inquiry which could enable one to explore, understand, explain, interpret, and justify the mathematical and pedagogical content in the original sources (Barnett et al., 2008). At the end of the work, each group shared their experiences in their own work, and they pointed out various advantages and disadvantages of using the historical modules in the existing format. Considering that the target group was actually pre-service middle school mathematics teachers, the

participants made valuable suggestions for the improvement of the design and implementation of the historical teaching activities.

DISCUSSION AND SUGGESTIONS

In this section, we present the discussion and suggestions made at the end of the workshop, and develop criteria for preparing the accompanying tasks for the original sources. One of the main topics of discussion was the *originality* of the original sources. The experts on Euclid's *Elements* noticed that the excerpts from Potts (1871) were slightly changed by himself for teaching purposes. Hence, they suggested using another translation which was not edited in this manner. They also added that comparing such a school edition of *Elements* with the original one may reveal the pedagogical aspects behind writing that school edition. Consequently, this may have inputs for the pre-service middle school mathematics teachers' mathematical knowledge for teaching and their beliefs about mathematics teaching. Another discussion topic was the need for using (i) brief and appealing introductions for the original sources rather than long pages of historical information (e.g., the preface by the translator), (ii) contextualizing the original sources into the relevant culture, (iii) more detailed instructions throughout the study of the original sources, and (iv) more open-ended questions following the extracts in order to clarify the *inner* and *metaperspective issues of mathematics* hidden in the original sources (e.g., "Describe al-Khwarizmi's geometric method for solving quadratic equations in your own words."). All of these needs uttered the necessity of employing a particular approach (such as *the genetic approach* or, *the hermeneutic approach*) for using original sources in our context of pre-service mathematics teacher education. Lastly, there was a debate about whether to use or not to use the marginal notes by the translators when presenting a historical excerpt. In our opinion, this decision depends on the approach adopted for the use of original sources.

Criteria for Preparing the Accompanying Tasks for the Original Sources

In the light of the above discussions and suggestions, and the practice of the workshop in general, we set the following essential criteria for preparing the accompanying tasks for the original sources.

- A genuine historical account containing the relevant mathematics and culture of the time under interest should contextualize the original sources. This account can be provided by pictures, biographies, anecdotes, and so on. It seems important that such an account has to be concise in order not to exhaust the pre-service teachers.
- It is crucial that the selected original sources are in agreement with the actual purposes of the historical modules. In other words, the purpose of the modules should be reconsidered in giving the final decision about whether the selected

sources are really helpful or not (as our experience on the need for choosing the appropriate translation of the *Elements* indicates).

- Instructions should be clear and understandable for the pre-service teachers' studies on the tasks accompanying the texts selected from the original sources. For example, putting a prescription about what the pre-service teachers are expected to do in a task appears to be helpful.
- The questions asked in the tasks should be able to make the pre-service teachers critically think about the potential *inner* and *metaperspective issues* of mathematics embedded in the texts. Moreover, the questions should have a potential to put pedagogical issues on display.
- Instructor plays an important role in guiding the pre-service teachers' readings of the original sources as well as their studies in the accompanying tasks.

The above mentioned criteria for preparing the tasks for the original sources and the discussions and in the workshop have led us to think about adopting *guided reading* (see, for example, Barnett et al., 2008; Knoebel et al., 2007) as a methodological approach for the dissertation. In particular, *guided reading* seems to be coherent with our intention of using original sources from the aspects of (i) situating the sources in their authentic historical context, (ii) supporting the pre-service teachers' access to the source content by giving the right amount of support (i.e., guiding), and (iii) creating a discussion environment among the pre-service teachers. The workshop experience has contributed to the dissertation also in selecting proper original sources for the modules and revising the questions asked in the tasks.

NOTES

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