Oral Presentation

PARALLELS BETWEEN PHYLOGENY AND ONTOGENY OF LOGIC

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The principle of a parallel between the ontogenetic and the phylogenetic development of knowledge is a known principle in mathematics education (see Schubring 1978). If teaching omits some of the stages, that were important in the historical development of the particular discipline, it can become an obstacle for students understanding. Logic is in this respect a very special discipline. Its origins are linked to ancient philosophy; during the Middle Ages it became an instrument for theological disputations. It was not until the late 19th century that a specific kind of logic, which we today call mathematical, started to develop. In our research we try to find out whether there are any parallels between the historical development of logic and the spontaneous growth of logical thinking in children.

THE GENETIC PARALLEL

The idea of a parallel between the ontogenetic and the phylogenetic development is established internationally (Schubring 1978) as well as in the Czech didactics of mathematic (Hejný 1984, Hejný & Jirotková 2012). Also P. Erdnijev's words are often cited:

> "The growing of the tree of mathematical knowledge in mind of each person will be successful only if repeats (to a certain extent) history of growing of this science." (Erdnijev 1978, p. 197)

Freudenthal expressed the same idea more precisely:

> "Children should repeat the learning process of mankind, not as it factually took place but rather as if would have done in people in the past had known a bit more of what we know now." (Freudenthal 1991, p. 48)

If the teaching process does not respect some important developmental stages of a particular discipline or topic, students can have problems to understand it. As one example for all, we can take the calculus. The history of this mathematical discipline is linked with Newton's and Leibniz's theory of infinitesimals. But nowadays teaching of derivatives typically starts with the so-called ε-δ calculus, which is mathematically more correct, but much less intuitive (Toeplitz 1949). Many students then have problems to grasp the essence of this topic; they learn to repeat theorems and proofs only in a formal way.

Logic is a very special discipline in this respect. As we indicated in abstract, the historical development of this discipline has crossed the spheres of philosophy, theology and other humanities. And only a short period of its modern development is connected with mathematics. But – contrasting to that – the first (and often the only) logic students meet is the mathematical one; Boole's truth-functions and Frege's quantifiers. These logical lessons take place typically during the first year of higher secondary education.

RESEARCH QUESTIONS AND AIMS

The idea of the genetic parallel seems to be ignored in teaching/learning logic. Should we see this as a problem? What is the level of logical abilities of students leaving lower secondary education? Is there a significant progress in logical abilities during the lower secondary education? And finally: Are there any parallels between the historical development (phylogeny) and the ontogeny of logic? For example, is it possible to say, that a child, that cannot solve a syllogism, will be unable to understand the idea of implication?

These questions and all of them are rather broad and so it is not possible to give precise and clear answers. This is a beginning of our research and we hope to formulate some more specific questions (and answers) in our future research and experimental work.

THE THEORY OF ONTOGENETIC DEVELOPMENT

Why do we think, that we should be able to recognize a development of logical abilities of children during the lower secondary education? One of the reasons is Piaget's theory of stages of cognitive development. According to him, the last stage of cognitive development, which is called "stage of formal operations", starts about the 12th year of age.

> "Subject starts to be able to draw logical consequences from *possible truths*. (...) He/she should be able to use new propositional operations, such as implication (*if then*) disjunction (*either or*)..." (Piaget 1971, p. 98)

The important part is the "possible truths". It means that children in this stage should be able to determine the truth value of statements only on the basis of the logical form of the sentences.

RESEARCH TASK EXAMPLES

We will discuss two concrete tasks and examples of their evaluation from our research. The first task is concerned on the very idea of implication. From the ontogenetic point of view, it is generally known, that children (but not only them) do not distinguish the relation of logical implication from that of equivalence. There are many researches on this topic, e. g., Shapiro & O'Brian (1970), Hoyle & Küchemann (2002). This tendency even received its own label; it is known as "child logic".

Focusing on the phylogenetic approach, implication has been a big issue since ancient times. There were some alternatives to the classical material form of implication, e. g.

Diodoros or Chrissipos attempts (Kneale & Kneale 1962), but all of them used modal logical functors. A few centuries later, C.I. Lewis deals with this topic in the realm of modern logic; his conception of conditionals is widely called “a strict implication” (Lewis & Langford 1932). This concept, just like those of Diodoros and Chrissipos, is situated in the field of intensional logic.

From the practical point of view, a very important issue is the influence of context. In the wider sense – we have no other possibility than to use our natural language to describe the task situation. And there problems could appear caused by the ambiguity of some terms of natural language. And in a more strict sense context is closely connected with the issue of motivation. More information about the role of context in logical tasks can be found in O’Brian, Shapiro & Reali (1971).

The concrete form of our implication-task was inspired by the Wason selection task. It is a very famous task from the area of psychology, when researchers tried to disprove Piaget. Wason selection task was submitted to different groups of adults, but almost in all cases the number of correct answers was only about 10 %. A very interesting study about using this task in the rather specific population of mathematics teachers and students is in Inglis & Simpson (2006).

Let us formulate the first task:

> A brave Prince entered a mysterious castle and after a while he found himself in a special room. There were no windows and the light of a few torches fell upon a large book that lay on a pedestal in the middle of the room. The book was opened on the first page and the prince read:
>
> Brave visitor, the door can hide great danger!
>
> Choose well: **If there is a tiger behind the door, there is the letter T on this door.**
>
> In addition to the door by which the prince entered the room, there were three doors:
>
> (1) door with the letter D,
>
> (2) door with the letter T and
>
> (3) door without any letter.
>
> Make a decision for every door whether:
>
> a) *there is a tiger,*
>
> b) *there is not a tiger,*
>
> c) *we cannot decide, whether there is a tiger.*

To find the correct answer two mental steps are needed. At first – in the first and in the third doors cannot be a tiger, because in that case there was one, there would be a “T” on these doors. From the logical point of view, it is the rule of *modus tollens*.

The second operation is much more difficult. About the second door we cannot decide, because in either case, if there is or is not a tiger behind the door, the rule (tiger → T) is not broken.

The correct answer is 1b-2c-3b. That means: behind the door with the letter D there is not a tiger, about the door with the latter T we cannot decide whether there is a tiger and behind the door without any letter there is not a tiger.

Table of respondents of our research follows:

Grade	5th	6th	7th	8th	9th
Number of respondents	38	47	51	55	37

Table 1: Numbers of respondents

In our research sample only 2 % of respondents chose the correct combination (1b-2c-3b). The semi-correct answer (1b-2b-3b or 1b-2a-3b) chose 20 %. The most common wrong answer was, as you can guess, 1b-2a-3c; chosen by 35 % of respondents.

But this is a very general classification of the answers. We tried to introduce a finer and deeper classification ascribing a score value for every part of all possible answers. This score value expresses the difficulty of the particular answer. In the following table you can see the scores and also the concrete percentage of given parts of the answers. In the table below there are the score values we assigned to these answers.

	Door (1)			Door (2)			Door (3)		
Answer	a)	b)	c)	a)	b)	c)	a)	b)	c)
Percentage	7 %	62 %	29 %	84 %	10 %	5 %	7 %	40 %	50 %
Score value	0	3	1	1	2	20	0	4	1

Table 2: Percentages and scoring values for task no. 1

Let us explain how we ascribed the score values. The simplest is the explanation of the score values 1. For the second and the third door we used this value for answers, which are the most intuitive. In the first door we used this value for the answer 1c, which is logically equivalent with the choice of 3c.

We decided to assign a slightly higher value to the answer 2b. It is still a wrong answer, but the respondent seems to suspects that the implication form cannot be reversed without loss of a generality.

Next higher values – 3 and 4 – we used for the correct answers in the first, respectively in the third door, which are logically equivalent. But the values are not the same, because the answer 1b is much more intuitive than the choice of 3b.

Finally we have to justify the high score value for the answer 2c. As we already mentioned, a very difficult mental operation is needed to do this choice. To determine the concrete value we compute how many "points" we gave in total in the first and in the third door. In both of these doors the total score was about 200 (205 and 210 respectively). To have a similar total score in the second door, we decided to use the value 20.

The zero values were used for answers, which are wrong and contra-intuitive; we are not able to identify the concrete mental operations leading to these answers.

After ascribing score values to each answer we can calculate the table of average scores in each grade. As can be seen from the table below, no significant progress can be seen there.

Grade	5th	6th	7th	8th	9th
Average score	6,03	5,11	7,53	5,85	6,94

Table 3: Average scores for task no. 1

The second task we used was based on the logical form of a syllogism. From the ontogenetic point of view, a syllogism is a very common logical form, but usually we use it implicitly, some of the premises are unspoken. To justify the choice of a syllogism from the phylogenetic point of view, we can, of course, mention Aristotle and his very impressive logical system. But we will cite the research of the Russian psychologist Alexander R. Luria (1976). In his study on the historical development of cognitive processes we read:

> "The emergence of verbal-logical codes allows to abstract the essential symptoms of objects ... leads to the formulation of complex logical apparatus. These logical devices allow getting the conclusions from the premises without immediate clearly achievable (known) reality. They allow acquiring new knowledge discursive, verbal-logical manner." (Luria 1976, p. 116)

To map the historical development of cognitive processes Luria did a research during the 1930-ties in the least developed areas of Soviet Union, on the territory of present-day Kyrgyzstan and Uzbekistan. People living here were usually illiterate, in most cases they have never left their native valley. And Luria gave them tasks, in which different levels of abstraction were needed. Some of these tasks also deal with syllogistic form. For example:

> "Cotton can grow only where it is hot and dry. In England the weather is cold and damp. There can grow cotton?" (Luria 1976, p. 122)

For most of Luria's respondents it was typical that their thinking very strongly bond to their common everyday concrete reality. Using modern terminology, they were unable to make a hypothetical judgment. We can quote one typical answer:

> "I do not know, I was just in Kaschgaria ... if there was a man who was everywhere, well he could have answered to that question." (Luria 1976, p. 122)

We expected similar phenomena when we gave tasks with syllogism to children. The context we were using was familiar to the children, so this kind of argumentation (from experience) was possible. But, of course, we hoped to see an increasing portion of use of logical argumentation. Let us formulate the task:

> In a class, for all boys the following two rules apply:
>
> 1st: Anyone who plays football can run well.
>
> 2nd: Somebody of those who plays hockey plays football too.
>
> Can we surely say that there is a hockey player, who can also run well in this class? Write why.

And there are two answers representing a kind of experience-argumentation. Both of them were written by 5th graders.

> "Yes, hockey player knows how to run, because if he could not, he cannot skate fast."
>
> "No, he is a hockey player, so he may not be able to run well, but he must skate well to be able to play the game."

Even if these two answers are, from the logical point of view opposite – both of them are on the same cognitive level. Let us describe the evaluation process of this task. After repeated reading of all the answers we sorted them to five, respectively six categories. These categories we ordered to according to cognitive performance needed to give this kind of answer and we assigned them corresponding values.

In the 0th category we included the respondents, who didn't give any answer.

The 1st category we used for students, who answered yes or no, but without argumentation.

Next category included the "experience-argumentation".

Category no. 3 we called "stepping out of experience". It is not really a judgment, but some hints can be seen in that direction.

Next – the 4th category – is a logical judgment.

And the last category is an if-judgment. Because the really right answer should include a condition of non-empty set of boys in that class and similarly a condition of non-empty set of football players.

Category	No answer	No argumentation	Experience argumentation	Stepping out of experience	Judgment	If-judgment	Average score inc. 0^{th} cat.	Average score exc. 0^{th} cat.
Value	0	1	2	3	4	5		
5^{th} grade	14 %	8 %	35 %	3 %	43 %	0 %	2,91	2,53
6^{th} grade	38 %	24 %	20 %	2 %	20 %	0 %	2,27	1,45
7^{th} grade	22 %	10 %	33 %	6 %	33 %	0 %	2,75	2,16
8^{th} grade	19 %	11 %	30 %	4 %	39 %	0 %	2,84	2,33
9^{th} grade	19 %	6 %	8 %	8 %	56 %	6 %	2,57	2,89

Table 4: Percentages and average scores for task no. 2

We used two averages in this task. If we want to know which of them fits better the situation, we need to know why some of our respondents didn't give us any answer. This can be because they didn't understand it or because they really didn't know. There many possibilities.

Seeking any trend in this table, we can compare the column titled "Judgment" and the last column, "Average score excluding 0^{th} category". In both of these columns we can see an increasing trend starting in the 6^{th} grade. But our data do not allow us to say whether these trends are statistically significant.

CONCLUSIONS AND FUTURE ORIENTATION OF RESEARCH

Our research, a small part of which was described here, consisted from seven tasks. We tried to map logical abilities of children in the several areas such as classification, negation, syllogism and implication. At first we were a really surprised by the absence of any remarkable trend.

Nowadays we believe that the idea of genetic parallel in logic needs to be grasped in another way. We have at least two two possible explanations:

a) We can find no continous ontogenetic development, because the phylogenetic development wasn't fluent too. After a very fruit-full era of ancient logic there were several centuries, in which the development of logic was incomparably slower. And from the end of 19^{th} century, this development again rapidly accelerated by the onset of modern logic.

b) We cannot rely on the spontaneous ontogenetic development, because the phylogenetic development wasn't spontaneous too. Aristotle's *Organon* emerged from his confrontation with the social problem of sophistic philosophy. The origin of modern logic was stimulated and conditioned by requirements of the development of mathematics.

This indicates that our approach to this topic has to be wider. Now we are studying possible connections between logic and cognitive sciences. We can see some concepts which can be very useful for our effort to describe development of logical thinking in children; e.g. conceptual metaphors and theory of embodied mind, that were introduced by Lakoff & Núñez (2000) in the book "Where the mathematics comes from" or the very impressive theory about cognitive tools, which was worked out for the field of logic by Novaes (2012) in her book "Formal languages in logic – A Philosophical and Cognitive Analysis."

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