Plenary Lecture

HISTORY IN THE CLASSROOM: EDUCATIONAL OPPORTUNITIES AND OPEN QUESTIONS

Adriano Demattè

Liceo Rosmini Trento & University of Genoa, Italy

Using the history of mathematics in everyday classroom activities is difficult because of various reasons, but it is an intriguing aim. This paper will report examples of activities, mainly inherent to interpretation of original texts, developed in my classes. Opportunities and problems, achievements and failures will be analysed.

With the aim of carrying out a critical analysis, theoretical considerations will be taken into account. The purpose is to introduce an ongoing discussion with regard to the complexity of everyday classroom activities. Ultimate answers are not the main aims of my analysis.

INTRODUCTION

It is widely recognized the importance of introducing history in mathematics teaching at all school levels, see (Furinghetti, 2012; Barbin & Tzanakis, 2014). In this paper I report on the activities I have recently carried out in my classroom. My motivations for the use of history rely on the conviction that history is a carrier of culture in student's view of mathematics (Jankvist, 2015) and that important educational goals of mathematics teaching may be achieved through history (Kjeldsen & Jankvist, 2011). In planning my activities I followed Janhke et al. (2000) who support the use of original sources in the classroom as a demanding task that may be carried out according to different perspectives. I considered group work among students and role of the teacher as tools for transformation of knowledge (Radford, 2011).

My paper will illustrate the theoretical background orienting my choices, the school context in which I realized my project, the main steps of the implementation into the classroom and the analysis of the outputs with some preliminary conclusions. My experiment will be presented almost as a narration to allow the reader hearing the voice of a teacher who tries to combine his educational goals in teaching mathematics with his passion for history. This narration is going to highlight the facts that, in my opinion, are really significant for discussing the issues related to the use of history and make my experiment transferable to other situations.

The problem of the transferability of experiments to different situations with different teachers is really crucial. In particular, when dealing with the introduction of history of mathematics in mathematics teaching, there is the problem of the teachers who do not believe that this introduction is possible or really suitable to reach their teaching goals. Many teachers are not familiar with history of mathematics and, even more, with original sources. All these difficult cases were discussed in the workshop I carried out during the conference ESU7 (see Demattè, to appear) on the ground of the paper (Siu, 2006). To meet the need and the perplexities of these teachers I devote a section of this paper to the presentation of materials and teaching sequences that may be developed in a mathematics laboratory.

In recent years, some authors, Jankvist (2009) for one, have raised the question of promoting empirical research to better understand the potentialities of this use. Through the description of my experiment and the analysis of the doubts raised by the results of my experiment I hope to offer materials for facing the following research questions:

- What kinds of activities are most suitable to involve teachers in using history of mathematics in their classroom?
- What educational goals regarding the use of originals could teachers consider relevant goals for their mathematics classes?

THEORETICAL BACKGROUND

Due to the fact that in my experiment I try to combine the need of achieving my educational goals and my confidence on the efficacy of history in my teaching, the theoretical underpinning of my work is inspired both by the research in mathematics education in general and by the particular research which concerns the relation between history and pedagogy of mathematics.

Classroom culture and mathematical discourse

Let me start from the educational side by quoting my personal experience as a young teacher. I remember an author whose works I got to know during a training course at the very beginning of my career. That is Carl Rogers (1951), the American psychologist who is considered the founder of the client-centred approach in psychology. Nowadays, I am able to quote only little of his thought, the following sentence for example: "A person cannot teach another person directly; a person can only facilitate another's learning". Some key words of this statement, or suggested by it, synthesize my ideal approach to teaching: facilitate, students' autonomy, learning with meaning and consciousness.

To explain my approach to teaching I start from the drawing of figure 1 where an Italian pupil of grade 3 answers the task: "Draw your mathematics class, that is the teacher and your classmates in a mathematics lesson. Use bubbles for speech and thought to describe conversation and thinking. Mark yourself (Me) in your drawing", see (Laine, Näveri, Ahtee, Hannula, & Pehkonen, 2012).

The student's perception of the atmosphere in the classroom expressed by this drawing is in line with the description made by Lampert (1990, p. 31) of the school experience in which: "*doing* mathematics means following the rules laid down by the teacher; *knowing* mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical *truth is determined* when the answer is ratified by the teacher." As a consequence of this school experience we have a cultural assumption which associates mathematics with the idea that knowing mathematics means to be able to get the right answer, quickly and following the rules given by an authority (the textbook, teachers). I try to challenge this common assumption by changing the roles and responsibilities of teacher and students in classroom discourse so that, as advocated by Lampert (1990), the practice of knowing mathematics in school becomes closer to what it means to know mathematics within the discipline.

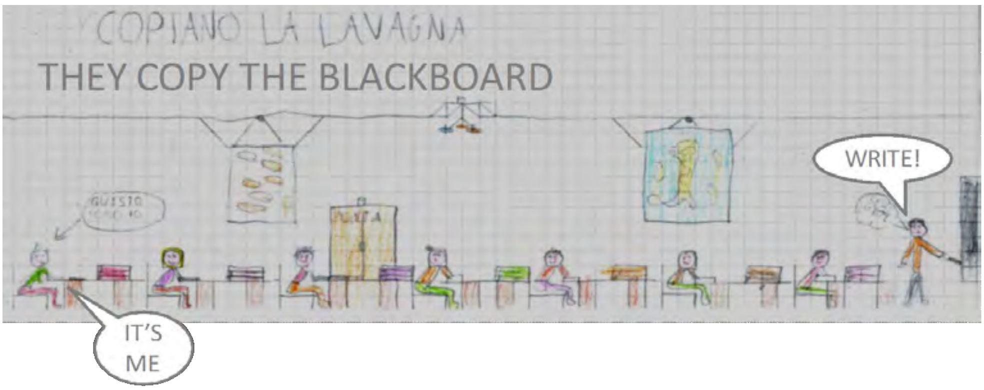


Fig. 1. Drawing by a pupil of grade 3

As a teacher I try to create a classroom culture in which student activity can occur through participation in the doing and learning mathematics so that students learn not only contents of their curriculum but also "what counts as knowledge and what kind of activities constitute legitimate academic tasks" (Lampert, 1990, p. 34). In the classroom discourse I implement teacher-student interaction and content, such as: learning as (re)discovery, group work, discussion among peers and between teachers and students, exploring mathematical phenomena, generating conjectures, verifying and, in case, refuting and refining them. Students become authors of their ideas and responsible of their intervention in the mathematical discourse. As discussed in (Demattè & Furinghetti, to appear), the laboratory is a good place where to realize my project since it allows actual participation to mathematics activities.

Using original historical sources

Taking into account the large amount of literature and my previous experiments, I decided that the history of mathematics is a good tool for realizing my ideas. In particular, as pointed out in the Introduction , original sources have shown their pedagogical efficacy in mathematics teaching, see (Furinghetti, Jahnke, & van Maanen, 2006; Pengelley, 2011; Jankvist, 2014).

Barbin (2006) has pointed out that there are different ways of reading original sources. My way, in line with previous experiments in the classroom, see (Bagni, 2008; Glaubitz, 2011a), is based on the hermeneutic approach. This approach changes the strategy of teaching/learning. Students use original mathematical documents and are asked to use the mathematics they have learnt, in a new way. Jahnke (2014, pp.83-84) outlines the basic guidelines of the hermeneutic procedure as follows:

> “(1) Students study a historical source *after* they have acquired a good understanding of the respective mathematical topic in a *modern* form and a *modern perspective*.
>
> The source is studied in a phase of teaching when the new subject-matter is applied and technical competencies are trained. Reading a source in this context is another manner of applying new concepts, quite different from usual exercises.
>
> (2) Students gather and study information about *context* and *biography* of the author.
>
> (3) The historical *peculiarity* of the source is kept as far as possible.
>
> (4) Students are encouraged to produce *free associations*.
>
> (5) The teacher insists on *reasoned arguments*, but not on accepting an interpretation which has to be shared by everybody.
>
> (6) The historical understanding of a concept is contrasted with the modern view, that is the source should encourage processes of reflection”.

The points of the guidelines may be grouped according to different types of action. (2), (3) and (6) concern history of mathematics in its “strong role” (Demattè, 2006a). In contrast with the ”weak role” that confines the use of history to mathematics, the “strong role” is based on didactical activities that are directly inherent to history and aims not only at learning the mathematical subject-matter. Then it requires an additional amount of time. Definitely, (2) refers to giving historical knowledge, while (3) may be integrated with mathematics teaching/learning since “the historical peculiarity” may regard the use of unusual procedures and concepts that reinforce previous students’ skills by provoking the *dépaysement*, that is the alienation and reorientation (see Janhke et al, 2000). The point (6) furnishes a synthesis of deep reasons for reading originals instead of doing ordinary exercises.

Developing points (4) and (5) means that through the originals it may happen that students’ capabilities and though of ancient mathematicians meet. Point (4) suggests that students should get some ways to become protagonists, autonomously with respect to the leading teacher’s role. What (5) suggests is far from the current situation in Italian schools. In order to be implemented, it requires a radical change in students’ assessment criteria. Judgments would be referred to individual advancement instead of to a set of abilities required by institution. Most teachers agree that students have to be involved in “reasoned arguments”, but they aim at contents and conclusions which have to be shared by all students. These teachers would not accept the proposal to use originals according to the point (5). Points (4) and (5) introduce a way to personalize teaching, suggesting students to build their learning directly on personal previous knowledge (this reminds us the ideal teaching we spoke at the beginning). However (4) and (5) do not suggest how to establish students’ levels of proficiency. On the contrary, class tests, national and international achievement inquiries require all students to achieve common competencies (PISA, for example). (6) could suggest that the teacher should establish common levels of proficiency during "the processes of reflection" on "the modern view" of mathematical subjects.

When the hermeneutic approach in Jahnke (2014) makes it difficult to cope with the requirements of the school situation, that is to reach a homogeneous level for all students, the teacher may resort to socio-cultural resources of interaction between peers and teacher, in line with Vygotsky (1978).

By combining the previous issues concerning theoretical background and practical requirements I pinpointed the following goals of my experiment:

- reinforcing abilities regarding solution of quadratic equations;
- using previous knowledge and abilities for discovering the familiar into the unfamiliar (Jahnke et al., 2000);
- using a text to gather mathematical information;
- knowing peculiarities of an original text (types, rhetoric aspects, choose of terms, lack of symbolism);
- viewing the document in broad sense including students' personal connections and remarks.

Since I assume, as Leatham (2006) does, that "teachers are seen as complex, sensible people who have reasons for the many decisions they make" (p. 100), in the present paper we will mainly concentrate on the reasons behind my decisions. In doing that, I feel I am in the situation so well illustrated by Donald Schön (1987) in his *The reflexive practitioner* where he highlights how professionals do not always feel at ease reflecting on their action because of the turbulence of environments like schools. In fact, reflection in action increases complexity of teachers' task and, as an extreme consequence, situations might even slip out of control. The other side of the coin is that teachers have to look for proper circumstances suitable to promote reflection on what they are doing, otherwise their work risks to become at least an unfruitful ritual.

A CLASSROOM EXPERIMENT USING ORIGINALS

My teaching context

I teach in a "Liceo delle Scienze Umane" [Human Science Lyceum]. Students can choose between two branches: a) 'human sciences', *strictu sensu*, b) 'social-economic' in which the study of human sciences is less widely treated, in favor of economics and mathematics. The school lasts 5 years. The students' score in mathematics is quite low. An 'entry test' is prepared by the school department of mathematics teachers and is handed out at the beginning of the first year. The average score is around 60% of correct answers. At the end of the second year a test provided by the National assessment organization INVALSI-Istituto Nazionale per la VALutazione del Sistema educativo di Istruzione e di formazione (National Institute for evaluating the educational system of instruction and education) is administered. It gives information to schools and teachers but, at the moment, do not officially certify students' levels. At the end of the fifth year, students take their final state national examination. Together with three or four other school subjects, mathematics is part of a one out of three written tests and with five other subjects is part of an oral examination.

The classroom activity with original sources

The activity took place in a third class of the socio-economic lyceum (students aged 17-18). The average level of proficiency in National achievement examination of these students was under the average score. Students were weak in algebraic manipulation and were having difficulties in problem solving so I was looking for new kinds of problems and I considered interpretation of texts a good option. In previous lessons they met history of mathematics in various circumstances: references to Archimedes works about levers, use of Arab combinatorial reasoning, arithmetic triangle in Chinese, Arab, Tartaglia's versions, solution of quadratic equations by al- Khwarizmi, a quadratic function from a medieval treatise etc. For some topics I used the materials in (Katz & Michalowicz, 2004).

An example of a document I proposed is in figure 2. It concerns an exercise which requires calculations less complicated then most of those required by students' textbook, but not trivial.

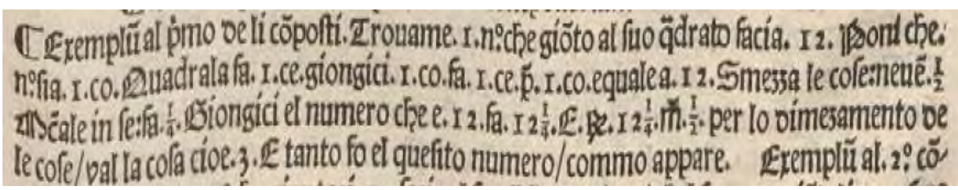

Exemplũ al pmo de li cõposti. Trouame. 1. nº che giõto al suo q̃drato facia. 12. Pongo che. nº sia. 1. co. Quadrala fa. 1. ce. giongici. 1. co. fa. 1. ce. p̃. 1. co. equale a. 12. Smezza le cose: neuẽ. ½ Mcale in se: fa. ¼. Giongici el numero che e. 12. fa. 12¼. E. R. 12¼. m. ½. per lo dimezamento de le cose / val la cosa cioe. 3. E tanto fo el quesito numero / commo appare. Exemplũ al. 2º cõ

Fig. 2: From Pacioli's *Summa de Arithmetica, Geometria, Proportioni et Proportionalita*, Venice 1523, edition of 1523 (first edition 1494), folio 145. "[…]

Find for me a number that, if joined to its square, makes 12. Imagine that the number be a thing. Square it. It makes 1 census. Join 1 thing. It makes 1 census plus 1 thing equals 12. Halve the things. It becomes ½. Multiply by themselves. It makes ¼. Joint the number which is 12. It makes 12¼ . And the square root of 12¼ minus ½, because of the halving of the things, equals the thing that is 3. And the required number makes this amount, as it appears. […]."

Giving to the students the short document from Pacioli's *Summa*, I asked them to read it and interpret it with respect to mathematical content. I underlined that the main goals that students were requested to achieve were to use personal resources and that the interpretation of the document aimed at: using previous knowledge and reinforcing abilities regarding quadratic equations, using a text to gather mathematical information, knowing peculiarities of that ancient text, formulating personal remarks, connecting with Italian literature whose origins students were treating in class.

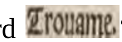
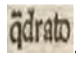
I considered that history allows introducing humanistic aspects in mathematics teaching (mainly those regarding communication) according to the peculiarities of school (a human sciences lyceum). I reflected on the opportunity to use a different version of the original, changing, in case, some aspect but I decided to maintain all the following characteristics:

- types, because they show that it is an ancient document, at a glance;
- use of abbreviations, because it poses questions regarding printing in 15th and 16th Century;
- archaic Italian words, because they show example of evolution of language;
- absence of separation between problem and solution, because it recalls a feature present in mathematical treatises since early Middle Ages;
- redundancies, because also the manner to communicate mathematics has changed;
- unusual manner to write rational numbers, because it shows an example of "the familiar found inside the unfamiliar" in elementary mathematics.

The students worked in pairs. I asked them to translate the document in modern Italian language, to conjecture about the meaning of the full document and its specific parts and to compare personal explanations of the mathematical passages in order to write a shared version of the interpretation. After a few minutes, I listened to their questions and I answered through hints or other questions that could help them to reflect upon the document and the (partial) explanations they had found at that moment. In order to give further opportunities for better understanding and for reviewing, I requested them to compare their explanations in this manner: standing in front of mates, most of them read their interpretations; they could also briefly criticize interpretations of other students.

Actually, the activity based on Pacioli's document showed controversial outcomes. Supplementary explanations by the teacher followed the group activities and in the written test students got an average score similar to previous tests, some of them even better. Students considered it as a meaningful experience. On the contrary, the part of the activity in which students were requested to work autonomously showed unsatisfactory results. The use of the written text did not fully succeed, with respect to the goal regarding solution of quadratic equations and use of previous knowledge and abilities for discovering familiar concepts into the unfamiliar document. The document highlighted students' incapability to use their mathematical knowledge to interpret the text or, in the same sense, to link their previous abilities to the content of the document. After the experiment, I met other class situations that had similar outcomes (even if regarding different kinds of documents such as an Euler's excerpt, or a graph). I have argued it could depend on the operational nature of my students' mathematical conceptions (Sfard, 1991) so that their knowledge was not actually at their disposal for interpreting the text. Many students required supplementary explanations about algebraic skills, which they were able to apply in routine exercises but they did not think to use for interpretation.

Here it is a list of difficulties that several students had.

1. Typefaces. For example: “What does  mean, inside the word ?” The current Italian word *Trovami* suggests the right answer: “**T**”. At the same time,  appears as a different manner to write ***v***.
2. Contractions. In the word , “quadrato”, two letters, that is ***u a***, are omitted and the specific mark  highlights this fact.
3. Exposition of the statement. “Where is the question? Where does the solution begin? No modern symbols!”. The sequential exposition in the document conflicts with the modern formula which shows all operations together.
4. From words to symbols. “Find a number that, if joined to its square, makes 12: what equation can I obtain?” [By a really weak student].
5. Unknown mathematical procedure. “Why do I have to halve the ‘things’ (coefficient of the linear term)? It is not even!”
6. Search for information to guess meanings (general meta-cognitive competency). Some students looked lost in front of difficulties 1) and 2): they did not think to read the text again, in order to find in the document the specific meaning of letters and words.
7. Meta-cognitive competencies regarding mathematical tools. Many students did not think to use the modern formula in order to interpret the document (to understand the meaning of specific elements like the letter  for “Root”, as well as passages in the reasoning).

Comments

I shortly describe my students’ difficulties as lack of willingness to guess, to produce conjectures, or to check them autonomously searching reasons inside the document. It seems that they have not internalized the *hermeneutic circle* which concerns the idea that the interpreter’s understanding of the text as a whole is established by reference to the specific parts and his/her understanding of each individual part by reference to the whole. I considered that students had experienced the hermeneutic circle using different kinds of texts, in various disciplines.

About the lack of disposition to make conjectures, we can identify a diagnostic role of hermeneutic approach. Conjecturing shows one's competences; students who produce conjectures reveal their being. It is really different from repeating a piece of the teachers' lesson! These remarks are based on the works of an Italian author, Bertagna (2000). He notes that from Parmenides and Aristotle until Heidegger, Fromm and Marcel an anthropological dilemma regards the distinction between *to be* and *to have*. The somebody’s *being* is her/his *essence* or *substance*. The Italian term "capacità" (*capacity*), in psycho-pedagogical context, recalls individual potentialities. The term *competence* has been used by Chomsky in contrast to *performance*: when teachers want to see whether students know a procedure, they create a task that requires a performance. A valid performance sometimes hides a lack of underlying competence. Both capacity and competence are inherent to the being of a student. On the contrary, *knowledge* and *skill* belong to the *having*. It is remarkable that from the Latin verb *habeo* (to have) derived: *habitus* (to behave), *habilis* (to do something well), *habitare* (to live in a place). About *knowledge*, we know *that*, *where/when/why*, *how*. The meaning of *skill* is strictly connected with that of the Greek *techné* (craft or art).

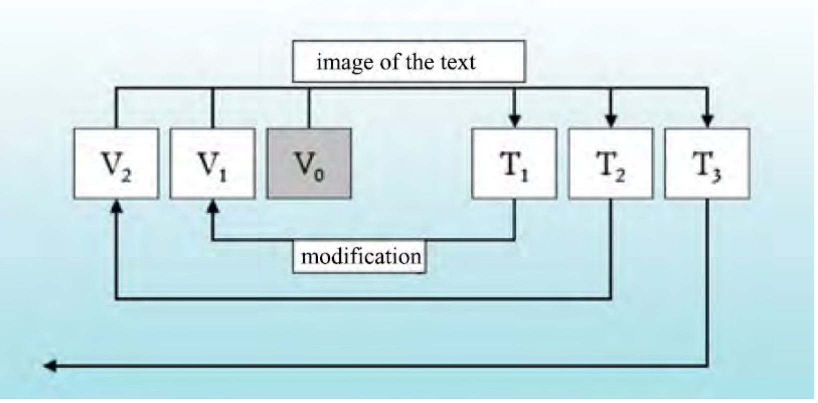


Fig. 3: Interpreting a text; from (Glaubitz, 2011b).

In interpreting a text, "you start with a certain image of the text reflecting your expectations about what it might be about. Then you read the text and realize that some aspects of your image do not agree with what is said in the source. Thus, you have to modify your image, read again, modify and so on until you are satisfied with the result or simply do not like to continue […] the hermeneutic circle can be considered as a process in which a *hypothesis* is put up, tested against the source, modified, tested again and so on until the reader arrives at a satisfying result" (Jahnke, 2014, pp.84-85), with reference to figure 3. Notice that the term "hypothesis" is used. I agree with Lampert (1990) who, quoting Lakatos, indentifies a conjecture with a "conscious guess". I believe that making hypothesis could be an unconscious guessing, also in the case in which students interpret originals.

When students are requested to interpret the text without any suggestion by the teacher, as in the first part of the experiment, they have to take a risk and guess or make conjectures, because they do not have the opportunity to choose the right specific knowledge or the skill they have got by the teacher. They have to abandon the reference to what has come to them from outside, and instead use their inner sources (the fact that they know to be requested to interpret the text with respect to its mathematical content does not significantly influence the situation). We call these inner sources *competences*.

I presume that only self confident students, i.e. those who believe to have a chance to give a "rewarding" answer to questions asked by teacher (a right one or, in any case, one well accepted by the members of class), have the willingness to conjecture. The students involved in the experiment got quite low test scores. Therefore I was not completely surprised when they did not conjecture trying to interpret Pacioli's original. As a teacher, this fact has posed me the problem on how to help students. Beyond individual cognitive difficulties, I think that they are influenced by other factors, mainly by the *didactical contract* discussed by Brousseau (1984), so that students consider more rewarding to give a performance quickly, rather than to "waste" time in personal efforts. It mostly happens in written tests. In addition, students could consider that leaving interpretation to the teacher has positive consequences: the quality of performance will be better, so they will get a better mark.

These reflections cannot be general because of the way in which the experience had been realized. However, some characteristics of my students are the same of other Italian students: they attend a state school which has similar curricula with respect to other kinds of secondary schools, the performance standards in mathematics and literature are similar to a significant percentage of Italian students, almost all of them have personal interests especially in new technologies etc. This leads to guess that some of their difficulties could be common to many other students.

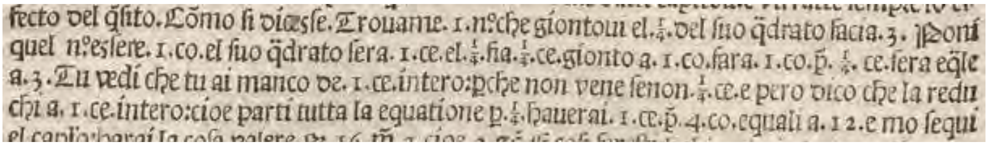
HISTORY IN THE MATHEMATICS LABORATORY: A PROPOSAL TO THE TEACHERS WHO DO NOT USE HISTORY; A WAY TO ACT ON THE *ZONE OF PROXIMAL DEVELOPMENT*

Reflection on the questions reported in Introduction about teachers' reluctance to use history in their mathematics teaching, led me to design proposals of activities that are now collected in the textbook (Bergami & Barozzi, 2014). Almost every chapter of this textbook contains a section called *historical laboratory* divided in three parts: the first one is printed in the paper book as input; the second regards "supplementary exercises" and is on-line, like the third which is devoted to cultural context (where and how mathematicians operate, mathematics and other subjects or applications, a research on the internet). As an example I describe the activity entitled "Equations with Friar Luca Pacioli".

EQUATIONS WITH FRIAR LUCA PACIOLI

[In translating into English I kept the reduction of old words used in the original passage: "co" literally means "thing"; "ce" means "census".]

[The students are requested to read the following historical document.]


fecto del q̄sito. Como si dicesse. Trouame. 1. n°che gionto ui el. $\frac{1}{4}$. del suo q̄drato facia. 3. Poni quel n° essere. 1. co. el suo q̄drato sera. 1. ce. el. $\frac{1}{4}$. fia. $\frac{1}{4}$. ce. gionto a. 1. co. fara. 1. co. p̄. $\frac{1}{4}$. ce. sera eq̄le a. 3. Tu vedi che tu ai manco de. 1. ce. intero: p̄che non vene se non. $\frac{1}{4}$. ce. e pero dico che la reduchi a. 1. ce. intero: cioe parti tutta la equatione p. $\frac{1}{4}$. hauerai. 1. ce. p̄. 4. co. equali a. 12. e mo sequi

"[…] Find for me a number such that, if ¼ of its square is added, makes 3. Let that number be 1 co; its square will be 1 ce. Its ¼ be ¼ ce which added to 1 co will make 1 co p[lus] ¼ ce, it will be equal to 3. You see that you have less than 1 whole ce because it results ¼ ce, but I say that you [can] reduce it to 1 whole ce, that is divide all equation by ¼; you will have 1 ce p 4 co equals 12 [...]"

Luca Pacioli, *Summa*, p. 146.

Let's interpret the document together.

[In the original worksheet, the problem "Find for me a number such that, if ¼ of its square is added, makes 3" is written in contemporary Italian here.]

a) Search the problem inside the original. How is written the word "quadrato" [square]? How is shown the omission of letters?

b) Write the associated equation.

c) Delete denominators [using equations' properties].

d) Use the answers a), b) to interpret the other lines of the document; then, answer the three following questions:

I. What do "co" and "ce" mean?

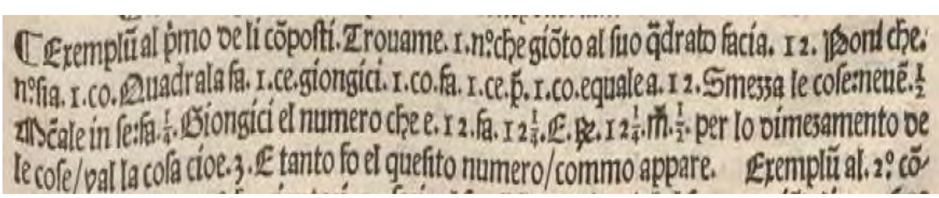
II. Divide the equation you wrote (previous point b) by ¼: what do you get? Check that you are right looking at the original text.

How does Pacioli write the addition sign?

Extra exercises

1. Use the reduced formula to solve the equation you have got from p.146 of the Luca Pacioli's *Summa*.

2. What we name "reduced formula" describes a procedure friar Luca used even when the linear coefficient is not an even number. Analyze the following original [students are requested to use the same formula to interpret the document].



Exemplũ al p̃mo de li cõposti. Trouame .1. n° che giõto al suo q̃drato facia .12. Pon che
n° sia .1. co. Quadrala fa .1. ce. giongici .1. co. fa .1. ce. p̃ .1. co. equale a .12. Smezza le cose: neuẽ .½
Mẽcale in se: fa .¼. Giongici el numero che e .12. fa .12¼. E. R̃. 12¼. m̃. ½. per lo dimezamento de
le cose/ val la cosa cioe .3. E tanto fo el quesito numero/ commo appare. Exemplũ al .2° cõ

[See the English translation in figure 2].

Luca Pacioli, *Summa*, p. 145.

a) Focus on the problem.

b) Write the corresponding equation

c) Solve it using the “reduced formula” (the linear coefficient is equal to 1 and can be written as 2 ½).

d) Compare the reckonings you have made to the part of the document which starts from “Smezza le cose” [halve the things]: first of all, find the manner Pacioli uses to write the square root and the minus sign, then calculate the value of 12 ¼.

e) Take note that Pacioli obtains only one solution: which one doesn’t he consider?

[The correct answer to the e) question is that Pacioli does not consider the value -4 as a solution of the equation. After that answer I propose to students the following historical remark, in order to explain the reason of this fact.]

In millenarian tradition, the solution of equations was based on geometric figures.

Research activity

IS MATHEMATICS THE SAME EVERYWHERE? [This is the main question of the task that is explained in the following two points.]

1) Babylonians, Greeks (Euclid), Indians, Arabs, Europeans (for us, Luca Pacioli): this is a short list of peoples who have made the history of algebra, particularly of quadratic equations. Search further details in the web or in books. Consider that these peoples are from different places so, in this case, mathematics looks the same everywhere.
2) On the contrary, point out the fact that different peoples had their own specific mathematics, with their own connotations.

Keywords for web research

Luca Pacioli, quadratic equations in the history of mathematics [in English also in the original for search on English sites], Babylonian-Arab-Indian mathematics, al-Khwarizmi, completing the square.

Suggested readings

Demattè Adriano, *Vedere la matematica – Noi con la storia*, UNI Service Trento, 2010 (http://www.uni-service.it; some pages in http://books.google.it)

Joseph G. Gheverghese, *C'era una volta un numero*, Il Saggiatore Milano, 2003. [Joseph G. Gheverghese, *The Crest of the Peacock: Non-European Roots of Mathematics*].

Suggested sites

www.e-rara.ch/zut/content/titleinfo/2683230

http://www.matematicamente.it/tesi-didattica/Lungo-Equazioni.pdf

http://www-history.mcs.st-and.ac.uk/HistTopics/Quadratic_etc_equations.html
[In the Publisher’s website, students can find the answers to the previous questions]

Fig. 4: From the textbook (Bergamini & Barozzi, 2014)

The role of questions posed by the teacher

In the *historical laboratory* (figure 4), some questions for students accompany the document to specify with respect to what issues interpretation is required. They are aimed at facilitating the task and they:

- take into account the hypothesized students' prerequisites
- construct prerequisites (like in the case of the original we analyzed at the beginning of my presentation with respect to the other);
- address the main points of the document;
- suggest how to create a bridge between modern solution and ancient solution;
- propose situations so that students could infer by themselves the meaning of specific elements in the document (for example, question d) I.);
- could be used for tests.

Asking questions to help pupils to acquire knowledge reminds us of the Socratic maieutics as a pedagogical method based on the idea that truth is latent in the mind of every human being. I consider it a fundamental part of the deep, ancestral teachers' role. It is a way to act on the *zone of proximal development* of Vygotsky (1978).

An example of asking questions can be found also in (Pinto, 2010) and regards the description of a workshop based on Pedro Nunes' shadow instrument (figure 5). The author asked four questions to participants so that they could "understand and validate the functioning of this instrument":

"1. Show that the triangles [*S'TS*] and [*S'TO*] are congruent and that <*SS'T* = <*OS'T*.

2. Show that <*OS'T* = <*AOX* .

3. Show that the plan *SS'T* is perpendicular to the horizontal plan.

4. Show that the angle that the sun rays make with the horizontal plan is equal to <*AOX*, i.e. equal to the angle marked in the circle by the shadow of the hypotenuse of the triangle."

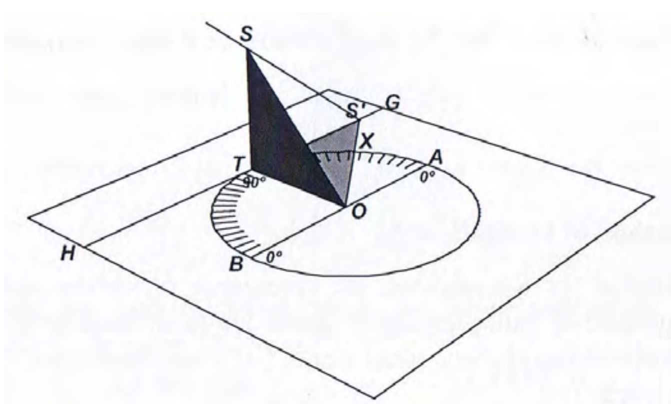


Fig. 5: The shadow instrument

Pinto's workshop, such as my laboratories for students, engages participants in a double level of text analysis: the original and the questions. In my opinion, questions can facilitate the understanding but not remove all obstacles (in case students are not good at reading, for example). Students have the obligation to follow a supplementary reasoning, just the one sketched by questions. It, paradoxically, requires no interpretation. More precisely, the students' attempt to refine their image of the text constituted by questions would introduce an "impossible" task, because they would have to understand the reasoning made by experts in mathematics like the teacher or the writer who prepared the written material.

Jahnke (2014, previous quotation) suggests that there could exist many kinds of reasoning in interpreting a document. In fact, expressions like "certain image of the text", "expectations about what it might be about", "satisfying result" implicitly suggest that, for example, a mathematics historian and a student do not have the same image and expectation about the Pacioli's document they never saw before. In addition, he states that "different readers with their different backgrounds arrive at different interpretations". As a teacher I know that students with low motivation consider satisfying the result that is unsatisfying for other mates; students like those in (Demattè, 2006b) got the same result but paying attention to different steps of reasoning. I have to precise that, in this paper, the term "reasoning" refers to individual processes aimed to acquire mathematical notions, nothing saying about mathematical objects.

In different classes of mine, using different documents accompanied by written questions, it happened that some students required supplementary explanations just regarding those questions. In this case, we can not say that every question facilitated their understanding. Another problem derives from the fact that questions usually regard specific parts of the text but students have to understand the whole meaning of the document, according to the concept of *hermeneutical circle*. In (Demattè, 2006b), I reported a case study of a student (grade 12) who focuses on specific parts of an original regarding al-Khwarizmi's graphical resolution of quadratic equations. This way, he did not get the whole view of the document. I argued that he was worried by reckoning and by dealing with algebraic passages, so that he did not acquire the capability to reason and operate with reference to an aim, that is to establish connections among data into the final geometric figure, in which also the solution is represented. Differently, a female student of the same class operated trying to interpret the figure: she was able to explain it with reference to the meaning of each part and to remedy her reckoning mistakes. In general, I consider that the reference to an aim is a manner to reconstruct a mathematical reasoning in an "almost narrative way", an "elementary" but necessary way to understand, because abstract reasoning is based on it. Aims inside mathematical reasoning suggest a track for identifying the whole meaning of a document (see Solomon & O'Neill, 1998; Thomas, 2002; Zazkis & Liljedahl, 2009).

FINAL REMARKS

In this paper, examples of class experiences as well as proposals of activities are described. I have considered this plenary an opportunity to let know a teacher's point of view. I am aware that not many teachers use history in their classes, but I believe that every teacher could agree about the relevance of educational problems like, for example, the way to involve students or to help them in learning, which have been analysed with reference to the history in mathematics class. With respect to educational research, I hope that the classroom episodes I have described in these pages could be useful for discussing the role of history in mathematics teaching, specifically for discussing what kind of mathematical or interdisciplinary abilities history can contribute to develop. I consider that this is one of the main contributions that teachers who participate in ESU-HPM Group activities can give, according to the goals we can find in the history of the Group written by Fasanelli and Fauvel (2006):

- “To produce materials which can be used by teachers of mathematics to provide perspectives and to further the critical discussion of the teaching of mathematics.
- To facilitate access to materials in the history of mathematics and related areas.
- To promote awareness of the relevance of the history of mathematics for mathematics teaching in mathematicians and teachers”.

REFERENCES

Bagni, G.T. (2008). Historical references and mathematics education: an hermeneutic perspective. In: M.V. Thomase (Ed.), *Science Education in Focus* 291-306. New York: Nova Publishers.

Barbin, E. (2006). The different readings of original sources: an experience in pre-service teaching. Mini-workshop on studying original sources in mathematics education. *Oberwolfach Reports 3*(2), 1301-1303.

Barbin, E., & Tzanakis, C. (2014). History of mathematics and education. In: S. Lerman (Ed.), *Encyclopedia of Mathematics Education*.

Bergamini, M., & Barozzi, G. (2014). *Matematica multimediale.* Bologna: Zanichelli.

Bertagna, G. (2000). Le parole dell’essere: capacità e competenze. Le parole dell’avere: conoscenze e abilità, *Scuola e Didattica,* XLVI, 1 and 2.

Brousseau, G. (1984). The crucial role of the didactical contract in the analysis and construction of situations in teaching and learning mathematics. In H. G. Steiner (Ed.), *Theory of mathematics education, Occasional paper 54* (110-119). Bielefeld: I.D.M.

Demattè, A. (2006a). A questionnaire for discussing the “strong” role of the history of mathematics in the classroom. In: F. Furinghetti, S. Kaijser and C. Tzanakis (Eds.), *Proceedings of HPM2004 & ESU4*, 218-228. Uppsala Universitet.

Demattè, A. (2006b). Historical images, narrative and goals in mathematical thinking. *Proceedings of CIEAEM 58*, 283-288. Plzeň: University of West Bohemia.

Demattè, A. (to appear). Yes, I do use history of mathematics in my class (*workshop*). In *Proceedings of ESU7*.

Demattè, A. & Furinghetti, F. (to appear). History in the mathematics laboratory: an exploratory study. *Education Sciences* (Special Issue 2014).

Fasanelli, F., & Fauvel, J. (2006). The International Study Group on the Relations between the History and Pedagogy of Mathematics: the first twenty-five years, 1976-2000. In F. Furinghetti, S. Kaijser, and C. Tzanakis (Eds.), *Proceedings HPM 2004 & ESU 4* - Revised edition (pp. x-xxviii). Iraklion: University of Crete.

Furinghetti, F., Jahnke, H.N. & van Maanen, J. (2006). Mini-workshop on studying original sources in mathematics education. *Oberwolfach Reports 3*(2), 1285-1318.

Furinghetti, F. (2012). History and epistemology in mathematics education. In: V. Lundsgaard Hansen & J. Gray (Eds.), *History of Mathematics*, in Encyclopedia of Life Support Systems (EOLSS), Developed under the Auspices of the UNESCO, Eolss Publishers, Oxford, UK, [http://www.eolss.net] [Retrieved June 20, 2015].

Glaubitz, M.R. (2011a). The use of original sources in the classroom, in E. Barbin, M. Kronfellner & C. Tzanakis (Eds.), *Proceedings of the 6th European Summer University*, 351-361. Vienna: Verlag Holzhausen GmbH / Holzhausen Publishing Ltd.

Glaubitz, M.R. (2011b). *Mathematikgeschichte lesen und verstehen. Eine theoretische und empirische Vergleichsstudie*. Dissertation, Universität Duisburg-Essen.

Jahnke, H.N. with Arcavi, A., Barbin, E., Bekken, O., Furinghetti, F., El Idrissi, A., da Silva, C.M.S., & Weeks, C. (2000). The use of original sources in the mathematics classroom. In: J. Fauvel and J. Van Maanen (Eds.), *History in mathematics education: The ICMI Study* (Luminy, Marseille, 1998), chapter 9, 291-328. Dordrecht Boston London: Kluwer Academic Publisher.

Jahnke, H.N. (2014). History in mathematics education. A hermeneutic approach. In: M. Fried and T. Dreyfus (Eds.), *Mathematics & mathematics education: searching for common ground*, 75-88. New York: Springer, Advances in Mathematics Education series.

Jankvist, U.T. (2009). On empirical research in the field of using history in mathematics education. *Relime (Revista Latinoamericana de Investigación en Matemática Educativa)*, *12*(1), 67-101.

Jankvist, U.T. (2014). On the use of primary sources in the teaching and learning of mathematics. In: M.R. Matthews (Ed.), *International handbook of research in history, philosophy and science teaching*. Vol. 2, 873-908. Chapter 27. Dordrecht: Springer Berlin Heidelberg New York.

Jankvist, U.T. (2015). Changing students' images of "mathematics as a discipline". *Journal of Mathematical Behavior*, *38*, 41-56.

Katz, V. & Michalowicz, K.D. (Ed.) (2004). *Historical modules for the teaching and learning of mathematics*, The Mathematical Association of America. http://www.maa.org/publications/ebooks/historical-modules-for-the-teaching-and-learning-of-mathematics.

Kjeldsen, T.H. & Jankvist, U.T. (2011). New avenues for history in mathematics education: mathematical competencies and anchoring. *Science & Education*, *20*(9), 831-862.

Laine, A., Näveri, L., Ahtee, M., Hannula, M.S. & Pehkonen, E. (2012). Emotional atmosphere in third-graders' mathematics classroom - an analysis of pupils' drawings. *NOMAD, 17(*3-34), 41-56.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: mathematical knowing and teaching, *American Educational research journal*, *27*(1), 29-63.

Leatham, K. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education*, *9*, 91-102.

Pacioli, L. (1494). *Summa de arithmetica, geometria, proportioni et proportionalita*. Venice: Toscolano.

Pengelley, D. (2011). Teaching with primary historical sources: should it go mainstream? Can it? In: V. Katz and C. Tzanakis (Eds.), *Recent developments on introducing a historical dimension in mathematics education*. 1-8. MAA Notes 78. Washington: The Mathematical Association of America.

Pinto, H. (2010). The history of mathematics in the classroom. Some activities. In: E. Barbin, M. Kronfellner and C. Tzanakis (Eds.), *Proceedings of the 6th European Summer University*, 245-257. Vienna: Verlag Holzhausen GmbH / Holzhausen Publishing Ltd.

Radford, L. (2011). Book review: classroom interaction: why is it good, really? Baruch Schwarz, Tommy Dreyfus and Rina Hershkowitz (Eds.) (2009) Transformation of knowledge through classroom interaction, *Educational Studies in Mathematics, 76*, 101-115.

Rogers, C. (1951). *Client-centered therapy: its current practice, implications and theory.* London: Constable.

Schön, D. (1987). *The reflective practitioner, how professionals think in action*. San Francisco: Jossey-Bass.

Sfard, A. (1991). On the dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin, *Educational Studies in Mathematics*, *22*, 1-36.

Siu, M.K. (2006). No, I don't use history of mathematics in my class. Why? In: F. Furinghetti, S. Kaijser and C. Tzanakis (Eds.), *Proceedings HPM & ESU4* – Revised edition. 268-277. Iraklion, Greece: University of Crete.

Solomon, Y. & O'Neill, J. (1998). Mathematics and narrative, *Language and Education*, vol. 12, n. 3, 210-215.

Thomas, R. (2002). Mathematics and narrative, *The Mathematical Intelligencer*, *24*(3), 43-56.

Vygotsky, L.S. (1978). *Mind in society*. Harvard: University Press.

Zazkis, R. & Liljedahl, P. (2009). *Teaching mathematics as storytelling*. Sense Publishers, Rotterdam/Taipei.