
Oral Presentation

TRAINING TEACHER STUDENTS TO USE HISTORY AND EPISTEMOLOGY TOOLS: THEORY AND PRACTICE ON THE BASIS OF EXPERIMENTS CONDUCTED AT MONTPELLIER UNIVERSITY

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The new context of Master's level courses for initial teacher education led in France to the development of history and epistemology courses on a larger scale, in interrelation with didactics of mathematics. We report on this phenomenon and especially on an experiment conducted at Montpellier University that aimed at training teacher students how to use history and epistemology tools. This is overall an opportunity to discuss the interrelations of epistemology and didactics, in the French culture.

I. INTRODUCTION

Due to several recent reforms of the initial teacher education system, the situation with regard to history and epistemology of mathematics for trainee teachers in France changed since the publication of the ICMI study (Fauvel & van Maanen, 2000). Indeed, the reform of 2010 offered a unique opportunity to introduce history and epistemology courses on a larger scale as Master's degrees in teacher education were created in order to raise the academic level of teachers. The national competitive examination (CAPES [1]) that regulates positions in secondary education was postponed to the second year of the Master's degree. As a consequence, a full year was gained in the training of teacher students, which gave time to teach fundamental concepts of didactics, but also history and epistemology of mathematics in more depth, in response to an increased awareness of its importance by the institution. For instance, official guidelines for Grade 10 (Terminale in France), published in 2011, make it clear that:

elements of epistemology and history of mathematics fit in naturally in the implementation of the curriculum. To know the names of a couple of famous mathematicians, the period in which they lived and their contribution to mathematics are an integral part of the cultural baggage of all students taking scientific education. Presentation of historical documents is an aid to the understanding of the genesis and evolution of certain mathematical concepts ([2]).

The purpose of this article is to give an overview of the situation by taking Montpellier University as an example. The author will elaborate on the basis of his experience as a University lecturer in epistemology and didactics of mathematics as well as researcher in mathematics education. In particular, we will analyze teacher training in history and epistemology at Montpellier University using the whys and

hows classifications available in the literature (the ICMI Study, Fauvel & van Maanen 2000; Jankvist 2009).

The core of this paper is an experiment that the author conducted with second year Master students that aimed at teaching teacher students by the practice how to use history and epistemology as a tool in the classroom. The students' project work led to the elaboration of “didactic source material” that, “compared to primary and secondary source material, seem to be the most lacking in the public domain” (ICMI Study p. 212). It is therefore also an interesting question to discuss the methodology to produce this type of resource. What kind of interactions between history, epistemology and didactics of mathematics can we expect and aim for? This is a deep question that is often debated inside the HPM community (Barbin 1997) and certainly cannot lead to a unique and definite answer. We will contribute to the debate by giving a detailed account of the epistemological and methodological domains used for the project work as well as the successes and drawbacks that we met using this approach.

II. HISTORY AND EPISTEMOLOGY AT MONTPELLIER UNIVERSITY: FROM 2010 TO 2014

About the goals and the context of initial teacher training

The many reasons for integrating history of mathematics in mathematics education have been carefully reviewed in the ICMI Study (Fauvel & van Maanen 2000). The reader will find in Annex 1 the result of our attempt to synthesize and arrange the why arguments in a table. Considering that the education community is composed of students, teachers and didacticians (in a simplified model), we also indicated the protagonist who was mainly concerned for each of the argument. Our teacher students are both students (at University) and teachers (in the classroom for their practice work or apprenticeship). They may also adopt the posture of the didactician when they are given the task of elaborating some (simple) didactical engineering. Therefore, most of the arguments presented in Annex 1 may apply to them.

On the occasion of the 2010 reform of the teacher education system in France, local and national committees debated on the goals of initial teacher training in history and epistemology of mathematics. Arguments needed to be formalized in order to obtain from the French Ministry of Education the accreditation of the new Master's track. Several dimensions were thus combined and put forward: an epistemological, historical and cultural approach of scientific knowledge; a didactical approach of the construction of concepts in a teaching and learning environment; minimal knowledge on history of education; a practical and reflexive approach on the way a teacher may introduce a historical perspective in his lessons.

Specification and clarification of competencies with regard to history and epistemology in initial teacher education has been carried out in great detail by a local group of lecturers and researchers in history of science that piloted the new Master's track at University Paris 12 ([3]). We will list these competencies below and

relate them to our typology of whys (between brackets, see also Annex 1; the sign ~ indicates an approximate matching).

- To develop one's scientific culture (~11)
- To understand how scientific knowledge is developed (6)
- To situate one's discipline in a larger context (~4)
- To acquire proficiency in the written language through the reading and studying of ancient scientific texts (5)
- To identify, in their epistemological and historical context, concepts, notions and methods met in the teaching of science at a given level (1)
- To appropriate different didactical options in the integration of a historical perspective in scientific teaching (2)
- To apprehend transverse competencies (reading, argumentation, writing, etc.) that history may work with in scientific education (5)
- To have knowledge on the history of education and the place of scientific education
- To get initiated to the questions and methods of epistemology and history of science as research fields

Initial teacher training at Montpellier University

At Montpellier University, two 50 hours-courses were devoted to history and epistemology of mathematics during the period 2010-13. The first course, a first-year Master's course, focused on tertiary level mathematics (abstract algebra, topology, probability theory, etc.) and aimed at promoting reflexive thinking on mathematical objects and methods (what is a mathematical proof, the problem of definitions and axioms, the axiomatic method, mathematical structuralism, the meta-notions of rigor, evidence, error, etc.). Students were trained to analyze and comment on a corpus of documents including primary sources or essays written by historians or philosophers. The second course was a second-year Master's course directed towards secondary level mathematics with a view to articulating history and epistemology of mathematics with didactics of mathematics. Conceptions that appeared in history were therefore connected to conceptions identified by didacticians in a learning context.

Unfortunately (or not), a second reform that took place in 2013 (after the French Presidential elections) affected the Master's degree. In an attempt to make the teaching career more attractive, the competitive examination (CAPES) was taken back to the first year of the Master, in order to facilitate a progressive entry in the career, no later than 4 years after the French Baccalauréat, with a 9 hours a week of practical training during the second year of the Master's course. As a consequence, teaching time dedicated to history and epistemology as well as fundamentals of didactics had to be diminished, which also resulted in a stronger articulation between these fields.

Nowadays in Montpellier are offered two 50 hours first-year master courses in history-epistemology and didactics of mathematics as combined subjects: a first

course shares time equally between the two and corresponds more or less, as far as history and epistemology is concerned, to the previous 2010-13 course deprived of the discussion of advanced mathematics. A second course reviews the didactics of the main mathematical domains in secondary education (geometry, algebra, analysis, probability and statistics, etc.) with integrated elements of history and epistemology. Finally, a third course (24 hours) during Master 2 is focused on practical issues: learning to use history and epistemology tools in the classroom, elaboration of pedagogical scenarios and their implementation. Although 2013-14 was more of a transition year, it offered an opportunity to experiment such a course. We will report on this experimentation in the sequel.

III. THE STUDENTS' PROJECT WORK: A CASE STUDY

We will now describe in detail the tasks that were assigned to teacher trainees so that they may learn how to use history and epistemology as tools in the classroom. We will analyze a few students' productions and comment on the difficulties that they encountered in the completion of such tasks.

Description of the project work

Teaching design has been chosen as an activity for students to learn by doing how to use history and epistemology tools. This makes sense in our context since second year Master students in 2013-14 had a teaching duty of 6 hours a week, which motivated an emphasis on classroom practices. This work, carried out in groups of 3-4 teacher students, also served as a project-based assessment for the course. There are of course several possible and different ways of integrating history of mathematics in mathematics education. What are the choices made by educators at Montpellier University?

Referring to the classification of 'hows' presented in the ICMI study (Tzanakis & Arcavi pp. 208-213), students were asked to follow a "teaching approach inspired by history" (loc. cit., p. 209). Nevertheless, in comparison to such an approach, less emphasis was made on a "genetic approach to teaching". To rephrase it more properly, our approach may be characterized as:

- a teaching approach supported by an epistemological analysis
- articulating history and epistemology with didactics of mathematics
- using history and epistemology as tools in the classroom
- distilling elements about the nature of mathematics or mathematics as a cultural endeavor
- producing some "didactic source material", that is a "body of literature which is distilled from primary or secondary writings with the eye to an approach (including exposition, tutorial, exercise, etc.) inspired by history" (loc. cit. p. 212).

This approach certainly fits in the French didactical culture, in which design has always been given a central role, through the notion of "didactical engineering"

(Artigue 1992). Epistemology plays an important role, which was again acknowledged by the French school of didactics of mathematics since its foundation (Artigue 1991).

To be precise, the project work was presented to the students in the following programmatic terms:

1. Choose a theme and set up mathematical and didactical goals (parameters: the curriculum, known didactical phenomena, keeping in view the use of history and epistemology tools)
2. Epistemological analysis of mathematical notions
 - 1st goal: identify cognitive roots in history and reflect on it
 - 2nd goal: identify primary or secondary historical sources that may be used in the classroom or identify a crucial epistemological anchor point to work as a lever in the classroom

Method: reviewing the literature in history and epistemology and also in didactics of mathematics

1. Set up epistemological goals (useful to meet didactical goals or as a combined goal: aspects related to the intrinsic/extrinsic nature of mathematical activity, the nature of mathematics/mathematics as a cultural endeavor)
2. Didactical engineering
 - History as a tool: motivation by historical questions and problems, contextualizing as a meaning-providing activity, etc.
 - Or Epistemology as a tool: through the meta lever (see epistemological domain below)

Produce a worksheet + detailed scenario + comments on didactical choices

1. Write down the *a priori* analysis
2. Classroom testing and *a posteriori* analysis (if possible)

In the context of this project work, history and epistemology are used as tools since they contribute to meet mathematical and didactical goals. The reader may also note that both tools are carefully distinguished. To our knowledge, epistemology as a tool in the classroom is given little attention in the literature. This may be due to the fact that elements related to the nature of mathematics are usually seen as history and epistemology as a goal. Moreover, epistemology may be more demanding than history to work out as a tool since it can seldom rely directly on a source document but always involves reflexive and critical thinking.

Epistemological and methodological domains

According to Radford, “the linking of psychological and historico-epistemological phenomenon requires a clear epistemological approach” (ICMI Study p. 162) as well as an adequate methodology for the design of historically or epistemologically based classroom activities. This statement is illustrated by figure 5.1 (loc. cit. p. 144) which certainly deserves to be reproduced here:

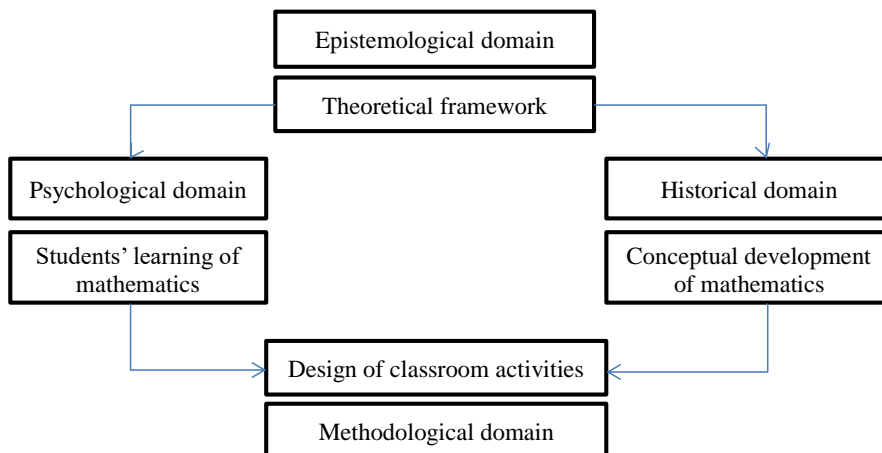


Figure 1: linking of psychological and historico-epistemological domains in the design of classroom activities, according to Radford.

Our epistemological domain may be described by the following formula:
Brousseau TDS + socio-cultural inputs + the meta lever

Brousseau's theory of didactical situations in mathematics (Brousseau 1997) is based on the idea that mathematical knowledge makes sense to the learner whenever it may be perceived as an optimal solution to a given system of constraints, in a problem-solving activity. Brousseau also incorporated in the field of didactics Bachelard's idea of epistemological obstacle. The didactical action is therefore centered on the organization of an adequate student/milieu relationship and the elaboration of "teaching situations built on carefully chosen problems that will challenge the student's previous conceptions and make it possible to overcome the epistemological obstacles, opening new avenues for richer conceptualizations" (ICMI Study, p. 163). It should be pointed out that, in this perspective, the articulation between students' learning and conceptual development of mathematics in history is not recapitulation or parallelism. History plays a role in so far as it may suggest or inspire fundamental situations as well as inform on possible misconceptions and potential steps in the conceptualization process. But any assumption should be confronted with the reality of classroom experimentation, in other words with the didactical phenomena.

In order to ponder Brousseau's paradigm, we also follow Radford and acknowledge the input of a socio-cultural perspective: mathematical knowledge is better understood "in reference to the rationality from which it arises and the way the activities of the individuals are imbricated in their social, historical, material and symbolic dimensions" (loc. cit. p. 164).

In order to make this rationality explicit, teacher students are asked to employ the meta lever (Dorier & al. 2000), that is "the use, in teaching, of information or knowledge *about* mathematics. [...] This information can lead students to reflect, consciously or otherwise, both on their own learning activity in mathematics and the

very nature of mathematics”. Although a large part of the meta is usually taken in charge of by the teacher, we encourage the devolution to the student of some part of this reflection, which may require a piece of didactical engineering (see examples below).

As far as the methodological domain is concerned, the project work, which leads to the production of didactical source material, uses a dedicated resource format, which may be seen as a didactical tool, since it helps students to clarify their thoughts and organize their work, to make explicit the choices that they make so that didactical action may be discussed, in particular the impact of history and epistemology and its functioning as tools. Mathematical, didactical and epistemological goals need to be carefully declared upfront and related to the curriculum, history and epistemology tools have to be described and commented: to which extent does it function as a lever? The text of the activity is complemented by a detailed scenario, and epistemological and didactical analysis are provided as annexes. Our resource format is in fact an adaptation of the SfoDEM resource documentation format (Guin & Trouche 2005), which has been designed for the purpose of collaborative elaboration, pooling and sharing of didactical material within a community of practice consisting of about 300 mathematics teachers in secondary education.

In other words, referring again to the typology of hows given in the ICMI Study and precisely the typology of examples of classroom implementation, our teacher students are building a “historical package” (Fauvel & van Maanen pp. 217-218): “focused on a small topic, with strong ties to the curriculum, suitable for two or three class periods, ready for use in the classroom”; a self-contained package “providing detailed text of activity, historical and didactical background, guidelines for classroom implementation, expected student reactions (based on previous classroom trials)”. In our case, such an extensive documentation is motivated by the development of professional skills but also by pooling and sharing since teacher students will communicate about their group-work during oral presentations in front of the assessors and their peers.

Examples

Several examples have been given to students in order to illustrate a functioning implementation of the historical or epistemological lever. We will present below a teaching sequence engineered by a team of teachers and educators at the IREM of Montpellier (Hausberger 2013, annex 3, pp. 120-158).

This sequence is devoted to a further discussion of the notion of mathematical demonstration, at the entrance of the Lycée (age 15-16). Students have already made acquaintance with standard Euclidean demonstrations during the last two years of Collège. In the sequel, history and epistemology will be called for meaning-producing activities as we tackle the following questions: what constitutes a mathematical demonstration compared to other types of argumentation? Why did mathematicians choose to set up these rules?

During a first activity, students are assigned the following tasks:

- Look up in the dictionary for definitions of the verbs “to show” ([4]) and “to demonstrate” ([5]). Give synonyms for each word. Bring into light the differences to be made between the two.
- For each of the following documents, identify the statement which is asserted and rewrite it if necessary. Is the argumentation of the statement a mathematical demonstration? If so, can you explain the different steps of the reasoning? If it isn't, can you write down a demonstration or demonstrate that the statement is false?

The documents submitted to the students include Lafontaine's poem “the wolf and the lamb” (doc. 1), the values of $n^2 - n + 17$ for $n \in \{0, 1, 2, 3, 4, 11\}$ and the statement “ $n^2 - n + 17$ is prime for any natural number n ” (doc. 2), “let us show that the square of an odd number is also odd” and a proof based on the algebraic development of $(2n + 1)^2$ (doc. 3), the graphical representation of the function $f(x) = 10x^3 + 29x^2 - 41x + 12$ and the statement that “the equation $f(x) = 0$ has two solutions since the curve intersects the x axis in two points” (doc. 4), a puzzle inspired from Chinese mathematics that establishes that a square inscribed in a right triangle of sides a and b (apart from the hypotenuse) has side $c = ab/(a+b)$ (doc. 5), and finally another puzzle by Lewis Carroll that leads to the erroneous conclusion that a square of side 8 and a rectangle of width 5 and length 13 have equal area (doc. 6).

This is an example of epistemology as a tool in the classroom: an implementation of the meta lever, involving reflexive thinking and devolution of meta-discourse to the students. No historical contextualization is given at this stage. Although a few mathematical competencies may be developed through this activity (for instance, refuting a universal statement by providing a counter-example or working out the factorization of a function), the goal is the development of competencies that our IREM team decided to set apart and explicitly describe as epistemological: being able to identify and characterize a mathematical demonstration, to distinguish induction and deduction, to distinguish the truth of a statement and the validity of an argumentation, etc. The role and status of the figure or representation is discussed with the students in the situations of “visual doubt” (doc. 4 to 6: these situations call into question the limits of our perception with the senses) as well as the validity of “cutting and pasting” procedures (doc. 5 and 6: the treatment of mathematical objects as material puzzles leads to an accurate answer in the first case but an erroneous result in the latter).

During a second activity, three historical primary sources are presented to the students: the Problem 41 of the Rhind mathematical papyrus, in which the scribe indicates how to compute the volume of a cylinder-shaped grain silo, the Yale Babylonian tablet 7289, which presents a very interesting approximation of $\sqrt{2}$ in sexagesimal numbers, and finally Euclid's demonstration of the irrationality of $\sqrt{2}$ (which requires quite a sophisticated pedagogical script in order to facilitate the reading and to make the devolution of the Greek context of magnitudes possible). Again, mathematical goals may be pursued, for instance on approximations and

algorithms by asking about the obtainment of the Babylonian value by empirical measurement and the introduction of Heron's (of Alexandria) method, or on logic and mathematical reasoning (implication, contrapositive, reduction to the absurd). Once such mathematical aspects have been worked out, the pedagogical scenario puts forward the following questions for investigation and discussion in the classroom:

- On the basis of these historical documents, what distinguishes Greek mathematics from Egyptian and Babylonian mathematics?
- What might have been, according to you, the reasons that led to the development of such Greek mathematics?

This is an illustration of both history (historical contextualization) and epistemology (reflexive thinking) as a tool, again through the meta lever. Elements about the nature of mathematical objects and mathematical activity as well as mathematics as a cultural endeavor may be addressed, since historians and epistemologists identified both internal and external reasons for the appearance of the mathematical demonstration in the Ancient Greece. This socio-cultural approach makes sense both epistemologically and didactically: indeed, as stated by Balacheff, “knowledge needs to be constituted in veritable theories and be recognized as such, which means accepted by a community that renounces to take anywhere the arguments that it may use. The mathematical demonstration relies on a body of knowledge highly institutionalized, whose validity is socially shared.” (Balacheff 1987, p. 160, our translation; [6]). Accordingly, the criteria for a valid argumentation in mathematics should be submitted to classroom discussion and connected to the practices of mathematicians, which is the very purpose of our teaching sequence.

Students' productions

We will now present and analyze the work of 3 groups of 4 teacher students who were involved in the project, out of a total number of 5 groups. Our main questions are the following: did they manage to implement a functioning lever? What kind of difficulties or pitfalls did they encounter? How does history-epistemology and didactics of mathematics interact in practice in the students' project works?

a) Group 1 decided to elaborate an activity dedicated to the introduction of Pythagoras's theorem at Grade 8 (4^{ème} in France). They used history as a tool and chose to put forward the following historical problem as a motivation: how have Karnak and Luxor temples been constructed, knowing that historians consider that the masonry set square only appeared in the 15th century? The Egyptian 13-knots rope was soon introduced as an historical object, which led the classroom into an experiential mathematical activity dedicated to the construction of right-angled triangles. The pythagorean triple (3,4,5) finally emerged together with a new problem: how to characterize pythagorean triples? In order to introduce the pythagorean relation $3^2 + 4^2 = 5^2$ and interpret it in terms of square areas, the scenario used a mechanical device in plexiglass (which had been manufactured by one of the students!):

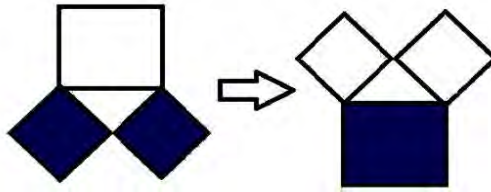


Figure 2: Students' mechanical device for the interpretation of the pythagorean relation

The blue liquid contained in the small squares flows to fill the bigger one when the device is turned upside down. Further investigations of the pythagorean relation were conducted afterward with the help of an interactive geometry software: “does other triangles fulfilling a similar relation seem to be right-angled triangles? What equality seems to exist between the sides of a triangle for it to be right-angled?”. The rest of the activity was devoted to working out a proof of Pythagoras's theorem by means of a contemporary version of the Chinese puzzle. Although it was the key to the justification of the Egyptian procedure, the reciprocal was admitted without proof.

As a conclusion, teacher students did a good job in the implementation of the history lever. Several cognitive representations or procedures that appeal to our senses and participate in the conceptualization of Pythagoras's theorem were introduced either using historical contextualization or an approach inspired by history. A weak point of the activity would be an insufficient epistemological clarification of the idea that the pythagorean relation *characterizes* right-angled triangles, which is visible in the two questions above. The current official guidelines instruct not to distinguish the theorem and its reciprocal, which troubled our teacher students. This is certainly an opportunity to work out an epistemology lever.

b) Group 2 named its project work “trigonometry and triangulation”. The genesis is the result of one student's personal encounter with the method of triangulation and the necessity to relate to the curriculum. They proposed the following situation:

About 600 years BC., Thales finds himself on a boat (point A) and wishes to know the distance to the coast. For that purpose, he sends two observers (B and C) on the (straight) coastline, separated by a known distance $BC=700\text{m}$, and gives them instructions to measure the angles from the coastline to the boat. The purpose is to help Thales compute the distance, the measured angles being 83.8° and 87.7° .

Teacher students described their situation in terms of an “open problem” (adidactical and non-routine task) and gave a decent *a priori* analysis. As far as history is concerned, they argued about motivation by a “historical” problem. It is of course concrete, practical... but is it historical? Contextualization is quite limited. Moreover, the mention of Thales is both historically and didactically misleading: such a question has been raised by Thales but the targeted method of resolution which involves the tangent function has nothing to do with Thales and Thales's theorem.

As a conclusion, this is diagnosed as a non-functioning history lever, both with regard to the articulation with didactics and on a social-cultural perspective. Teacher

students didn't manage to mobilize the results of their research in the history of trigonometry and the triangulation principle in the elaboration of the activity and pedagogical scenario.

c) Group 3 worked on proportionality and the linear model, with a project entitled “inappropriate linear reasonings”. The starting point, once the theme had been chosen in relation to the curriculum, to lesson planning and the possibility of classroom testing, was the review of didactical literature on proportionality. Teacher students therefore got acquainted with the notion of “illusion of linearity” by reading De Bock & al. (2008). They identified the presence of an epistemological obstacle and took note of Aristotle's famous error (speed and mass are proportional) pointed out by Galileo. They decided that Aristotle's error was as a historical situation appropriate for discussing in the classroom the misuse of the “linear model”, together with a geometric situation taken from De Bock & al. (to fertilize a square field of side 200m, the farmer needs 8h. How long will it take to fertilize a field of side length 600m?) which was first presented to the students so that they (or at least some of them) may experience the illusion of linearity.

As we can see, historical-psychological parallelism is pointed out by teacher students but they do not reflect on the goals of historical contextualization in the present context. It could be an opportunity to discuss with their students the place and role of errors, to connect these with the conceptions and methods in the historical context, and state that the linear model is often advocated by application of a principle of simplicity as a heuristic rule (“nature operates in the shortest way possible”).

Among the goals declared by teacher students, the latter mention “to clarify the concept of model: proportionality models a constraint (physical, logical or social) between at least two magnitudes (which makes them dependent) and describes a functional relation between their elements”. Yet, proportionality as a linear model remains an unmet epistemological goal. The epistemological problem which relates to validating/refuting a mathematical model remains implicit in the scenario. Validation criteria for the linear model are not discussed: the teacher is the validator in the geometric situation. The experimental refutation of Aristotle's assertion is also difficult in the classroom without an appropriate protocol, which was not known to our mathematics teacher students who experimented with rulers and rubbers (although it is well known to physics teachers: for instance, take two tennis balls and fill one of the two with sand). The pedagogical scenario therefore uses a “thought experiment” and appeals to students' aptitude to argue that the result is unrealistic. Teacher students were not clear on the point that the underlying epistemology of model validation is that of experimental sciences, not mathematics! Mathematical procedures of validation would be available if tables of values were produced, which was not the case in Aristotle's situation.

As a conclusion, the teacher students' approach was quite interesting but they didn't succeed in working out the history and epistemology lever, due to lack of hindsight, particularly on an epistemological point of view (insufficient understanding of Aristotle's context and lack of expertise to discuss the notion of model, also

interdisciplinarity issues in a context mixing mathematics and physics together with their specific epistemologies).

IV. GENERAL CONCLUSIONS AND PERSPECTIVES

Initial teacher education in France has been considerably impacted since 2010 by two consecutive reforms, which offered an opportunity to improve the training of pre-service teachers in history, epistemology and didactics as interrelated subjects. Courses at Montpellier University were more focused on history and epistemology as a goal during the first year of the Master degree, then as a tool in year 2 in relation to practical training. Nevertheless, the goal/tool distinction shouldn't erase dialectical aspects which were always present.

The methodology used to produce didactic source material has been carried out successfully by teacher students, although it required to search the literature in history and epistemology for connections with the curriculum, in relative autonomy, which was not straightforward. They produced simple engineering or appropriated existing ones themselves. History as a lever has been more successfully implemented than epistemology. An analysis of the didactical material that they produced reveals quite a few epistemological issues, which suggests that epistemology as a tool should deserve further investigation within our community of practice.

Our 20 students were asked after completion of the project work the following question: “what are, according to you, the benefits of an approach that uses history /epistemology as a lever?” Their answers were interpreted and dispatched on our grid of whys (see Annex 1) as follows:

1	Re-contextualization as a meaning-producing activity	16 (students)
2	Historical genesis/ artificial genesis	7
3	Psychological motivation	6
4	Interdisciplinarity	0
5	Linguistic and transverse competencies	0
6	Nature of mathematics	5
7	Obstacles and conceptions	9
8	Illusion of transparency	1
9	Teacher's dogmatism	0
10	Humanization of mathematics and human qualities	2
11	Mathematics as a cultural endeavor	1

Figure 3: Motivations for the integration of history and epistemology of mathematics into classroom teaching, as perceived by the students after the project work

As we can see, re-contextualization as a meaning-producing activity is very well perceived. Whys connected to Brousseau's paradigm (2,7) are also reasonably acknowledged, which isn't surprising (epistemological framework of the project,

French culture of didactics). Accent has not been made on the socio-cultural perspective.

To summarize, our approach is characterized by the intent that history/epistemology should explicitly meet didactical goals. For a happy and fruitful marriage, didactics of mathematics should certainly develop more specific and dedicated tools in order to integrate history and epistemology as full partners.

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NOTES

1. Certificat d'Aptitude au Professorat de l'Education Secondaire.
2. Des éléments d'épistémologie et d'histoire des mathématiques s'insèrent naturellement dans la mise en œuvre du programme. Connaître le nom de quelques mathématiciens célèbres, la période à laquelle ils ont vécu et leur contribution fait partie intégrante du bagage culturel de tout élève ayant une formation scientifique. La présentation de textes historiques aide à comprendre la genèse et l'évolution de certains concepts (Bulletin officiel spécial n° 8 du 13 octobre 2011, p.1 ; http://www.education.gouv.fr/pid25535/bulletin_officiel.html?cid_bo=57529).
3. (1) Développer leur culture scientifique (2) Comprendre comment se construisent les connaissances scientifiques (3) Situer sa discipline d'origine dans un contexte plus large (4) Travailler la maîtrise de la langue écrite au travers de la lecture et de l'étude de textes scientifiques anciens. (5) Identifier dans leur contexte épistémologique et historique les concepts, notions et méthodes rencontrés dans l'enseignement des sciences à un niveau donné. (6) S'approprier différentes pistes didactiques permettant d'inclure une perspective historique dans l'enseignement des sciences (7) Appréhender les compétences transversales que font travailler l'usage de l'histoire dans l'enseignement des sciences (lecture, argumentation, écriture...) (8) Connaître l'histoire du système scolaire et de la place qu'y occupe l'enseignement des sciences (9) S'initier aux méthodes et aux problématiques propres à l'épistémologie et l'histoire des sciences (A. Besnard, A.S. Genin, A. Mayrague, Proposition pour des formations en épistémologie et histoire des disciplines d'enseignement scientifiques et technologiques, dans le cadre des masters et parcours de masters 'enseignement' ; http://f.hypotheses.org/wp-content/blogs.dir/873/files/2014/06/08_10_15_DOCT-7-MAQUETTE-EPISTEMO-HIST-SC.pdf).
4. « montrer » in French.
5. « démontrer »: note that the two verbs differ only by a prefix in the French language, the etymology being very enlightening.
6. Elles [les connaissances mathématiques] doivent être constituées en une véritable théorie et être reconnues comme telle, c'est-à-dire acceptée par une communauté qui ne s'autorise plus à aller chercher où elle veut les arguments qu'elle utilise. La démonstration en mathématiques s'appuie sur un corps de connaissances fortement institutionnalisé, ensemble de définitions, de théorèmes, de règles de déduction, dont la validité est socialement partagée.

Annex 1: why arguments in favor of the integration of history in mathematics education (ICMI Study, Tzanakis & Arcavi, pp. 202-207)

Why argument	Description of the argument	Student Teacher (T) or Didactician (D)	Ref. to ICMI Study
1 Re-contextualization as a meaning-producing activity [didactical transposition leads to dehistoricization]	“The learning of a mathematical concept, structure or idea may gain from acquaintances with the motivations [questions and problems] and the phenomena for which it was created”	S, T	First part of a1
2 Historical genesis/ artificial genesis	“ History suggests possible ways to present the subject in a natural way, by keeping to a minimum logical gaps and ad hoc introduction of concepts, methods or proofs”	T, D [didactical engineering]	Second part of a1
3 Psychological motivation	History as a resource that has the potential to “motivate, interest and engage the learner”	S	Last part of a2
4 Interdisciplinarity	History as a bridge between mathematics and other subjects, to decompartmentalize disciplines and put their interrelations into evidence.	S	a3
5 Linguistic and transverse competencies	“The more general educational values of history (reading, writing, documenting...)”	S	a4
6 Nature of mathematics Implicit assumption: reflections about the nature of mathematics may enhance mathematical literacy	“A more accurate view of mathematics and mathematical activity” that takes into account the role of “mistakes, heuristic arguments, uncertainties, doubts”, etc. i) evolutionary nature of mathematics ii) meta-concepts (proof, rigor, evidence, error, etc.) iii) also reflect on the form (notations, modes of expression and representation, etc.)	S	b)

7	Obstacles and conceptions	Identify epistemological obstacles (Bachelard) and collect conceptions. These may inform on and relate to obstacles and conceptions in the learning process.	D [epistemological obstacles, conceptions], T	c2 i)
8	Illusion of transparency	“Even when a subject may appear simple, it may have been the result of a gradual evolution”.	T, D	c2 ii)
9	Teacher’s dogmatism	“Exercise sensitivity and respect towards non-conventional ways to express and solve problems through the deciphering of correct mathematics whose treatment is not modern”	T	c5
10	Humanization of mathematics and human qualities	“Mathematics is an evolving and human subject, not a system of rigid truths”. To promote (through “role models”) and develop human qualities relevant to support the learning of mathematics (perseverance, creativity, etc.)	S	d
11	Mathematics as a cultural endeavor Underlying paradigm: teaching mathematics is not only teaching mathematical knowledge but also mathematics as a culture	History may highlight the non-utilitarian driving forces of mathematical development such as “aesthetical criteria, intellectual curiosity, challenges and pleasure”, the influence of social and cultural factors, and open up to cultural diversity.	S	e

Figure 4: description of the why arguments (loc. cit.) with an emphasis on the actors (student, teacher or didactician) in the motivating area under consideration