Oral Presentation

STATISTICS AND FREE WILL

Michael Kourkoulos & Constantinos Tzanakis

Department of Education, University of Crete, Greece

Abstract *In this paper we present our teaching work based on Quetelet's texts on "Moral Statistics" and Free Will aiming to motivate and stimulate relevant discussion with students. The work done allowed them to obtain significant insights on the Free Will debate, statistics and their relation. We provide evidence supporting the position that with adequate teaching design and implementation, it is possible to explore fruitfully existing links among statistics, probability and important philosophical issues, even with novice students in statistics.*

INTRODUCTION

All along the historical development of mathematics and philosophy, there have been deep links between them, developed and operating fruitfully in both directions. In particular probability and statistics are connected to concepts such as *uncertainty* and *chance* that also convey an important philosophical meaning (for the historical relations among probability, statistics and philosophy see, e.g., Hacking 1975, Porter 1986, Hald 2003, Chandler & Harrison 2012).

Though historically probability, statistics and philosophy have been strongly linked, their rich interrelations have been very little explored in the conventional teaching of these disciplines, and even less (or not at all) at an introductory level.

We argue that: (a) With adequately designed and implemented teaching activities, it is possible to explore links among probability, statistics and philosophy even with novice students in statistics and probability. (b) Exploring these links can contribute significantly to discuss with students deep philosophical issues, which are often related to important aspects of everyday life and in most cases are nontrivial for the students. (c) Appropriate teaching activities exploring such links for discussing philosophical issues may have an important motivational and emotional impact on the students, raising their strong interest and involvement. (d) Such teaching activities enrich students' concept image of what statistics is about, how it works and why it is interesting and meaningful. Moreover, combining (d) and (c) may also improve students' affective predisposition towards statistics.

To support (a)-(d) above, we present an example of teaching work realized during an introductory seminar on probability and statistics with prospective elementary school teachers.

HISTORICAL TEXTS USED

A key element in our teaching was the use of original historical texts that present statistical work and link it to a fundamental philosophical issue; namely, that of *Free Will* (FW). The texts we used with the students were chosen among Quetelet's works concerning Moral Statistics (MS)[1] and FW (we used mainly Quetelet (1847); but also Quetelet (1833), Quetelet (1842) Book 3, ch3 and Quetelet (1848)).

Quetelet's writings on MS and FW, as well as, the rest of his statistical works on the study of social phenomena are among the pioneering works that used statistics in social sciences. Educated as a mathematician and astronomer, Quetelet was familiar with probability theory of his time[2], as well as, with observational methods in astronomy, geodesy and meteorology and the associated error theory.

In analogy to these methods and the underlying theory, he thought that quantitative data of social phenomena could be understood as consisting of average values related to constant causes and variation around these averages related to accidental causes. Also, he thought that if one observes sufficiently large populations, then, because of the Law of Large Numbers (LLN) and the de Moivre-Laplace Central Limit Theorem, the cumulative influence of accidental causes on statistical figures is practically neutralised and thus it would be easier to identify regularities and relations between average values and the underlying constant causes. Quetelet believed that this approach involving statistics in social sciences has a great potential for uncovering regularities and relations concerning social phenomena (which he often called "*social laws*") and thus it would permit their much deeper understanding. He was able to find such important regularities on social phenomena, like marriages and crimes. However, he was often criticized of being over-optimistic concerning his opinion on the generality and validity in time of "*social laws*" that could be uncovered by the use of statistics, and, even more, for his vision that a coherent system of such laws – that he called "*Social Physics*" - could be established. Nevertheless, the promising results in his work, as well as, Quetelet's enthusiasm and energy, significantly inspired scientists to systematize the use of statistics in social sciences (Porter 1986 chs.4-6, Stigler 1986 ch.5, Stigler 1999).

In his works on MS, from 1829 till 1869, Quetelet pointed out that events like crimes, suicides and marriages present a remarkable statistical stability from year to year, provided that social conditions in a given country or state remained approximately stable. This stability allowed a quite accurate anticipation of statistical results for the years to come, on the condition of social stability as well. On the other hand, in such events, human FW plays an important role, and according to conceptions at the time, events in which FW is involved, should escape any possibility of prediction. Quetelet considered that the observed stability of statistical results and the resulting possibility of prediction, point to the existence of restrictions on the large-scale ("macroscopic") influence of human FW and call for revising existing ideas on FW. These statistical results and Quetelet's interpretation stimulated at the time, the debate on human FW and its limitations and Quetelet was criticised that his work promoted ideas close to

fatalism and materialism. He worked hard to refute this criticism by elaborating on his argumentation further and providing new statistical results to support it (Lottin 1911, Seneta 2003, Porter 1986 ch.6).

In addition to their interest of linking statistics to philosophy, Quetelet's texts we chose, have significant educational advantages,: (i) the mathematical treatment of statistical data is simple enough and thus adequate to be discussed with novice students in statistics; (ii) the proposed interpretation of statistical results is explained in detail, often accompanied by illustrative examples; and (iii) the texts themselves reflect the enthusiasm, passion and excitement that usually accompany new important and promising discoveries.

OUTLINE OF COURSE WORK

Our teaching work was realized during an introductory seminar on probability and statistics (with classroom meetings 3 hours per week; however, see note 5 below) for 29 3rd and 4th-year students (26 girls and 3 boys) of our Department of Education. Their sole previous education on probability and (descriptive) statistics was some rudiments they have been taught in high school, so the first three weeks were devoted to revise and complete this knowledge.

From the 4th to the 8th week, Quetelet's paper on the statistics of marriages (Quetelet 1847) was discussed. There he presented his point on the link between the observed stabilities of statistical figures and the limitations of men's FW. This was the first part of classroom discussion. For the second part, the teacher asked students to look for different philosophers' positions and ideas about FW and to present in the classroom elements of their personal study, thus enriching the classroom discussion in connection with results of the first part. This second part of the discussion lasted from the 8th week until the end of the course (12th week) [3].

Moreover, the teacher asked each student to prepare a written essay, of at least 6000 words, that should be delivered one month after the end of the classroom meetings and in which they should present and comment both on elements of the classroom discourse and of their personal study concerning philosophers' positions on FW.

After the end of the classroom meetings, the teacher interviewed each student individually, focusing on what they found (or did not find) interesting and attractive in the course, as well as, their motivations and feelings about the work they had done.

Since the first part involves more work on statistics, and at the same time, is essential for the reader to understand our approach, in this paper we present in more detail elements of the first part of the classroom discussion, while elements of the second part are presented more briefly due to space limitations.

BACKGROUND INFORMATION

As already mentioned, our students' previous education on probability and statistics consisted of only some rudiments of descriptive statistics and probability that they had been taught in high school[4]. This knowledge was revised and completed during the first three weeks[5]. We talked about data organization and their (graphical and numerically tabulated) representation, measures of central tendency (mode, median, mean) and variation (range, interquartile range, mean absolute deviation and standard deviation), the shape of a distribution and skewness. We also talked about the probability multiplication and addition laws, the binomial distribution and examples of its applications (e.g. chance games, newborns' sex, simple insurance models) and the LLN and the normal distribution accompanied by adequate examples.

THE FIRST PART OF THE CLASSROOM DISCUSSION

Introducing the problem

During the 4th week, the teacher gave information on the important scientific developments in the 19th century and the corresponding intellectual atmosphere and enthusiasm. In this context he explained the great interest of the scientific community on probability and statistics, whose main successful applications at the beginning of the 19th century were in astronomy and geodesy, while later on, their use was extended to all natural and social sciences. Then he presented elements on Quetelet's education and work, paying due attention to its pioneering character in social sciences, and Quetelet's point of view and ideas about the virtue and possibilities offered by the use of statistics in the study of social phenomena. Furthermore, he discussed with students the concept of accidental and non-accidental causes of variation, together with adequate examples[6]; a key concept in the development of 19th century statistics, as well as, in Quetelet's ideas for the use of statistics in social sciences (Stigler 1986).

Then, the teacher presented Quetelet's introduction in his 1847 paper (Quetelet 1847[7]), where he remarks that moral statistics is criticized for attempting to measure man's passions and inclinations, which is not only impossible, but also absurd; moreover, that this is an effort "…to chain up (men's) future in an inflexible mathematical formula.." (Quetelet 1847, p.135). For those studying only individual cases - he wrote - FW acts in a way so capricious, disordered and unpredictable that it seems absurd to suppose regularities and laws in the series of facts realized under its influence. However, he remarks, when observing large populations, the influence of peculiarities of individuals' FW vanishes and the series of general facts because of which society exists and lasts become dominant. When a large population is observed the effects of peculiarities of individuals' FW on statistical results are mutually neutralized and fall under the category of effects due to purely accidental causes[8]. This fundamental property of human FW allows establishing moral statistics and obtaining useful results. Moreover, Quetelet emphasizes that this property is also remarkable from a philosophical point of view, since it informs us that the influence

of individual peculiarities of man's action is limited in a sphere such that the underlying laws of nature escape from it forever (Quetelet 1847, p.136); moreover, it points out that conservation laws may exist in the moral world, as they exist in the physical world. Quetelet remarks that a main question is to prove this fundamental property of human FW, and that in previous works he had pointed out that the neutralization of effects of peculiarities of individuals' FW is indeed observed when the examined data concern a sufficiently short period of time so that social conditions remain essentially unchanged. Then, he mentions his works on criminal acts, in which a remarkable stability and regularity of statistical figures is observed from year to year and notes that in the present work the same question is examined for marriages, based on data from Belgium.

Discussing variation and effects of peculiarities of individuals' Free Will

Subsequently, Quetelet provides examples of data in support of the existence of a remarkable stability of statistical figures of marriages for 1841-1845, a period of social stability in Belgium. The teacher asked students to examine these data and formulate their own considerations and opinions about their stability.

Quetelet presents the annual number of marriages among widowers and widows, which were for the towns 231, 221, 224, 244, 226 and for the rural communities 498, 474, 492, 482, 514. Students calculated the average, the differences maximum - average, average - minimum, the range and the Mean Absolute Deviation (MAD)[9], first for towns, and then for rural communities. After that, they calculated the variation measures as ratios (percentages) of the corresponding average values. So they found that the maximum and minimum values for towns differ from the average by 6.5% and 3.6% of the average, with the corresponding percentages for rural communities being 4.5% and 3.7%, and that MAD is 2.9% of the average for towns and 2.3% for rural communities. Given these results, students agreed that Quetelet was right to consider that there is a small variation, hence stability in the annual number of this category of marriages.

Then students worked on the second example given by Quetelet; the number of men and women 25 to 30 year-old, married in towns. For the period 1841-45, they are 2681, 2655, 2516, 2698, 2698 for men, and 2119, 2012, 1981, 2120, 2133 for women. They found that for men, the differences maximum-average, average-minimum and the MAD are 1.8%, 5% and 2% of the average value, and for women the corresponding value are 2.9%, 4.4% and 3%. Once again students expressed the opinion that these results indicate a small variation and thus considerable stability. Subsequently, they continued to work on other data from Quetelet's paper (the table on p.143), where he gives the distribution of marriages per year according to the grooms' and brides' age category. In the four larger categories (with average values between 2495 and 12752) the ratios of variation measures divided by the corresponding averages were not far from those observed in the previous examples[10], and for the total annual number of marriages these ratios were about the same size[11]. In the smaller categories, however, there were cases for which these ratios were

larger than the aforementioned ones, especially in categories with average value less than 150. Quetelet remarks on this, that for small categories it is more likely that accidental causes destroy stability and thus show a larger (relative) variation.

In the follow-up discussion, students expressed the opinion that the data examined so far, were compatible with Quetelet's interpretation that in a period of social stability, the variation of men's FW generates a small variation in the large categories of population concerning the annual number of marriages and thus considerable stability is observed in these statistical figures.

Then the teacher posed the question: Even in a period of social stability there are many economic, emotional or social reasons, because of which individuals may change their will and disposition from year to year about getting married. What do you think it may happen so that, despite all these possible reasons, the annual number of marriages in the country did not change substantially? Students assumed that compensation processes were at work and proposed relevant examples such as:

- In a given country in the course of a year, some people loose their job, which may affect their will to marry that year; however, if stable social conditions are prevailing, about an equal number of people will find a job, affecting their will to marry the other way round. So, although in both groups there are changes concerning people's will to marry, the annual number of marriages may very well remain practically unaffected.
- In a big country under stable social conditions, it may be assumed that each year about the same number of unmarried people are in grief because of their parents' death, but they are not the same individuals each year, since some enter a state of grief and approximately the same number leaves it. Concerning their will to marry, the first are affected negatively, while, the others positively. However, it is likely that the annual number of marriages remains unaffected.

The teacher remarked that, under stable social conditions, these compensation processes produce variations of the kind Quetelet called accidental variations. He also discussed with students about accidental variation and the fact that compensation processes, like those mentioned above, work better concerning statistical figures of large categories or groups of population (e.g. a big city), than for small ones (e.g. a village). This happens because there are accidental causes of variation which in small groups can easily produce important variations compared to the corresponding average values, while, this is not likely to happen in large groups. Once again, students proposed a variety of adequate examples such as:

- In a village in which 10 marriages per year happen on the average, divorces of engaged couples or job loss can easily yield a 10% or 20% decrease of the number of marriages from year to year. But, in a big city, this cannot happen. In fact, in big cities, especially under stable social conditions, the number of divorces of engaged couples or of men that loose or find a job does not change much from year to year …

Comment: This discussion allowed students to form a better qualitative understanding of accidental variation and how it works in social phenomena; in

particular, concerning its influence on the stability of statistical figures of large and small groups. Also, the teacher noted common aspects between accidental variation and variation of random samples that obey the LLN. Moreover, in this way students came to better understand Quetelet's position that in conditions of social stability the variations in time of individuals' FW do not produce important variation of statistical figures for large populations.

After that, the teacher noted that Quetelet's remark that the period 1841-1845 was socially stable in Belgium does not mean that this should be understood too literally, but only as an approximation. This means that it would be possible for some important social factors to exhibit small, though significant changes, without however disturbing the overall image of social stability. Such changes may lead to small, but non-accidental variation of the statistical figures of marriages in that period. Hence, part of the observed variation could be non-accidental. Furthermore, with the aid of adequate examples, the teacher remarked that the LLN does not apply to non-accidental variations, which often do not depend on the size of the observed population categories.

Then, he invited students to search if there are indications of such non-accidental variations in the data. Students, despite their limited formal background in probability, were able to make some sensible relevant observations. Notably: (i) They observed that three out of the four maximum values of the four larger categories previously examined concern 1841. They considered that this was an unusual result if the year of maximum for each category was determined randomly in the five-year period. (ii) The total population constitutes a category far larger than the four large categories examined previously, so students thought that if all the observed variation was accidental, then the measures of relative variation of the total population should be substantially smaller than those of the four larger categories. However students considered that the ratio of the difference average-minimum over the average did not decrease as expected (see endnotes 10 & 11) and that this could be also an element indicating the existence of non-accidental variation[12].

People's tendency to follow social habits and requests

Quetelet writes that people have the strong tendency to follow customs, habits and requests of the society to which they belong and that this is a main element influencing their will in general and on marriage in particular, and determining the relevant statistics[13]. This is the second main element he proposes for interpreting the observed statistics in connection to men's will. Moreover he provides empirical evidence to support and illustrate his position, such as: (i) The modal age of brides differs as much as two years from one province to another and the difference is observed each year. This difference, he remarks, is due to the difference of customs of different provinces and not to individuals' FW. (ii) The number of marriages between young and aged people is small, but quite stable from year to year. Quetelet comments on this arguing that a man less than 30 married with a woman more than 60 did so not because of fate or blind passion; he was in a position to think about it

and to fully use his FW; however, he finally decided to pay his debt to the needs of the existing social organization. This kind of debt, he remarks, is paid each year more regularly than the taxes paid to the State.

The teacher presented Quetelet's position and examples regarding the influence of social factors on men's will to marry and then asked students to express their thoughts and opinions on this issue. They identified a large variety of such factors. Based on their knowledge and experience, they presented a considerable number of examples to illustrate the influence of these factors, which can be classified as factors concerning: familial environment, economic situation, social environment and in particular other people's opinion, and education. Students remarked that through these factors, social habits and ethics, as well as, moral and religious beliefs are often strongly expressed, and underlined the importance of education in cultivating men's ability to critically evaluate and consider the influence of these factors. Many students described the influence of social factors in terms of pressure to which men's will yields, or is subordinated. Other students reacted to this, noting that for many people, their FW is in agreement with ethics, social habits and common moral; hence, in this case, there is no question of such yielding or subordination. Others remarked that people from their earliest days are subjected to strong influences from their family, education and social environment, creating stereotypes and beliefs that determine their future will on issues like marriage. So, even if they are willingly in agreement with ethics and social habits, it is questionable if this will is free will. These remarks led to the following important question: To what extent a man creates and controls his own will?

This question was raised during the 6th week, but was discussed mainly in the 7th. Many students thought that a large part of ideas and beliefs determining men and women's will are determined by social factors, but there is also a significant part which is their own. Others remarked that even referring to their own crucial decisions[14], they could not identify any important ideas or beliefs underlying these decisions that were completely theirs. They said they found ideas and thoughts that they initially considered being their own, but upon deeper examination they found that these were strongly influenced by preexisting ideas and beliefs which in turn, were formed under the strong influence of their family, education and social environment. They agreed that this is a difficult issue to clarify, but that it is important to keep on trying, because any clarification may be important for revising possible illusions on men being masters of their own will.

SECOND PART OF THE CLASSROOM DISCUSSION

In the 6th week the teacher said that it would be interesting to read about other scholars and philosophers' ideas on FW, proposed some reading sources on the past and current debate on FW, and mentioned some key personalities, who have significantly contributed, like St Augustine, St. Thomas Aquinas, Newton, Hume and Kant. He further suggested to start with an overview of the subject, but that the students should feel free to continue focusing on one or more philosophers or lines of

thought that they would find interesting and attractive in relation to their own ideas and thoughts. The students actively worked on this task as they found the subject very attractive. So, from the 8th week on, till the end of the course (12th week), they orally presented in the classroom, elements of their study and their own comments that substantially enriched the discussion there.

Ideas of St Augustine and St. Thomas Aquinas were often presented and commented. An important element introduced in this way was the discussion on the interrelations among FW, personal responsibility of one's own actions, and the aim and role played by punishment and reward. This covered a considerable fraction of the second part of the seminar. Students brought in this debate, ideas of many other philosophers; either classical, like Hume, Kant, Schopenhauer, or modern – hence less known to the wider public - like Frankfurt, Strawson, Kane and others (cf. the next section as well). This is a strong indication that although students had no specialized knowledge on philosophy, they were strongly stimulated by the issues raised in Quetelet's text and their elaboration in the classroom discourse, and they intensively worked on them, searching into the existing literature by themselves. Below we describe some characteristic aspects of the classroom discourse:

St Augustine and St Th. Aquinas remark that FW is not the only condition for attributing moral responsibility; it is also necessary that one is aware of the consequences of his/her choices. In particular, they stressed that children and fools cannot be held responsible for their actions because of lack of this awareness. This was vividly discussed among the students, who remarked that very often someone could not have any satisfactory knowledge of the long-term consequences of its choices because of existing objective and/or subjective uncertainties. Some students said that moral responsibility should be attributed to a person according to its knowledge of the consequences of its choices. Other students remarked that this is not the only thing to be taken into account; social conditions that have played a determinant role on the formation of a person's will and character must also be taken into account. Moreover, some students referred to and commented on elements of Quetelet's work on crimes (excerpts from Quetelet 1833, 1842, 1848). Quetelet observed a remarkable stability in time of the statistical data of the different kinds of crimes, as well as of suicides, though he found important differences among different provinces and countries. One aspect of Quetelet's interpretation that students underlined, is that the different kinds of crimes and their frequency are determined by social conditions and organization, while criminals are just the tools for realizing these crimes[15]. These elements fed the discussion on extenuating circumstances that should be considered and some students argued that in fact it is very difficult to fairly attribute moral responsibility to someone for its choices and actions. Later on, many students considered as satisfactory moderate answers to this question contained in the so-called compatibilist ideas of Hume and Kant.

Another interesting issue is that students presented ideas about Newton and Laplace's hard determinism physically based on Newtonian mechanics, and indeterministic ideas stemming from Quantum Theory. According to Newton and Laplace, the future

is fully determined once the initial conditions are given; hence, there are no alternative possibilities and therefore, both uncertainty and FW are only illusions. In this context, theories and ideas on FW, as well as probability theory, are only conceptual models for managing parts and aspects of our ignorance. The teacher remarked that although uncertainty objectively exists according to Quantum Theory, our ignorance is also a reality, and part of the use we make of probability theory is due to our ignorance and not to any objectively existing uncertainty. In this sense Laplace's conception of probability is partly valid. Some students thought that a similar idea holds also for FW; although FW may very well exist, part of the potential we attribute to it, is due to ignorance about restrictions on its influence, as well as, to lack of awareness of the influence of social factors determining our will[16]. Students were not convinced by Newton and Laplace's deterministic ideas, but the fact that it was these great men, who supported these ideas, strengthened their quest on the limitations of human FW and, for some, their quest about the very existence of FW.

ON THE STUDENTS' ESSAYS

The teacher asked each student to provide a written essay of at least 6000 words, within a month after the end of the classroom meetings, presenting and commenting on aspects both of the classroom debate and of their own study of other philosophers' positions on FW. He also encouraged them to feel free to develop their own thoughts and ideas on them.

In all essays, students discussed limitations of the influence of men's FW, as well as factors that influence the formation of men's will. Some of them considered the discussion on these limitations and factors in the context of their own quest about the central question of the existence of FW. There were also students who discussed the importance of critical awareness of these limitations and factors and the role played by education, family and society in the development of a person's critical thinking. Six students focused on the relation between FW and personal responsibility, and three on the relation among uncertainty, chance and FW.

Below we list the philosophers/scientists, whose ideas were more frequently discussed in students' essays (the number of essays referring to a philosopher's ideas follows his name): Quetelet, 26; St Th. Aquinas, 11; St Augustine, 10; Kant, 9; Hume, 7; Hobbes, 6; Aristotle, 3; Newton, 3; Frankfurt, 3; Laplace, 2; Fichte, 2; Schopenhauer, 2; Everett, 2; Steiner, 2; Strawson G., 2; Kane, 2. Another 23 philosophers' ideas were mentioned, though each one of them appears in one essay only.

It is also worth mentioning students' positions in the essays on the question of the existence of men's FW:

(a) 10 students were not convinced for its existence; (a_1) 2 of them expressed the opinion that it is an illusion; (a_2) the other 8 were skeptic about its existence;

(b) 19 were convinced that it exists, but that there are also important restrictions about it. (b_1) 7 of them emphasized that there are people without FW on essential

points (like people who are subjected to systematic totalitarian or manipulative education from childhood). (b_2) The other 12 did not emphasize what is mentioned above in b_1.[17]

FINAL COMMENTS

On the discussion with students about Free Will

The philosophical discussion on FW has been lasting more than 2000 years and has formed part of the central philosophical debate on the basic characteristics of man as an individual and as a social being. Hence, the seminar could not aim at formulating and discussing any definitive answers, but rather, at raising questions and bringing to light issues that till then, students had considered little, or not at all. Quetelet's statistical data and his interpretation was an important asset for posing such questions and stimulating a debate that motivated students' further study and thinking.

In the first part of the classroom debate, students identified and discussed at an initial level, both limitations on the influence of men's FW and on factors that form and determine men's will. Students gradually realized that (i) this is a complex and deep issue; (ii) they had little knowledge and had thought little about it; (iii) gaining knowledge and insights of it, is not only interesting for philosophical and social questions, but also important for personal fulfillment. These three elements together, generated a strong motivation for students to work on and look for this issue further. In the second part of the debate, ideas of all philosophers' points of view discussed, underlined the importance of critical awareness on this issue that further enhanced students' interest and motivation to search for it. It is because of this motivation and interest that, in many cases, students' work by far exceeded the course's typical requests (for a sample of students' opinions in the final interviews see Appendix, excerpts 1-3).

On Statistics and Free Will

On the one hand, the work done allowed students to improve their understanding on specific issues in statistics; in particular, the distinction between accidental and non-accidental variation and on how variation works in social phenomena in connection with the LLN. On the other hand, it allowed students to enrich their concept image on what statistics is about and how it works. More specifically:

(i) Students realized that statistics is not just the technical treatment of data, but it may concern issues like FW, which is not only a fundamental philosophical issue, but also has important implications on everyday life and personal behavior and attitude.

(ii) They had the opportunity to realize that on issues such as FW, statistics can provide macroscopic information of critical importance, which cannot be accessed if one is limited to examining the subject only at the individual (microscopic) level.

(iii) They realized that an important part of statistics is the interpretation of statistical results and how it works. During their work on interpreting data, students linked statistical results with ideas and beliefs they disposed, as well as with elements of

their experiential background. This linking often led to the evolution of ideas and beliefs, the emergence of new ideas and the rise of new questions. In fact, interpretation work was for students the most interesting part of statistical work that made statistical results meaningful.[18] [19]

Furthermore, because of the conjoint presence of the three aforementioned elements many students' poor affective disposition and opinion on statistics was improved.

APPENDIX: Excerpts from students' final interviews by the teacher[20]

(1) Maria: I never believed that a course on statistics could refer to such interesting issues; I mean not just academically interesting, but interesting for each one of us personally... There are all these decisions and choices that I thought to be my own, and then, after discussing and thinking about them, I realized that there are so many influences that determine our will! I did a lot of work wondering which of my choices and decisions are really mine, and which responsibility is really mine. This is a difficult question, but it is also important to find at least some answers; I mean, it is important not just philosophically, but personally… Because of these questions, I did a lot of work voluntarily and not because of the course's requests.

(2) Katherina: I found the discussion on the restrictions and potential of our FW very important… For example, Quetelet is very right saying that we tend to follow and do what our environment and others say. This point, that is, "do what the others like", is an issue to which I devoted a lot of thought, not only in general, but also examining myself, my own behaviour and attitude… Also, Quetelet's statistics points out that despite our FW, society - like a "well-oiled" machine - produces the same results each year. His statistics confirm that our FW has a "limited sphere of influence", as Quetelet says. Based on the whole discussion and study on this issue, I came to believe that this "sphere of influence" is small. But how small? This is an important question that still remains unanswered to me… The discussion during the seminar posed questions and burning issues that are not going to be extinguished any time soon. In fact, as far as I understand, we have just started struggling with these questions.

(3) Anna: …. Another important issue was the one on FW and responsibility. On the basis of the discussion, I realized how complex and difficult is to judge people for their choices and actions justly; still, we keep doing it easily and superficially every day. But, by doing so, it is very probable that we become unfair without even being aware of it. This is not just an academic discussion. If one succeeds to understand better this issue, it is very likely that he will change his attitude while judging others…

(4) Photini: …I had never thought that statistics could be so interesting. I mean it is so interesting because it is linked to the issue of FW, which - as we have seen - is important philosophically and socially and personally. Additionally, it was the way that we worked on the statistical results. At school, most of the time devoted to statistics, we were finding averages and graphs only, so I believed that statistics is a very boring subject. Here we discussed a lot on the statistical results, trying to explain them. We discussed examples and individual cases in connection with the statistical results. There were Quetelet's statistical

results but there were also Quetelet's ideas for explaining them and then we brought in our own ideas and we could discuss our examples and even our related personal experiences. And then, with all this, we started grasping the problem of FW; I mean we did not find any definitive answers, but we raised deeper and wider questions and saw aspects of the subject that we did not even suspect they exist. … This was a really exciting course; if this is doing statistics, then statistics is far more interesting than I had thought.

(5) Eva … With all these causes and factors that may influence and change a person's will about getting married, I could not guess the existence of such an annual stability in the number of marriages; not just in general, but for each category and in each region. And the stability of the number of crimes; this is even more impressive. What Quetelet says in his text is important; that without statistics, people could believe that peculiarities of the individuals' FW can produce important changes in the number of marriages, or crimes from year to year. But this is a wrong idea, which overestimates the power and potential of an individual's FW; moreover it is an idea that describes society more disordered than really is. This is what I found important with statistical results; they allow to clarify things and to avoid certain important wrong ideas…

[1] Quetelet considers *Moral Statistics* to be the domain of statistics that concern phenomena, like crimes, suicides, marriages, which are phenomena that may be subjected to moral characterization (Hankins 1908 ch 4., Lottin 1911).

[2] Two key elements being the Law of Large Number (LLN) and the DeMoivre-Laplace Central Limit Theorem (CLT), both permeating explicitly, or implicitly his work. In view of our students' elementary knowledge of Statistics however, his papers selected for the seminar's purpose are mathematically more elementary, hence not directly referring to the latter.

[3] In the second part students also presented elements of other works of Quetelet; in particular, of his statistical works on crime and suicides.

[4] We note that high school teaching gave students the impression that statistics is mainly the technical treatment of data (computations, creation of graphs etc). As a result, many of them considered it as an unattractive subject.

[5] Classroom work lasted for 6 (teaching) hours in the first week, 3 hours in the next, and so on (so on the average it lasted 4.5 hours per week).

[6] E.g. the analysis of error measurements that are due to accidental and systematic errors; or the analysis of reparations paid by insurance companies, as due to constant causes determining the average values of the reparations and to accidental causes responsible for deviations from these averages.

[7] Quetelet's text is available online (http://www.edc.uoc.gr/~tzanakis/Quetelet1847Marriages.pdf)

[8] When Quetelet refers to men's FW, he actually means the peculiarities of individuals' FW, not the common aspects of men's will. Obviously, the effects of these common aspects are not mutually neutralized and do not vanish for a large population. As Lottin puts it, Quetelet considers FW as a reaction force that creates peculiarities and individual specificities (Lottin 1911). On the other hand, students used the term FW in an ordinary sense that encompasses both individual peculiarities and

common aspects of human will. Therefore the teacher clarified this point to avoid misunderstanding Quetelet's text.

[9] Students chose to use MAD as a global measure of deviation instead of the Standard Deviation (SD), because they felt it is simpler and better understood than SD. Though the teacher accepted this choice, on several occasions later, he asked them to compute also the SD so that they gradually became acquainted with it, and benefited from the comparative consideration of MAD and SD.

[10] The four ratios concerning MAD were between 2.2% and 4.3%, those concerning the differences maximum - average were between 3.2% and 5.5%, and those concerning the differences average - minimum were between 3% and 5%.

[11] The ratio for MAD was 1.4%, that concerning the difference maximum-average was 2.6%, and the one for the difference average-minimum was 3.1% (the average of the total annual number of marriages was 29131).

[12] Then the teacher remarked that it would be interesting if they could find a way to estimate the size of variation measures that it is probable to result by accidental variation and then to compare it with the variation measures calculated from Quetelet's data. Six students worked on this issue with teacher's assistance, in activities independent of the rest of the course. They did interesting work using - among other things - large numbers of random samples as informal tools to answer questions that were raised. Their work is not presented here, because of space limitations.

[13] Furthermore, Quetelet relates this strong tendency to people's inherent sociability, which leads humans to voluntarily cede part of their individuality in order to become members of the society.

[14] E.g. selecting a subject for their tertiary studies and a profession.

[15] Furthermore, students provided recent statistical data pointing out a very important increase (about 30%) of suicides in Greece during the current economic crisis. They considered this to be also in line with Quetelet's interpretation.

[16] Moreover, other students made an interesting analogy: Though Quetelet considered macroscopic stability of statistical figures, he thought that, at the individual level, there are capricious and unpredictable peculiarities of human FW, which, however, are not powerful enough to destroy this stability. Students considered that there is an analogy between this and the deterministic regularity of macroscopic phenomena as described by the laws of Newtonian mechanics and the uncertainty of physical phenomena at the microscopic level inherent in Quantum Theory. It is worth noting that Herschel's presentation of Quetelet's research was inspiring for Maxwell, who thus conceived an analogy close to the aforementioned that stimulated him to introduce a statistical approach to microscopic phenomena; in particular, his introduction of the normal distribution to derive the molecular velocity distribution of gases, a key step for the systematic development of Statistical Physics and Kinetic Theory since then (Porter 1986, pp.115-116, 118, 121, 123).

[17] However, we should note that many students remarked that their position in the essay was a first one, susceptible to change after further study.

[18] See some of students' own opinions; appendix, excerpts 4, 5.

[19] This is a point that deserves to be examined as a more general characteristic of learning (other topics of) mathematics: work on understanding and interpreting selected data (e.g. measurements of

physical or geometrical magnitudes) acts as a strong motivation for learning a particular subject and modifying positively the learner's affective disposition to it.

[20] Names have been changed.

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