
Oral Presentation

A CONCEPTUAL AND METHODOLOGICAL FRAMEWORK ANCHORED IN SOCIOCULTURAL APPROACHES IN MATHEMATICS EDUCATION FOR THE INVESTIGATION OF *DÉPAYSMENT ÉPISTÉMOLOGIQUE*

David Guillemette
University of Ottawa

Since the mid 90s, the dépaysment épistémologique has been a recurring concept in literature regarding the use of history in mathematics education (Barbin, 1997, 2006; Jahnke et al., 2000). My research project's goal is to describe the dépaysment épistémologique experienced by a group of six pre-service secondary teachers who took part of seven historical texts' reading activities. Data were collected during video recordings, individual interviews and a group interview. I will focus here on how a conceptual framework anchored in sociocultural approaches in maths education (Radford, 2011, 2013), as well as a methodological framework articulated with dialogic perspective (Bakhtin, 1977, 1929/1998), helped me obtain descriptive elements of the dépaysment épistémologique experience.

THE CONTEXT

For decades, many researchers have explored the contributions of history of mathematics in teacher education. In parallel, the presence of mathematics history has established itself considerably in curricula around the world. An attempt to “humanize” mathematics is increasingly present in the curricula of mathematics worldwide (Barbin, 2006; Fasanelli et al., 2000).

In the Quebec province in Canada for instance, the Ministry of Education even now prescribes the use of mathematics history in the classroom. The curriculum (both at primary and secondary level) highlights the importance for students to recognize the contribution of mathematics to science, technology and culture on societies and individuals. Cultural and historical elements form an integral part of the program implementation. This insertion of cultural references in teaching is new and characteristic of this program (Charbonneau, 2006).

These requirements concerning the presence of history in the mathematics classroom, however, raise many questions as teachers and their ability to conduct such activities and mobilize the historical aspects in their teachings. For over 20 years, the presence of mathematics history teachers in training environments has increased substantially in many countries. However, despite the various objectives associated with it, this presence, implicit or explicit, take the form of specific initiatives for each institution. Thus, the objectives and the means employed are not subject to a widely established consensus and the status of history in mathematics teachers' training does not yet seem clearly defined (see Schubring et al., 2000).

From the research side, several discourses emphasize on the positive contribution of the study of the history of mathematics, particularly in teacher education. In this context, a recurring concept is that of *dépaysement épistémologique* (Barbin, 1997, 2006; Jahnke et al, 2000). In this regard, researchers are saying that the history of mathematics “astonishes” and “troubles” our everyday customs on the discipline and highlights its cultural-historical dimension. This important experience of *dépaysement épistémologique* could bring a critical look at the fundamentally social and cultural roots aspect of mathematics.

Overall, this *dépaysement épistémologique* emphasize the historicity of mathematical objects with the astonishment of the learner facing a posture, a framework, a process or a particular argument, far from those of today. In this context, the history of mathematics is a source of encounters whose catalytic effect pushes the learner to question a naïve vision of the discipline and its objects, a vision in which they transcend eras and cultures keeping shape and immutable sense. Introducing the history of mathematics replaces the usual by the different, it makes the familiar unusual. As it occurs when someone is in a foreign context, after an initial phase of confusion, there are recovery attempts, a search and reconstruction of meaning.

These considerations about the *dépaysement épistémologique*, however, have not yet been the subject of systematic researches that truly give voice to the actors in training environments (Guillemette, 2011; Siu, 2007). Thus, I ask for my research two broad questions: “How does this *dépaysement épistémologique* appear and how is it manifested during training activities based on reading historical texts?” and “How does it go with the development of the ‘becoming a teacher’ of students?”.

HISTORY OF MATHEMATICS AND MATHEMATICS TEACHER’S TRAINING

My focus on these questions began with socio-cultural theories. From this perspective, new discourses have recently emerged in favour of the introduction of mathematics history in the teacher training program. From this point of view, mathematics history is a special place where it is possible to overcome the particularity of our own understanding of mathematical objects, which is limited to our own personal experiences and the sociocultural context in which we live this understanding. In other words, history of mathematics “provides tools for dialogue with other understandings [...] with those who preceded us” (Radford, Furinghetti & Katz, 2007, p. 109). It provides opportunities for meetings with ways of doing and being radically different in mathematics, ways that are historically or culturally distant from us. It is important to understand that this perspective is not carrying individuals and personal self-centred and self-sufficient discourse of empowerment and opportunities, but it is carrying the opportunity for students to explore with others new ways of being-in-mathematics and open the space of possibilities for mathematical activities that occur in classrooms.

This discourse on the potential of the history of mathematics is part of a redefinition of the teaching-learning put forward by an emerging theory in mathematics education: the theory of objectification (Radford, 2007, 2011, 2013).

ON THE THEORY OF OBJECTIFICATION

Inspired by Vygotsky's perspective, theory of objectification is a contemporary sociocultural theory of teaching and learning mathematics. It calls for a non-mentalist conception of thought. Both sensitive and historical, thinking is considered here as “a mediated reflection of the world in accordance with the mode of activity of individuals” (Radford, 2011, p. 4, my translation), that is to say, mediated by the bodies, signs, artefacts and cultural meanings.

In this context, mathematical knowledge is perceived as “movement”. Knowledge is abstract and is a “set of ingrown historically and culturally process that is constituted of reflection and action” (Radford, 2013, p. 10). It is constantly changing, constantly moving. It shows itself and makes sense only through men activity, taking inevitably the trace of this cultural and historical activity.

This apparition of knowledge in the activity suggests that it is not owned nor constructed by the learner, but rather frequented. It is then expected that learners “meet” the knowledge in the classroom. Also, as we will discuss, learners can transform this knowledge, and see themselves transformed by it.

On one hand, it is a process of objectification because it is made of acts of meaning that emphasize the appearance of something that revealed itself. On the other hand, it is a process of subjectification because consciousness is also changing during learning. Thus, learning also means becoming, that is to say, the creation of a unique and particular self. They are two inextricable dimensions of learning maths.

In such context, it is now impossible to consider the class as a neutral space in which learners act according to general and invariable mechanisms of adaptation. Indeed, the classroom activity, which is centered on social interaction, does not fulfil an adaptive, facilitative or catalytic function, but is “consubstantial to learning” (Radford, 2011, p. 10, my translation). In other words, learning mathematics is not just learning how “to do mathematics”, but learning ways of “being-in-mathematics”. Mathematical activity, as a cultural form, is a particular way of “being-with-others”.

NEW PERSPECTIVES ON DÉPAYSEMENT ÉPISTÉMOLOGIQUE... AND NEW QUESTIONS

From this perspective, how can we think the *dépaysement épistémologique* (Barbin, 1997, 2006; Jahnke et al., 2000) that is associated with the encounter with the history of mathematics experienced by students during their initial pre-service teacher program? The theory of objectification probably sees this encounter as an eminently social phenomenon encouraging people to take a critical look at the social aspect of mathematics to better understand the historical and cultural mechanisms of their production, to understand that mathematics are not ideologically neutral knowledge, and that all knowledge is part of ethical issues for which we need to develop our sensibility.

And what about the sphere of being, ethics and otherness here? What can mean this meeting with the history of mathematics for future math teacher? How the study of the history of mathematics and the encounter that it raises can make sense for

students that are becoming teachers? And most importantly, how to give the students a voice about this experience?

Armed with a conceptual framework from the theory of objectification, while being inhabited by these epistemological questions, my research objective is to describe the experience of *dépaysement épistémologique* lived by future high school mathematics teachers in the context of training activities that involved mathematics history.

APPROACH(ES)

The need to clarify the meaning of a particular experience for the learners and the focus on the lived experience of the individual in the description of the phenomenon led me to choose a phenomenological approach. The phenomenological approach in human sciences has been developed particularly in psychology (Giorgi, 1975, 1997) and education (van Manen, 1989, 1994). It obtains, from individual testimonies, specific descriptions of the participants' experience. Descriptive and comprehensive, the phenomenological approach focuses on "the experience of the individual and his subjective experience" (Anadón, 2006, p. 19). It highlights the significant elements of the internal living world. In addition, it brings the researcher into a welcoming attitude and openness towards participants' lived experience, searching to avoid a reified, reducing or sterile description of *dépaysement épistémologique*.

That being said, the socio-cultural perspective that carries the theory of objectification on the *dépaysement épistémologique* invites me to question the kind of description that I'm searching for and the way to construct such a description. Indeed, the sociocultural perspective implies a particular view of knowledge, learning and the self. For instance, through different authors such as Bakhtin, Levinas and Heidegger, the theory of objectification emphasizes on the possibility of a divided and multiplied self. Historical, we are thrown into a world that asks us answers. As it has been discussed, learning mathematics, as a process of objectification and subjectification, is inevitably learning-with-others and implies the development of an ethical subject.

To provide a description consistent with this view, I settled up various ways "to mesh" the participants' views and to recognize the common living world that emerged from their experiences. Without rejecting the phenomenological approach that seems, at first glance, focusing on the individual and his subjective experience, I was searching for ways to articulate it to the conceptual framework. This articulation appeared to be possible through the development of a particular form for the general description of the phenomenon proposed in this study.

In other words, I was searching for a description of *dépaysement épistémologique* that tries to maintain the plurality of discourses and emphasizes on their "permeability", how they respond to each other and let them being transformed by the others. The phenomenological approach leads to a general description of the phenomenon from specific descriptions obtained by the testimony of each participant. In fact, how can we get an overview from specific descriptions? How can we avoid the simple observation of redundancy, as if, by accumulation, a general and final description could appear? This could reduce participants to simple exemplarities, culminating in statements such as; "This one, he lived it like that", "that one otherwise", "that one

stands out by this”, etc. Inhabited by the epistemological assumptions underlying the theory of objectification, and the prospect of “being-in-mathematics” questioned earlier, I felt the need to look for the multiplicity of students’ experiences. A multiplicity that does not seek to present side by side, in rows, the experiences of each participant, but to truly provide the “common world” that emerged during the trials that took place.

LOOKING FOR COMMUNE LIVED EXPERIENCES: HELP FROM LITERARY CRITICISM

Mikhaïl Bakhtin, one of the main references in the theory of objectivation, rightly said that any movement of consciousness is itself dialogical, that is to say, penetrated by those of others, and therefore, cannot be discussed without taking into account other movements of consciousness that respond to it, and make it respond. Discourses, intimately related to consciousness here, are then perceived as “dialogic”. Indeed, this dialogism “goes far beyond the relationship between the built replicas of a formal dialogue [...] it is universal and goes across all human speech, in general everything that has meaning and value” (Bakhtin, 1929/1998, p. 77). Very broadly, we can speak of dialogues both in language and in terms of ideas and social horizons.

Going further on dialogism, Bakhtin developed the concept of the “polyphonic narrative” (*ibid.*). A scientific, literary or philosophical work can be called “polyphonic” if it offers a strong plurality of discourses and understandings of the world. Bakhtin profoundly highlights an example: the novel *The Brothers Karamazov* written by Fyodor Dostoyevsky. The novel is considered emblematic of the polyphonic work. Dostoyevsky portrayed here many characters inhabited by singular personalities that take finely established roles (the bourgeois, liberal atheist, scientist, etc.). They are characters acting as “spokespersons of world views” (Sabo & Nielsen, 1984, p. 80) that are constantly in dialogue. These strong individual speeches, which escaped the author “control” through the narration, highlight the existential, ideological and socio-historical thickness of the reality. For Bakhtin, it is this polyphonic aspect of the novel that allows the readers to account for the reality of Dostoyevsky, in this case Russia after the 1860 reforms.

In a polyphonic work, “the hero and the author jointly express [...] the speech works openly, despite having two faces, like Janus” (Bakhtin, 1977, p. 198). I am myself as a writer/researcher inevitably involved in this web of meaning that bind all the “actors” of the events of the *dépaysement épistémologique*. Therefore, it is important, as Bakhtin points out, to join “the accents of the heroes (participants) and those of the author (me as a researcher) within a single linguistic construction” (*id.*, 214).

For my study, polyphonic narrative appeared as a way to stage this world in common that emerged with the participants. Then, it will be possible to bring the “knowing-with-others” that emerged, the collective experience, the fabric of shared meaning on the study of the history of mathematics.

Searching for ways-of-being-in-mathematics as it claimed by the theory of objectivation, my research inscribes itself profoundly in sociocultural approaches in mathematics education. With the constitution of a polyphonic narrative as a

methodological strategy to grasp the world-in-common that has arisen during these experiments, it goes deeper in research itself and looks for consistency and coherence.

“CONTEXTS” AND “DATA”

The participants’ selection was conducted among those registered in the *History of Mathematics* course offered in the secondary school mathematics teachers program at the University of Quebec in Montreal. During winter 2013, I stepped in the classroom activities by providing seven reading activities (90 minutes each) of historical texts. Those texts were constituted of the writings of mathematicians associated with different eras discussed in class:

1. A'hmosè: Rhind papyrus, problem 24.
2. Euclid: Elements, proposition 14, book 2.
3. Archimedes: The quadrature of the parabola.
4. Al-Khwarizmi: The Compendious Book on Calculation by Completion and Balancing, types 4 and 5.
5. Chuquet: *Tripartys en sciences des nombres*, problem 166.
6. Roberval: *Observations sur la composition des mouvements et sur le moyen de trouver les touchantes des lignes courbes*, problem 1.
7. Fermat: *Méthode pour la recherche du minimum et du maximum*, problems 1-5.

These classical texts were read in small groups (2 or 3 students). Both synchronic and diachronic lectures (Fried, 2008) were performed. Trying, first, to understand the mathematics involved and to bring it to a modern understanding, and, second, to read the text with the worry to keep the author in his historical, social and mathematical background.

For Fried, teachers and mathematicians too often reinforce the synchronic reading of mathematical objects. In this context, the role of the teacher should precisely be to constantly switch the learner between these two visions. It is this back-and-forth work that is continuously needed and that is creating the emergence of an awareness of its own conceptions of mathematics in the learner, its individuality toward the subject and the possibility for him to confront constructively with those of others. These considerations were taken into account during the reading activities implementation, continuously trying to, not only translate the texts in modern language, but also stay with the author in his historical and mathematical background.

Six students in the group were recruited to participate in individual in-depth interviews (approximately 90 minutes) and a group interview at the end of the study session. Video recordings of classroom activities and transcripts of interviews constituted the data of my study.

Individual interviews focused on three topics: their overall experience of the course, their experience of readings historical texts and, specifically, their experiences of cultural and epistemological *dépaysement épistémologique*.

The same set of themes was taken for discussion during the group interview. This time, the goal was to encourage participants to share their experiences. Therefore, the point was not necessarily to seek consensus, but rather to refine their description of their experiences through listening to those of others. Participants were asked to respond to the comments of their colleagues in order to possibly recognize themselves or to assert their differences.

ANALYSIS (S)

Phases of analysis given here are seen as steps in writing. These analyses have allowed the collect of notes for the construction of the polyphonic narrative.

Video recordings show how activities affect learners. Students in learning situations do not know in advance how to guide their quest for knowledge. In this sense, the reading of historical texts “affects” students, and can leave them with frustration and both positive and negative emotions, because students “suffer” the objects of knowledge (Roth, 2011). Video recordings yield descriptive elements of the encounter with the history of mathematics. It could be gestures, reactions or particular expressions that emerged during the reading of historical texts. In addition, having fully participated in the readings activities as an animator, I do not exclude myself from the descriptions.

Concerning the analysis of individual interviews, they explicitly give voice to the study’s participants. The goal here is to get closer to the participants, to go meet “them”. Analyses of written transcripts of individual interviews were done in two steps: the extraction of meaning units and the construction of the specific descriptions. Concerning the extraction of meaning units, most phenomenologist researchers generally include four phases (Deschamps, 1993). (1) Making a general sense of the entire description of the phenomenon. (2) Identify the meaning units that emerge from the description. (3) Exploring the meaning of these units by assigning a specific category. (4) Establish the phenomenological experiences associated with meaning units. Thereafter, a summary text will be produced for each participant. This summary is called the specific description.

These phenomenological analyses recognize more accurately the experience of each participant of the study. In this particular phase of analysis, I tried to trace the process of subjectification associated with the activities of reading historical texts. As noted above, the conscience is also changing during the learning process. Learning means to frequent knowledge, but also means “becoming”. This is what phenomenological analysis is pointing on.

This phenomenological approach seems appropriate here, despite the distance between the perception of the subject (including consciousness) in the phenomenological perspective and in the theory of objectification. It is not a matter of establishing facts, but to investigate the participants’ experiences. I borrow to

phenomenology a method, an approach, a style of analysis, but I also borrow its openness, its a-theoretical mind, and the need to leave in indecision as long as possible the establishment of the significance of the participants' experience. It is this attitude that allows to perceive the participants, not as thought by science, but as subjects received throughout the concreteness of their experiences, with all the texture, nuance and density that is implied.

TOWARDS A POLYPHONIC NARRATIVE

The transcription of the group interview forms the basis for the final description that takes the form of a polyphonic narrative. This narrative will derive its density of two previous phases of analysis. The narrative/description allows me to bring out tensions between points of view on *dépaysement épistémologique*, which overlap and influence each other, creating a sort of siphonophore, both singular and plural. Unlike the positivist position that tries to eliminate alternative discourses on the phenomenon and the subjective position of the researcher, my study rather seeks to integrate them. This narrative will be the "results" of the study. It is a way to provide the community with a rich and open description of the *dépaysement épistémologique* that occurs during the study of the history of mathematics in the context of pre-service teacher training program, a description that is consistent with the underlying epistemological theory of objectification posture.

In this perspective, my research is asking theses questions: how to stay here "on the wire" and keep a form of dialogue between individual and community, between isolated subject and multiplied subject, between singular and shared learning of participants, between inner space and group activities...?

SOME "RESULTS" FROM THE PHENOMENOLOGICAL ANALYSIS

When adopting a phenomenological stance, major themes emerge from the analysis. Two of them are the experience of *otherness* and *empathy*.

Students are saying that they are trying very hard to understand the mathematic depicted in original texts. They show great difficulties concerning language, notation, implicit argument, style, definitions, interpretations, typography, etc. Literally, they "suffer the texts". The experience of *otherness* seems brutal, from a cognitive and affective point of view, it sometimes includes shocks and violence.

From Levinas, I learned that violence is a "thematization of the Other", a reification of the Other, a way to make the Other a Mine, and that to understand something is to control it, make violence at it. I saw a few acts of violence during my experimentation, for instance, someone said: "Fermat was doing this or that".

That's why *otherness* is linked with *empathy*. Again with Levinas, and also with Bakhtin, *empathy* could be heard as an effort of a non-violent relation with the Other, in this case, a way of keeping alive the subjectivity of the authors, keeping it fragile and mysterious. The question is how to accompany the students in this ordeal, in this hardship of the experience of *otherness*? How to maintain an *empathic* relation with the authors?

REFERENCES

- Anadón, M. (2006). La recherche dite « qualitative »: de la dynamique de son évolution aux acquis indéniables et aux questionnements présents. *Recherches qualitatives*, 26(1), 5-31.
- Bakhtin, M. (1977). *Le marxisme et la philosophie du langage*. Paris: Minuit.
- Bakhtin, M. (1998). *La poétique de Dostoïevski*. Paris: Seuil. (Original work published 1929)
- Barbin, E. (1997). Histoire et enseignement des mathématiques: Pourquoi? Comment? *Bulletin de l'Association mathématique du Québec*, 37(1), 20-25.
- Barbin, E. (2006). Apport de l'histoire des mathématiques et de l'histoire des sciences dans l'enseignement. *Tréma*, 26(1), 20-28.
- Charbonneau, L. (2006). Histoire des mathématiques et les nouveaux programmes au Québec: un défi de taille. In: N. Bednarz & C. Mary (Eds.), *Actes du colloque de l'Espace mathématiques francophone 2006* (pp. 11-21). Sherbrooke: Éditions du CRP et Faculté d'éducation de l'Université de Sherbrooke.
- Deschamps, C. (1993). *L'approche phénoménologique en recherche*. Montréal: Guérin.
- Fasanelli, F., Arcavi, A., Bekken, O., Carvalho e Silva, J., Daniel, C., Furinghetti, F. & Zhang, D.Z. (2000). The political context. In: J. Fauvel & J. van Maanen (Eds.), *History in mathematics education: the ICMI study* (pp. 1-38). Dordrecht: Kluwer Academic Publishers.
- Fried, M.N. (2008b). History of mathematics in mathematics education: a Saussurean Perspective. *The Montana Mathematics Enthusiast*, 5(2), 185-198.
- Giorgi, A. (Dir.) (1975). *Phenomenology and psychological research*. Pittsburgh: Duquesne University Press.
- Giorgi, A. (1997). De la méthode phénoménologique utilisée comme mode de recherche qualitative en sciences humaines: théorie, pratique et évaluation. In: J. Poupard, J.-P. Deslauriers, L.-H. Groulx, A. Laperrière, R. Mayer & A.-P. Pires (Eds.), *La recherche qualitative: enjeux épistémologiques et méthodologiques* (pp. 341-364). Montréal: Gaëtan Morin.
- Guillemette, D. (2011). L'histoire dans l'enseignement des mathématiques: sur la méthodologie de recherche. *Petit x*, 86(1), 5-26.
- Jahnke, H.N., et al. (2000). The use of original sources in the mathematics classroom. Dans J. Fauvel & J. van Maanen (Eds.), *History in mathematics education: The ICMI Study* (pp. 291-328). Dordrecht: Kluwer Academic Publishers.
- Radford, L. (2007). Towards a cultural theory of learning. In: D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of the fifth congress of the european society for research in mathematics education (CERME 5)* (pp. 1782-1797). Larcana, Chypre: CERME.

- Radford, L. (2011). Vers une théorie socioculturelle de l'enseignement-apprentissage: la théorie de l'Objectivation. *Éléments*, 1, 1-27.
- Radford, L. (2013). Three key concepts of the theory of objectification: knowledge, knowing, and learning. *Journal of Research in Mathematics Education*, 2(1), 7-44.
- Radford, L., Furinghetti, F., & Katz, V. (2007). Introduction: the topos of meaning or the encounter between past and present. *Educational Studies in Mathematics*, 66(1), 107-110.
- Roth, W.-M. (2011). *Passibility and the limits of constructivist metaphor*. New York: Springer.
- Sabo, K. & Nielsen, G.M. (1984). Critique dialogique et postmodernisme. *Études françaises*, 20(1), 74-86.
- Schubring, G., Cousquer, E., Fung, C.-I., El Idrissi, A., Gispert, H., Heiede, T. & Weeks, C. (2000). History of mathematics for trainee teachers. In: J. Fauvel & J. van Maanen (Eds.), *History in mathematics education: The ICMI Study* (pp. 91-142). Dordrecht : Kluwer Academic Publishers.
- Siu, M.-K. (2007). No, I don't use history of mathematics in my class: Why? In: Fulvia Furinghetti, S. Kaijser & C. Tzanakis (Eds.), *Proceedings HPM 2004 & ESU 4 (revised edition)* (pp. 368-382). Uppsala: Université d'Uppsala.
- Van Manen, M. (1989). Pedagogical text as method: phenomenological research as writing. *Saybrook Review*, 7(2), 23-45.
- Van Manen, M. (1994). *"Doing" phenomenological research and writing: An introduction*. Edmonton: University of Alberta.