Oral Presentation

ANCESTRAL CHINESE METHOD FOR SOLVING LINEAR SYSTEMS OF EQUATIONS SEEN BY A TEN YEARS OLD PORTUGUESE CHILD [1]

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Inspired by the simplicity of the fangcheng method, for solving linear systems of equations, presented on the Chinese ancient mathematics book "The Nine Chapters on the Mathematical Art", we decided to test the viability of a Portuguese ten years old child understand, reproduce and apply the method. We created a task, transposing didactically that method, to be presented to the child. By research design, the child solved the task in parts, outside the classroom and without relation with his classes. The analysis of the collected data allows us to affirm that this young child was able to use and apply this method in an a-didactical situation.

INTRODUTION

Mathematics has developed with humanity throughout the ages and in different spaces. It can be seen that within all civilizations have emerged manifestations of mathematical nature. Looking to the framework that exists today about the mathematics developed by some of the ancient civilizations is possible to infer about the mathematical knowledge of ancient civilizations of Mesopotamia, India, Egypt and China (Katz, 1998/2010, p. 4).

The ancient Chinese mathematics can be understood as an independent mathematics that was configured as a distinct area of knowledge and which was perpetuated over generations through writing (Martzloff, 1987/1997, pp. 3-13). This mathematics has remained virtually unknown to the Western world until the second half of the nineteenth century.

Swetz (1988, p.8; 1994, p.2) argues that the history of mathematics in general, and the history of ancient Chinese mathematics in particular, can provide many fruitful and challenging problems from the pedagogical point of view.

The use of history of mathematics as educational resource has been the focus of several academic studies in recent times. The available literature presents several favorable arguments to the integration of history of mathematics in mathematics education, arguing that the teaching of mathematics can be enriched through this integration (Jankvist, 2009a). Tzanakis et al. (2002) point out that these improvements can be significant in the learning of mathematics, in the development of views on the nature of mathematics, in the affinity of students with mathematics and in the social and cultural aspects of mathematics. The history of mathematics can be seen as a storehouse of issues, situations, problems and examples that may contribute to the diversification of didactic resources and consequently student engagement.

Despite favorable arguments to integrate the history of mathematics, there are few empirical studies on this true integration into mathematic teaching (Jankvist, 2009b, 2011).

Focusing on ancient Chinese mathematics, the opinion of Leng (2006) is consistent with the previous argument, when he says that the ancient Chinese mathematics has been the focus of many studies in a historical perspective, but a little has been done to investigate the role that mathematics could have on teaching and learning mathematics.

The *fangcheng*, an example of ancient Chinese mathematical method, is presented in the 8th chapter of *The Nine Chapters on the Mathematical Art* and is used to solve problems which now could be associated to a linear system of equations. It uses tables of numbers and elementary arithmetic operations between the numbers in the columns of these tables.

In his book, Martzloff (1987/1997, pp. 249-258) presented a description of this method. He starts with a problem, presented in *The Nine Chapters on the Mathematical Art,* which could be associated to the linear system of equations below

$$\begin{cases} 3x+2y+z=39 \\ 2x+3y+z=34 \\ x+2y+3z=26 \end{cases}.$$

Problem solving using the *fangcheng* method involves the distribution of the numbers that arise in the problem by columns. After identifying the first condition of the problem, the distribution of these numbers is made in the rightmost column. Then, it’s made the distribution of the numbers of second condition in the left column of the first and so on. In ancient China it was used counting rods (Martzloff, 1987/1997, p.210) in the resolution of such problems. The original representation of the problem would be as shown in fig.1:

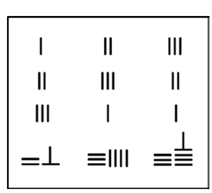

Fig. 1 – Representation of the problem using counting rods (*in* Martzloff, 1987/1997, p. 253)

The application of the *fangcheng* method produces the following sequence of tables using Arabic numeration (see Fig.2), resulting from elementary arithmetic operations on the columns, and allows us to find the solution of the problem.

1	2	3
2	3	2
3	1	1
26	34	39

1	6	3
2	9	2
3	3	1
26	102	39

1	3	3
2	7	2
3	2	1
26	63	39

3	0	3
6	5	2
9	1	1
78	24	39

0	0	3
4	5	2
8	1	1
39	24	39

0	0	3
20	5	2
40	1	1
195	24	39

0	0	3
15	5	2
39	1	1
171	24	39

0	0	3
10	5	2
38	1	1
147	24	39

0	0	3
5	5	2
37	1	1
123	24	39

0	0	3
0	5	2
36	1	1
99	24	39

Fig. 2 – Application of the *fangcheng* method to the previous system (*in* Martzloff, 1987/1997, p. 254)

In Portugal, linear systems of equations play an important role in the articulation of concepts from the domains of algebra and geometry. In Mathematics for Basic Education the content linear systems with two equations and two unknowns is taught to students with about thirteen, in 8^{th} grade (Ministério da Educação e Ciência [MEC], 2013). This topic also arises in Mathematics for Secondary Education in the 11^{th} grade, (Ministério da Educação - Departamento do Ensino Secundário [ME-DES], 2002) taught to students with about sixteen.

The method of Gaussian elimination for solving linear systems of equations is usually taught to students from eighteen, in Higher Education, in the first year of many courses.

We believe that *fangcheng* method is simplest than Gaussian elimination method because, as far as we know, that method doesn't uses algebra symbolism. Unlike this, it uses the context of the problem and arithmetic relations. Comparing the simplicity of *fangcheng* method with the Gauss elimination method we wonder if this method could be used, with advantages, by younger students. We ask ourselves if it is possible that a ten years child understands and appropriates the *fangcheng* method to solve linear systems of equations.

METHODOLOGY

To understand how a young child would react to a task on the *fangcheng* method, we designed an exploratory case study (Yin, 2009). We made the didactic transposition

(Brosseau, 1986) of this method and we created a task partitioned in three parts, with increasing level of complexity and abstraction (Stein, 1998). We emphasize that to use the history of mathematics in teaching and learning is fundamental to adjust it in order to be understood by the students. We had this fact in mind. We decided to apply it to a young child in three different moments. At the 1st and 2nd parts of the task, one of the researchers acted like mediator/teacher and interacted with the child urging dialogues where the child was able to verbalize his thoughts and the procedures used in solving the problems proposed. In the 3rd part of the task problems were provide through a website created for the purpose, and the child solved them autonomously, without any intervention of the researchers and, from the point of view of the child, the situation was not connected with the previous task.

The child, Gabriel [2], was picked occasionally. He is known by researchers, had 10 years old and he had just finished the primary school, had good marks and talent to mathematics. He didn't know yet unknowns/variables or equations and systems of equations much less.

We did the video recording of the application of the task and all productions of Gabriel were filed. After the implementation of the three parts of the task we interviewed the boy.

THE *FANGCHENG* TASK: PRESENTATION AND DISCUSSION

We start presenting the task to Gabriel on the 28th October of 2013 and we completed de implementation of the task on 10th December of 2013.

First part of the task

The first part of the task was presented and discussed in *Potentialities on the Western Education of the Ancient Chinese Method to Solve Linear Systems of Equations* (Costa, Alves & Guerra, 2014). It is synthetized in this subsection.

The first problem (see Fig.3) was designed to be the motivation for introducing the ancient Chinese method.

Gabriel didn’t know linear systems, as we said before. He managed to find a solution by using other strategies. Nevertheless he showed some insecurity in solving the problem. After this, one of us presented the *fangcheng* method to Gabriel.

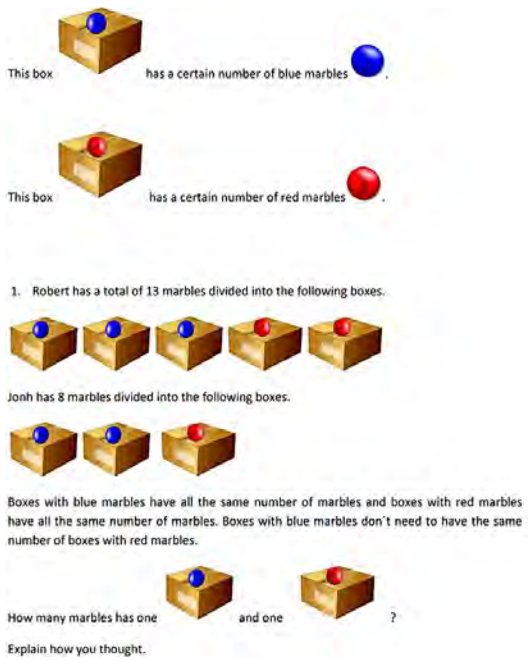

Fig. 3 - Statement of the first already solved problem presented to Gabriel that could be associated with the resolution of a linear system with two equations and two unknowns (translated to English) (Costa et al, 2014)

During this presentation of *fangcheng* method to Gabriel, we considered more appropriate didactically maintain the orientation of writing with which he was accustomed (from left to right), we used only non-negative integers and we illustrate the problem data with pictures alluding to the statement. After this initial motivation, we presented a new (but similar) problem (see Fig.4) to Gabriel and suggested him to solve it.

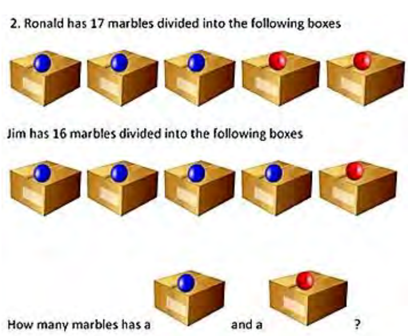

Fig. 4 - Statement of the second problem presented to Gabriel, similar to the 1st one, which could be associated with the resolution of a linear system with two equations and two unknowns (Costa et al, 2014)

Gabriel's resolution is presented in Fig. 5.

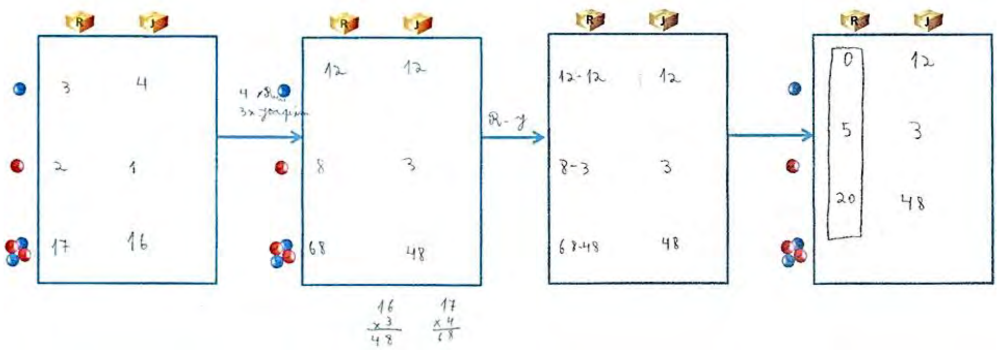

Fig. 5 – Gabriel´s resolution of the problem presented on Fig.3 (Costa et al, 2014)

After presenting these tables (see Fig.5), Gabriel wrote (in Portuguese):

5 boxes of red marbles = 20 marbles;

Red box has 20:5 = 4 marbles;

12 blue boxes + 3 red boxes = 48;

12 blue boxes + 12 red marbles = 48;

12 blue boxes = 36 marbles;

Each blue box has 3 marbles

Following we made it more difficult. This time we presented a problem with three unknowns and, following the same strategy, we present a resolution of this problem using the same method (see Fig.6).

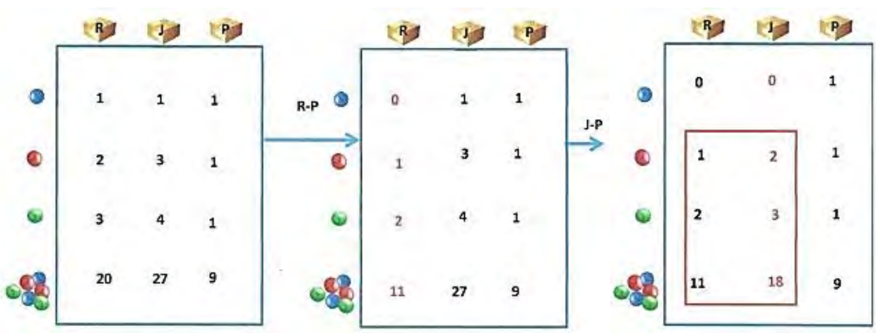

Fig. 6 - Resolution of a part of the problem that could be associated with the resolution of a linear system with three equations and three unknowns, presented to Gabriel (Costa et al, 2014)

After this, we suggest to Gabriel to solve a problem (see Fig.7), also with three unknowns.

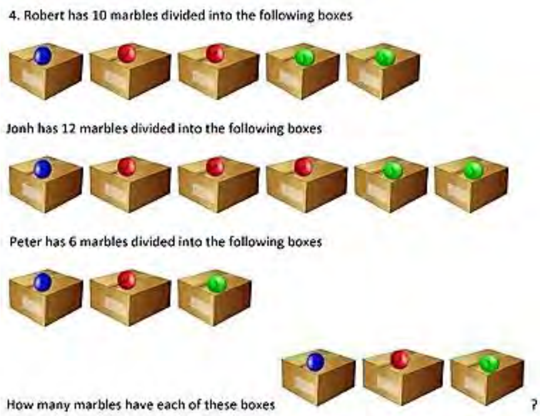

Fig. 7 - Statement of the 3rd problem presented to Gabriel, which could be associated with the resolution of a linear system with three equations and three unknowns (Costa et al, 2014)

Fig.8 shows the resolution made by Gabriel.

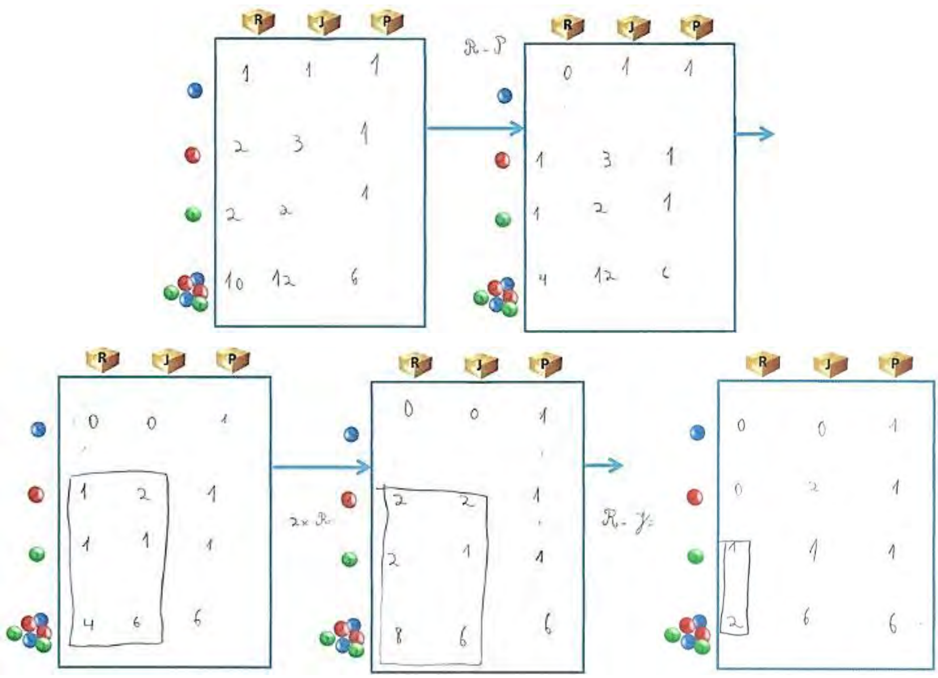

Fig. 8 – Gabriel´s resolution of the problem presented in Fig.6 (Costa et al, 2014)

After presenting these tables (see Fig.8), Gabriel wrote (in Portuguese):

1 box of green marbles = 2 green marbles;

2 red boxes + 1 green boxes = 6;

2 red boxes + 2 green marbles = 6;

2 red boxes = 4;

1 red boxes = 4:2 = 2 red marbles;

1 blue box + 1 red box + 1 green box = 6;

1 blue box= 2 blue marbles.

Second part of the task

The second part of the task was applied on 5th of December of 2013. In this second part we proposed to the child more formal statements (see Fig. 9), without pictures and therefore demanding higher degree of abstraction; however the registration tables were kept in the statement. Some images alluding to the study variables were replaced by letters (variables).

Robert wants to offer a bouquet of flowers to his mother on her birthday. He visited 3 flower shops.
The first shop had 1 bouquet of roses, 2 bouquets of tulips and 3 bouquets of daisies. Altogether, in this shop, there were 11 flowers.
The second shop had 1 bouquet of roses, 3 bouquets of tulips and 2 bouquets of daisies. There were 13 flowers.
The third shop had 1 bouquet of roses, 1 bouquet of tulips and 1 bouquet of daisies. Altogether, in this shop, there were 6 flowers.
It is known that bouquets of roses have all the same number of roses, the bouquets of tulips have all the same number of tulips and the bouquets of daisies all have the same number of daisies.
Can you find out how many flowers were in a bouquet of roses, in a bouquet of tulips and in a bouquet of daisies?

Fig. 9 - Statement of the 4th problem presented to Gabriel, which could be associated with the resolution of a linear system with three equations and three unknowns (translated to English)

Gabriel solved the problem doing the calculations presented in Fig. 10.

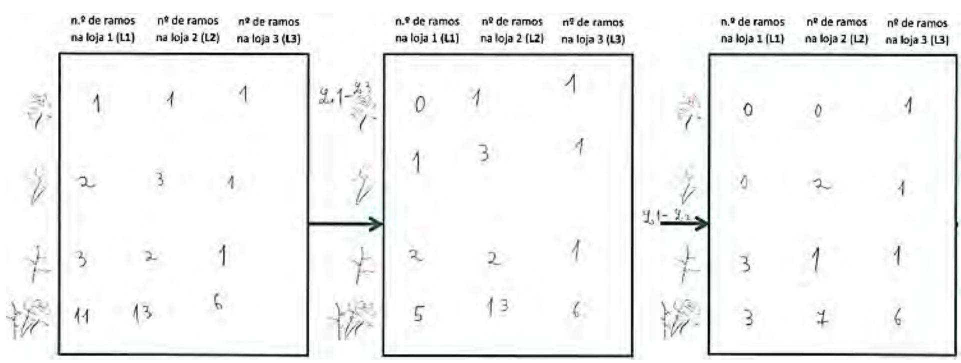

Fig. 10 – Gabriel´s resolution of the 4th problem presented in Fig.8

After presenting these tables (see Fig. 10), Gabriel wrote (in Portuguese):

1 bouquets of daisies = 1 daisy;

1 bouquets of tulips = 3 tulips;

1 bouquets of roses = 2 roses.

At that moment, it seemed that Gabriel understood and knew how to apply the algorithm associated with the *fangcheng* method. But we needed to verify if the method was learned and appropriated by him as a mental tool. For that purpose, we have carefully drafted an a-didactical situation to investigate if he mobilizes autonomously his knowledge in a different context.

Third part of the task

In creating this a-didactical situation we careful that, from the perspective of Gabriel, the situation was not connected with any of the above tasks and the problem presented could not be easily recognizable as likely to be solved through the ancient Chinese method.

The strategy we found took advantage of the Gabriel's beliefs in the existence of Santa Claus and his elves (with the consent of the child's parents).

A website (available at http://duendematematico.wix.com/concurso-natal-2013) has been created on the Internet, where a Mathematical Elf was presenting a competition for children aged between ten and twelve.

To participate, children had to solve three problems and send, by uploading on the contest website, the resolution of the problems and the video recording of task execution.

The three problems presented were similar but, only one problem (the second one) could be solved by the ancient Chinese method.

We applied the third part of the task on 10th of December of 2013. Gabriel starts solving the Mathematical Elf´s task applying knowledge gained at school or at home. When he saw the statement of the problem (see Fig. 11) which could be solved by the *fangcheng* method he didn't hesitate and immediately began to draw tables and put numbers on them.

Today I decorate 3 Christmas trees of different sizes.
I used 1 box of red balls, 2 boxes of golden balls and 3 boxes of silver balls to decorate the biggest one. This tree was decorated with a total of 21 balls.
I used 1 box of red balls, 2 boxes of golden balls and 2 boxes of silver balls to decorate the middle one. This tree was adorned with a total of 17 balls.
I used 1 box of red balls, 1 box of golden ball and 1 box of silver balls to decorate the smallest one. This tree was adorned with a total of 10 balls.
It is known that each box of each type has the same number of balls. How many balls has each box?

Fig. 11 – Statement of the second problem of the Mathematical Elf´s task. This was the only one, in the contest, that could be solved by the method under consideration.

In a few minutes Gabriel wrote his answer (see Fig. 12).

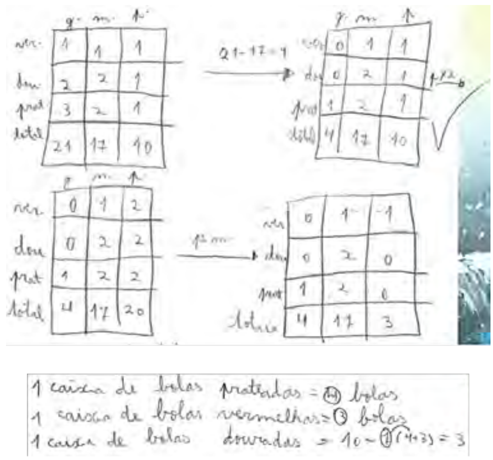

Fig. 12 – Gabriel´s resolution of the second problem of the Mathematical Elf´s contest

After presenting these tables (see Fig. 12), Gabriel wrote (in Portuguese):

1 box of silver balls = 4 balls;

1 box of red balls = 3 balls;

1 box of golden balls = 10-7=3 balls.

Afterwards, we interviewed the boy during 65 minutes. First we want him to explain his thinking, comparing it with his productions. In the second phase, it was required to him to solve problems with a higher degree of complexity, seeking to establish if the child is able to progress to more complex resolutions.

Focusing on the task of Mathematical Elf (a-didactic situation):

Researcher: How did you find out that you could use the Chinese method?

Gabriel compares the statements of the problems of the various tasks and concludes:

Gabriel: Because I thought so ... Today I decorated three Christmas trees... I think I remember something ... I thought this was familiar. I thought a bit and I remembered that the problem of the flowers began the same way.

Apparently Gabriel did not use the algorithm that has been taught and preferred to follow the resolution that seemed more appropriate and faster, since it facilitated the calculations.

Researcher: Gabriel, you used a method that is not exactly the same method you used in the other two tasks.

Gabriel: I invented a little.

Researcher: Why? Why did you change the method?

Gabriel: Because I think this method is easier than the Chinese method. It is more appropriate.

Researcher: Why is it more appropriate?

Gabriel: Because it's easier. I don’t need to do many calculations. It's faster.

Researcher: How did you know that the method you used is correct?

Gabriel: I didn’t know. I had my heart beating so fast!...

Researcher: And when did your heart stopped beating so fast?

Gabriel: When I started to check the problem ... When I verified that the results were correct.

Researcher: Do you think that this modified method works?

Gabriel: I think it works ... at all. I think that it always works...

Trying to analyse perception of Gabriel about the creation of *fangcheng*, the researchers were surprised. In order to answer, Gabriel associates the method to the greats of Western mathematics, like Euclid or Pythagoras, and takes them to another part of the world and a different culture, showing his vision of the universality of mathematics.

Researcher: How do you think that Chinese people discovered this method? How do you think they would thought?

Gabriel: It must have been a Pythagoras of China.

Researcher: A Pythagoras of China?

Gabriel: Yes, a Chinese Pythagoras! Or a Chinese Euclid or still a Maurits Cornelis Escher!

Researcher: In your opinion do you think kids in your age could learn this method in school?

Gabriel: Hmm... I don’t know.

Researcher: But, in your opinion, we could teach this method in maths classes for kids of your age?

Gabriel: Yes, if they are interested in learning it.

At the end of the interview the researcher suggested another problem, about “*Cakes and chocolates*”, to Gabriel. This time the statement was formal and without images.

The problem proposed could be associated with the resolution of a linear system with four equations and four unknowns however this didn’t seem to bother Gabriel. Fig. 13 shows Gabriel’s resolution.

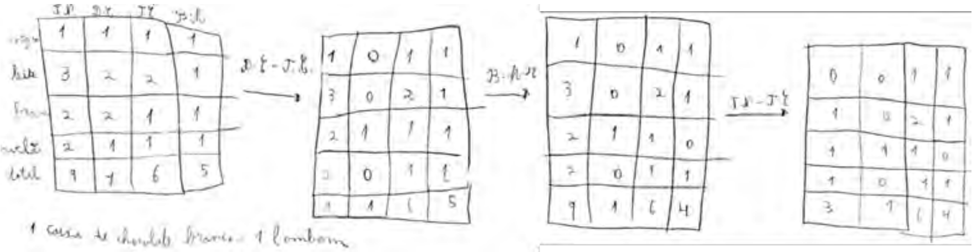

Fig. 13 – Gabriel´s resolution of the problem "*Cakes and Chocolates*" that could be associated with the resolution of a linear system with four equations and four unknowns

Gabriel concluded (in Portuguese):

1 box of bonbons of white chocolate= 1 bonbon

1 box of bonbons of milk chocolate= 1 bonbon

1 box of bonbons with hazelnuts= 1 bonbon

1 box of bonbons of dark chocolate= 2 bonbons

We hope that with the description of the *fangcheng* task and with the Gabriel's productions and comments at the interview, have reflected the way a ten years Portuguese boy sees the ancestral Chinese method for solving linear systems of equations.

CONCLUSIONS AND FINAL REMARKS

We consider that this experimental study design was adequate to the research question: Would a ten years old child understands and appropriates the *fangcheng* method to solve linear systems of equations? This exploratory case study is part of a wider investigation involving the construction and implementation of tasks based on ancestral Chinese mathematics.

From what has been presented it seems appropriate to present the initial findings of this study.

Gabriel, a ten years old boy, solves linear systems of two, three and four equations with two, three and four unknowns, respectively, using the *fangcheng* method. It should however be noted that some didactical transposition was performed in order to use only non-negative integers and to adapt the method to the occidental way of writing. We also highlight that Gabriel likes math and math challenges, which is not the standard in Portugal.

There are evidences that the *fangcheng* method had become part of the child's knowledge; meaning that Gabriel would be able to replicate it in similar situations and foremost use it in a-didactical situations.

This study shows that it is possible to learn the *fangcheng* method much earlier than in the first university year, as usually occurs in Portugal with the Gaussian elimination method.

This leads us to think that some curricular adjustments on this subject may occur, notably in the way we teach and when to teach.

We also think that this experimental case study illustrates the ideas of Swetz (1988, 1994) about the potential of the history of ancient Chinese mathematics from the pedagogical point of view. In this example it can be used to contribute to the development of pre-algebra contents, such us linear systems of equations, unknowns and matrices.

It seems to us that the case presented by us is also in the same line of thinking of the ideas of Jankvist (2011) when he says that by conducting empirical researches we can determine the true impact that the integration of history has in mathematics education. These empirical researches allow us to reassert theoretical conjectures and give new ideas for future lines of research.

From the articulation of these ideas with the feedback received during the oral presentation of this work we accept the future challenges to investigate how an ordinary class of 10 years old students would react of to this *fangcheng* task and how will Gabriel react to the introduction of algebra concepts.

NOTES

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2. Thanks are due to Gabriel who accepted patient and happily all our math challenges.

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