Panel Discussion 2

THE QUESTION OF EVALUATION AND ASSESSMENT OF EXPERIENCES WITH INTRODUCING HISTORY OF MATHEMATICS IN THE CLASSROOM

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INTRODUCTION

The question of Evaluation and Assessment that we were asked to consider contains many different aspects of general beliefs and principles, of personal didactic and pedagogic decisions, and of internal freedoms and external constraints. The use of history of mathematics in education and teaching of mathematics also concerns the broad cultural aspects of our subject surveyed by Alan Bishop (1988). Areas not considered explicitly in this short report are questions of equity and social justice, of race and gender, which are the concern of all sensitive educators.

This report is intended first to survey the contexts, options, possibilities, and situations surrounding the problems of assessment and evaluation, and to offer a number of questions that we all have to consider when we plan a course, and before we make an assessment of our students' work.

These aspects were considered by the panel members, and since each of us work in different contexts, our work situations and observations can be found in the statements at the end of this report.

Leo Rogers

EVALUATION AND ASSESSMENT: CLARIFYING THE TERMS

It is clear that the words *Evaluation* and *Assessment* are found in different contexts and have slightly different meanings in different languages, and these meanings are often confused.

The word *Test* is also used to mean some kind of assessment, and has its own particular contexts and intentions.

In UK English we use all three words:

a) Evaluation is about objects, ideas, entities, and beliefs. It indicates what it is about the subject matter, namely the history of mathematics, its use in education, and often about some mathematics, that we **value**.

Our values derive from our own philosophy of mathematics and of history of mathematics, and are inherent in any beliefs, principles or practices we hold when teaching students, or designing their assessment.

b) Assessment is about estimating quantity, or agreeing a 'measure', or finding out what students 'know' in some way and is generally qualified as:

i) Formative Assessment (sometimes called *Continuous* Assessment) which is about observing changes over time in relatively short-term periodic checks on a student's progress, like in-class discussions, weekly tests of facts, or short essays or projects.

These are often used to explore students' understanding of a concept or to *check on whether our own teaching has been effective*.

ii) Summative Assessment is the traditional assessment taken at the end of a student's course, like a test at the end of a semester or end of year examinations.

c) Test: usually means a single short summative assessment.

It could be argued that if we have used formative assessments during a course or semester, then we have enough information, and we do not need to ask the student to perform a summative assessment at the end of the period.

There is also the possibility of applying these aspects of evaluation and assessment in the context of assisting an individual to reflect upon their own progress as 'self-assessment' as noted below (Ipsative assessment).

BASIC QUESTIONS: BACKGROUND CONTEXTS AND PRINCIPLES TO CONSIDER

Value judgements govern answers to all these questions that can be considered and debated with colleagues.

- **Why** Assess? *Deciding* on effects or outcomes we expect or seek.
- **What** to Assess? *Becoming aware* of and deciding on what we are looking for.
- **How** to Assess? *Selecting* the method we regard as being more 'truthful' or 'fair' for different kinds of valued knowledge.
- **How** to Interpret? *Making sense* of observations, measurements and impressions gathered by whatever means we employ and explaining, appreciating and *attaching meaning* to 'raw' data.
- **How** to Manage the data? As expressed in words, numbers, statements, student profiles, personal interviews, etc.
- **How** to Respond? *Expressing and communicating* appropriate response sensitively to individuals and communities.
- In all contexts ***Feedback*** for those being assessed is important.

NORM REFERENCING AND CRITERION REFERENCING

Humanities and Social sciences generally use qualitative assessment methods whereas the ‘Exact’ sciences, like Mathematics and Physics tend to use quantitative assessment methods. However, with History of Mathematics we have to select what is appropriate; our work involves Essays and Projects as well as solving Mathematical Problems and following Calculations, so both Qualitative and Quantitative methods depend on the kinds of questions asked about the material being studied.

Norm Referencing

The essential characteristic of **norm-referencing** is that students are awarded their grades on the basis of their ranking within a particular group. This involves fitting a ranked list of students’ ‘raw scores’ to a pre-determined distribution for awarding grades. Usually, grades are spread to fit a ‘bell curve’ (a normal distribution), either by qualitative judgements or by statistical techniques of varying complexity.

Norm-referencing is based on the assumption that an approximately similar range of performance can be expected for any student group.

Criterion-referencing, as the name implies, involves determining a student’s grade by comparing their achievements with clearly stated criteria for learning outcomes and clearly stated standards for particular levels of performance.

Unlike norm-referencing, there is no pre-determined grade distribution and a student’s grade is not influenced by the performance of other students. Theoretically, all students within a particular group could receive very high (or very low) grades depending solely on the levels of individuals’ performances against the established criteria. The goal of criterion-referencing is to report student achievement against objective reference points that are independent of the group being assessed.

Ipsative assessment. (Self - assessment)

In this mode of assessment, a person's performance is **compared with their own earlier performance**, to determine whether any improvement has been made, or any 'added value' brought about. Such assessment might involve setting a learner pre-course, or post-course assessment or keeping track of how a student's average percentage mark or overall grade changes as they progress through an entire course. In all cases, however, the benchmark against which any change in performance is measured is the person's own previous performance - not the performance of other people. (Andrade & Valtcheva 2009)

Small Groups or Individual Students, and Peer-Assessment

Clearly, small groups of students will not fit into a formal pattern as implied above, and judgements on individuals may be made on the experience (often over a number of years) of the assessor. However, in such cases, more than one examiner, or an external moderator is usually involved. Topping (1998) shows that Peer Assessment is "of adequate reliability and validity in a wide variety of applications."

Once a decision about the type of assessment has been made, the actual tools - the test, the essay, the project or report (be it quantitative or qualitative) may be applied.

THE VALIDITY - RELIABILITY SPECTRUM

Validity (or truthfulness)

A valid assessment is one that measures what it is intended to measure. The assessment tools must be appropriate – for example a practical skill cannot be measured solely by a written test.

On the other hand, for a statement to be valid, it depends on personal, idiosyncratic, discursive, cultural, individual, and affective factors. Hence judgements about valid statements are very difficult, often subjective and raise questions about extension over time and space:

- Can assessment be extended over time and in different situations?
- Can predicting future results and behaviours become more robust?
- Can an individual retain a particular ability whilst maintaining the disposition to act in the same way over time?

Reliability (or consistency)

For a result to be reliable it needs to be objectively measureable, testable, appropriate, and repeatable. A reliable result is necessarily restricted to a narrow range of results. This is the scientific ideal. Even so, reliability involves the expectations of students and teachers, individual predispositions and attitudes, experiences and personalities, qualities of experience, and conceptions of abstract entities.

It is well known that two people can witness the same thing (a result, a process, or entity) but disagree about its meaning or significance. In our own experience we can find a wide variation in marks in test papers or essays, over time, and between individual assessors.

Our Problem is to *seek a path between these two concepts*, to balance the *nomothetic* demands of the (quantitative) mark scheme against the *idiographic* uniqueness of the (qualitative) student response.

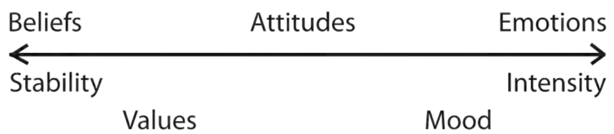
(Educational Studies in Mathematics, 2001; Smith et. al. 1996).

FRAMEWORKS, TAXONOMIES AND TEMPLATES FOR ASSESS-MENT

Bloom *et al.* (1956) published a taxonomy developed for educational assessment. It was originally designed for application to all school subjects, and provided definitions for each of the six major categories in the **cognitive** domain. The categories were *Knowledge, Comprehension, Application, Analysis, Synthesis,* and *Evaluation.*

At the time there were a number of mathematics educators who adapted it to their own views of assessment of mathematics, and many in mathematics education have shown it particularly ill-fitting for use in mathematics (Kilpatrick 1993). Later, a taxonomy of objectives for the **affective** domain was published (Kratwohl, 1964) which dealt with *beliefs, attitudes and emotions* as representing increased levels of affective involvement, with consequent decreased levels of cognitive involvement, increasing levels of intensity of response, and decreasing levels of stability of response. (Krathwohl, 2002, Evans, et.al. 2006, Hannula 2012.) Recent versions of Bloom's Taxonomy offer Characterising, Organising, Valueing, Responding, and Receiving as the main affective domain categories.

The Range of Affective Aspects



(Krathwohl 1964, 2002)

Richard Skemp's (1976) paper on *Instrumental* and *Relational* understanding had a significant impact on teachers' views about learning mathematics and influenced much research on assessing *mathematical thinking processes* rather than the production of results.

Generally, variations of Bloom's Taxonomy fail to identify *levels of learning* as opposed to designing different types of question (Freeman and Lewis 1998), and that its hierarchical nature is flawed, as certain levels in it may be considered interdependent (Anderson and Sosniak 1994, Kadijević, 2002). However, the most important difficulty with using taxonomies relates to the classification process itself, specifically:

(a) *It is difficult to put certain questions into just one category.* More involved questions can include routine aspects and procedural calculations as part of the solution process.

(b) *It is difficult to know what skills and thinking are employed by individual students to answer a question.* For example, when asked to prove a theorem, a student may learn a proof by rote and reproduce it from memory; or understand the principles and associated concepts and definitions, and use these to independently develop a proof when assessed.

(Darlington, 2013).

Competency-Based Learning and Assessment

In this category system we define a set of competencies (criteria), about what students should know and be able to do, and develop valid, reliable assessments for them. Similar to the taxonomy above, it defines a set of objectives that, however well intentioned, is open to the same problems, and we still have to make choices about what criteria are important and what ideas or principles we value in a given context. Some versions of this approach allow students to take examinations more than once and obtain feedback, so that they finally qualify when they have met all the criteria. A typical competency based situation is the assessment of teacher training which relies on cognitive skills, effective performance, affective rapport, and qualitative judgments.

ABILITY THINKING AND 'LEVELS' OF ABILITY

Ability thinking

- is ingrained in our educational systems.
- is an entity that determines 'how much' or 'how fast' an individual can learn
- describes similar levels of attainment; hence students with assumed similar ability are taught together
- leads to the common practice of grouping / setting / streaming in school environments
- influences interactions between teachers and learners and between learners themselves
- is the dominant discourse for teachers, pupils, parents, policy makers, curriculum planners, test writers. etc.
- we all use *levels of ability* – so it is 'obviously true'!

Ability it is never consistently defined and only understood in the sense that A is 'better' (in some particular skill, or group of skills) than B.

THE SCHOOL, COLLEGE, CURRICULUM, AND THE EDUCATIONAL SYSTEM

We find ourselves working inside a local educational system, in a particular institutional social context and choose to abide by its rules of governance. The situation imposes *constraints* which may limit our choices of teaching material and methods, but it could also offer *affordances* (Gibson 1977) namely, possibilities of choice, development, and action.

Guidelines, Constraints and the Curriculum itself are often politically motivated to some degree or other. We may follow the guidelines and test the constraints of the system, and explore the nature of the school or college curriculum; its opportunities and affordances which include regulations about assessment and evaluation methods. (Gresalfi, Barnes & Cross, 2012)

Our Pupils' or Students' age or position in the learning programme will determine the approach we have to the situation.

8 - 10 (Primary School)

10 - 18 (Middle and Secondary Schools)

16 - 20 $^+$ (High School and College)

20+ University students and *Teachers' Professional Development*

Curriculum Control

What we are able to achieve is subject to different kinds of control

(a) Professional control (what is valued) this concerns the content and nature of the subject matter; of children, pupils and students; of teaching and learning and understanding.

(b) Political control (what is ideologically desirable) justification of content – and methodology – system constraints and affordances ...

These controls depend on the philosophy (ideology) of both Administrators. Educators and Teachers. (Ernest, 1989, 1991; Furinghetti & Pehkonen 2002; Andrews, Paul 2007; Drew & Hannafin 2011).

Clarifying our Objectives and Alternatives

- For including the use of History of Mathematics as an element of mathematics courses at different levels
- For teaching the history of mathematics as a separate course.
- For choosing appropriate assessment methods
- For evaluating the process of 'use and assessment'

Range, Suitability and Significance of Historical Materials

The materials we use with our students can create a diversity of mathematical experiences**,** including cultural contexts and historical awareness. However, we have to be aware of the different possibilities of assessment modes available, and make our own judgments about their use.

(Ball, et. al. 1998)

- **Physical materials:** Original texts, documents, engravings, memorials, manuscripts, letters, diagrams,
- **Historical 'events':** Births, deaths, social-economic events, publications, and similar well-substantiated dates. How important is it to remember the date of an event?

- **Historical 'facts' and 'problems':** Theorems, propositions, conjectures, arguments, calculations, explanations, and improvements or refutations of these.
- **Interpretations, Influences**, and comparison of different accounts (past and present) – primary sources and secondary sources; 'history' books and translations of original texts.
- **Cultural contexts: e**xplores links between the cultural-historical dimension of mathematical practices and an individual's likely mathematical thinking.

All of these aspects have different values, qualities and affordances when used with different groups of students. Choosing an appropriate system for assessment will not only allow us to encourage the well-grounded and vigorous development of our students but also,

"…. investigating the process of how knowledge grows through researching historical materials themselves, and through evidence of the growth of mathematical ideas, and using this material either directly or indirectly in the classroom, reaches for similar understandings and operationally valid results as 'main-stream' educational theory." (Rogers (2014: 120).

STATEMENTS FROM PANEL MEMBERS

Janet Heine Barnett

Colorado State University, Pueblo.

I teach at a Mid-size Regional State University and I teach Undergraduate mathematics majors; mostly upper division courses, and Prospective teachers - mostly at both lower and upper secondary school. Guidelines for history of mathematics CBMS (2012) suggests:

For middle grades: *A history of mathematics course can provide middle grades teachers with an understanding of the background and historical development of many topics in middle grades.*

For high school: *The history of mathematics can either be woven into existing mathematics courses or be presented in a mathematics course of its own. ……*

"It is particularly useful for prospective high school teachers to work with primary sources. Working with primary sources gives practice in listening to ``wrong'' ideas. Primary documents show how hard some ideas have been, for example, the difficulties that Victorian mathematicians had with negative and complex numbers helps prospective teachers appreciate how hard these ideas can be for students who encounter them for the first time. Finally, primary documents exhibit older techniques, and so give an appreciation of how mathematics was done and how mathematical ideas could have developed."

I use history of mathematics in my teaching with Guided Reading Modules based on Original Historical Sources: (Barnett et.al. 2014)

"*Learning Mathematics and Computer Science via Primary Historical Sources"*

This involves joint work with colleagues at New Mexico State University Funded by US National Science Foundation with 33 existing modules available at www.cs.nmsu.edu/historical-projects

The typical Structure of a Primary Source Project (PSP) contains:

- Historical and biographical background
- Excerpt(s) from original source(s)
- A project narrative to guide student reading of excerpt(s)
- Student tasks based on excerpt(s)
- Concluding Comments / Epilogue

The primary goal is to support student learning of core material in contemporary undergraduate courses using classroom assessment of student learning of mathematics using:

Reading and Study Guides (Including classroom Preparation and Reading exercises); Written Homework Sets*;* Observations of Class Group Work and Contributions to Whole Class Discussions; Written Exams, including Comprehensive Final Student

Interviews.

Additional Goals: Motivate and support development of a deeper level understanding that reaches beyond basic content objectives

ASSESSMENT will include student comments on benefits of learning from original sources, and a theoretical perspective uses (Sfard (2008/2010)

There are Plans for a new project: "Evaluation with research" a component of a new NSF grant proposal (pending). Project Evaluator: Kathy Clark, Florida State University.

Ysette Weiss-Pidstrygach

Mathematical Institute, Johannes Gutenberg-University of Mainz. DE.

A Community of Practice

I teach courses in mathematics education for Mathematics student teachers for the gymnasium at the university of Mainz (Germany). There exist different forms of assessment and evaluations, like tests, oral examination, essays, coursework, seminar papers, presentations and homework assignments. But it seems that the biggest impact on self-concept and self-esteem of the Mathematics student teachers as future mathematics teachers are the written maths examinations. Today's students were brought up in a school system with normative approaches to human development.

They had to produce a required output in situations that are created and determined by others. A different approach is taken in the process of value creation in a community of practice (Wenger et al, 2002). The development of a community of practice starts with a university course in mathematics education. The use of historical and cultural perspectives in university mathematics education can support the development of self-esteem and maturity. It can bring together students with similar interests. In (Weiss-Pidstrygach & Kaenders, 2015) we present the concept of a seminar on the analysis of mathematical school textbooks and of learning contexts based on the consideration of historical excerpts. Such a seminar can become a starting point for a community of practice of student teachers, mathematics educators, historians, mathematicians, mathematics teachers and school textbook authors with the potential to develop social recognition and personal appreciation of the individual interests and talents of its members and their joint activities. We choose to work on historical excerpts in mathematical school textbooks, because for teacher students this topic is strongly related to their future practice: In Germany, there are a handful of schoolbook series that are used extensively in school. At present, most of them have historical insertions. Since the historical references that we deal with in the seminar stem from books that teachers use in their daily teaching, they constitute a link of this activity with the practice.

In countries where textbooks are not used in the classroom, the concept of the seminar can be adapted to other learning aids with historical references.

Weiss-Pidstrygach, Y., & Kaenders, R. Using historical School book excerpts for the education of mature mathematics teachers. In Proceedings of **CERME** 9, 4th-8th February 2015.

Wenger, E., McDermott, R.A. & Snyder, W. (2002). *Cultivating communities of practice: A guide to managing knowledge.* Harvard Business Press.

Frederic Metin

École Supérieure du Professorat et de l'Éducation, Université de Bourgogne**.**

I am a mathematics teacher trainer at the School of Education of the University of Burgundy in Dijon, and my major tasks are:

- to give literary students the opportunity of improving their skills in basic mathematics;
- to prepare students to take the competitive exam which will make them civil servants;
- then to train them into the construction of their own professional style

But in the various classes I teach, History of Mathematics is a minor subject, but can be the core of some courses, with for instance a special training session on how to use original texts in the classroom at middle school and high school levels.

Nevertheless I use History of Mathematics for both enlightening student's knowledge in mathematics and putting some distance between them and this knowledge, plus linking this knowledge to other disciplines. Of course the question of assessment is a difficult one: why? How? And does it even make sense to assess the historical aspects of a course on mathematics?

For example, when you try to make sense of recreational problems contained in a manuscript course of geometry from a 17^{th} century Jesuit college, what kind of the way do you have to make sure the students understood the contents and methods? A simple answer will be: give them items 1 and 2 as exercises and hide item 3, that you will keep for the special moment of assessment. The ideal original texts are the ones where the methods are obscurely described, not well explained or even not mentioned. The natural assessment will be the simple explanation of the mathematics in the text.

Take practical geometry: studying the usual theorems in their unusual but 'useful uses' of the past will provide a kind of *depaysement* which makes assessment obvious, or obviously irrelevant: you just check the understanding of the underlying mathematical thinking, but you might rather reconstruct it. To avoid the trap, you can ask unusual (for me) question as 'is that approximation accurate?' or 'what is your opinion about the notations?' or even 'how do you feel about the text?' The problem then is that there is no unique and impersonal answer, and you thus have to accept different points of view, which is quite unfamiliar in assessing mathematics.

David Guillemette

Université du Québec à Montréal.

From my part, I'll concentrate on experiences lived with my students that are pre-service secondary school teachers. Aiming at "disorientation" with the reading of original texts, I'll try to explain our account, to underline our perspective of disorientation argument and to say few words about the problematic of assessment in this context.

In my thesis, I manage to describe the experience of disorientation of my students involved in the reading of original texts. When adopting a phenomenological stance, major themes emerge from the analysis. Two of them are the experience of *otherness* and *empathy*.

Students are saying that they are trying very hard to understand the mathematics depicted in original texts. They show great difficulties concerning language, notation, implicit argument, style, definitions, interpretations, typography, etc. Literally, they "suffer the texts". For now, in this context, I see the reading of original sources as a 'hermeneutic extreme sport' … and without helmet. The experience of *otherness* seems brutal, from a cognitive and affective point of view, it sometimes includes shocks and violence.

From Levinas, I learned that violence is a “thematisation of the Other”, a reification of the Other, a way to make the Other a Mine, and that to understand something is to control it, make violence to it. I saw few acts of violence during my experimentation, for instance, someone said: “Fermat was doing this or that”.

That’s why *otherness* is linked with *empathy*. Again with Levinas, and also with Bakhtine, *empathy* could be heard as an effort of a non-violent relation with the Other, in this case, a way of keeping alive the subjectivity of the authors, keeping it fragile and mysterious. The question is how to accompany the students in this ordeal, in this hardship experience of *otherness*? How to maintain an *empathic* relation with the authors? I try to address these questions from a fundamental pedagogical point of view.

From these bases, the question of assessment in this context is for me a question of affectivity and a question of being-with-others. If assessment should support students, what are the actions that could support *empathy*? (Bakhtin, 1981, Levinas, 1985, 2010, 2011)

Discussion among the audience and the panel

There was a general discussion between the audience and the panel members clarifying points of view and contexts. Most of the audience concentrated on the university training of teachers and history in the context of teaching mathematics at this level.

However, the situation in the secondary school and some secondary teachers were present, and the following points were made by Ewa Lakoma on behalf of the situation in secondary education

Concering Secondary Education.

Ewa Lakoma

Institute of Mathematics, Military University of Technology.

In Poland there is a system of education: 6+3+3, starting with children at 7 years old. After each step of education there is outer examination, the same for the whole population of students at this level in Poland, leaded by the Central Examination Board [1, 2].

In fact students, when learning, are also preparing to sit these examinations. After the second stage (gymnasium) the examination opens the door to the Lycee. After the third stage the 'matura examination' is the entrance examination for the universities.

When we look at the textbooks that were presented in 2000 in the ICMI book of Fauvel and van Maanen (eds.), we can notice that many of these examples still exist in current textbooks but now they are treated rather as additional material for students. Sets of exercises preparing students for the exams are the most important.

Currently, in school practice, the history of mathematics is often present as mathematical projects, that students develop individually or in a small team and present to a classroom audience. They have to find some information on the Internet and then usually they prepare a multi-media presentation. What is most important from the point of view of assessment in this activity is the level of invention of students, their social ·competences, the level of using IT, the history of mathematics is an illustration of these activities. But it is also often evident that students are really interested in old historical materials, mathematical examples, and their solutions. They sometimes really learn something new in mathematics.

In the education system, the result of the final examination after a given level decides on a position for the student at the next higher level. Current school practice is that the history of mathematics is placed mainly in the context of better preparation of students for these examinations and has its value when it appears among questions and tasks in the examination.

An example of such a situation was found in the test after the first phase of education for 12 years old pupils in 2011.

Example 1 - the 'historical' context in the examination

The text to consider was the famous anecdote about the young pupil Karl Gauss whose teacher gave pupils the task of adding all the numbers from 1 to 40.

In the text there is also a presentation of the reasoning of Gauss.

Just below the text there is some short information:

Karl Gauss (1777-1855) - *German scholar, mathematician, astronomer, physicist; obtained the title of Doctor at the age of* 22. *In 1807 was* a *Professor. One of the greatest mathematicians of the world.*

And after that text there were eight multiple-choice exam questions (to select one correct answer among four statements): six questions consider the situation in the classroom from the point of view of the teacher, for example:

After checking the notebook the teacher realized that they needed to:

A. *move Karl to the next class;* B. *call his parents ;*C. *to develop his talent.*

D. teach him 'a *lesson '.* - and the last two questions were supposed to be 'mathematical':

When was this lesson? (i.e. How old was Gauss?)

A At the turn of the seventeenth and eighteenth centuries.

B. In the second half of the eighteenth century.

C. In the late eighteenth and early nineteenth century.

D. In the first half of the nineteenth century.

and

How old was Gauss, when he became a professor? A. 22, B. 30, C. 48, D. 78

We can pose many important questions such as:

Is this really considering historical material in a way that we prefer?

Was it really about the history of mathematics?

What is the conclusion from results of these 'mathematical' questions?

Example 2:

The History of mathematics as a theme for lessons in the Polish language

Surprisingly, the oldest book for geometry written in Polish by Stanislaw Solski, was titled: *The Polish Geometrist,* (1683), and has been known by using it at historical lessons in the Baroque Palace of the King Jan III Sobieski in Vilanov, Warsaw.

Students are able to attend at the historical lesson (real or virtual) and are able to read some pages of this book. The intention is linguistic - to recognise some old Polish words, but from the point of mathematics this is very important book, because in it we can find the creation of Polish names for the most fundamental mathematical notions.

CONCLUSIONS

As long as the history of mathematics is absent from the examination tasks, the status of considering historical materials will be still seen as 'an appendix' to the main stream of 'common' examination tasks.

For both teachers and students the use and consideration of historical materials must be clearly justified (otherwise teachers will claim there is *'no time'* to consider it)

It is a good idea to integrate areas of the history, language, culture and mathematics in order to place some historical original materials to consider, but it needs considerable cooperation between teachers of different subject areas. '

In any case, the history of mathematics is interesting for students and valuable from the point of view of their cognitive development.
The most important problem is how to profit from the short time between the examinations whose results decide the future career of young people.

This contribution points to the importance of the history of mathematics in our cultural education, and the problems about raising the awareness of history without trivialising the subject within the traditional structure of a formal examination.

This also reflects on the problems raised by the members of the panel above who, in university contexts have much more freedom to choose their mode of assessment. Clearly, what can be done in school depends upon the significant external constraints of the system, and wherever possible, the mode of assessment needs to be appropriate for the level of sophistication of the students.

REFERENCES

Anderson, L. and L. Sosniak, eds. 1994. *Bloom's Taxonomy: A forty-year retrospective.* Chicago: National Society for the Study of Education.

Andrade, H. & Valtcheva, A. (2009) Promoting learning and achievement through self-assessment, *Theory into Practice*, 48, 12-19.

Andrews, Paul (2007) The curricular importance of mathematics: a comparison of English and Hungarian teachers' espoused beliefs. *Journal of Curriculum Studies* 39, 3, (317-338)

Ball, N., G. Smith, L. Wood, M. Coupland and K. Crawford. 1998. Creating a diversity of mathematical experiences for tertiary students. *International Journal of Mathematical Education in Science and Technology* 29 (6): 827-41.

Bakhtin, Mikhail. (1981). *The Dialogic Imagination*. Texas. University of Texas Press.

Barnett, Janet, Lodder, J. & Penhelley, D. (2014) The Pedagogy of Primary Historical Sources in Mathematics: Classroom Practice Meets Theoretical Frameworks. *Science and Education* 23, 7-27

Bishop, A.J. (1988). *Mathematical Enculturation-a cultural perspective on mathematics education.* Netherlands: Kluwer Academic Publisher.

Bloom, B., M. Engelhart, E. Furst, W. Hill and D. Krathwohl, eds. 1956. *Taxonomy of educational objectives: the classification of educational goals, Handbook I: Cognitive domain*. New York: David McKay.

Conference Board of the Mathematical Sciences (CBMS) (2012). History of Mathematics in US Curriculum Guidelines: Mathematical Education of Teachers II. *Issues in Mathematics Education* Volume: 17

Darlington, Eleanor. (2013). Undergraduate Mathematicians' Approaches to Learning. Unpublished D.Phil Thesis. University of Oxford Department of Education.

Drew, Polly & Hannafin, M.J. (2011). Examining How Learner-Centered Professional Development Influences Teachers' Espoused and Enacted Practices. *Journal of Educational Research* 104, 2. (120-130)

Educational Studies in Mathematics. (2001) 46, 1-3, Special Issue: *Bridging the Individual and the Social: Discursive Approaches to Research in Mathematics Education*.

Ernest, P. (1989a) 'The Impact of Beliefs on the Teaching of Mathematics', in P. Ernest, Ed. *Mathematics Teaching: The State of the Art*, London: The Falmer Press.

Ernest, Paul (1989b). The Knowledge, Beliefs and Attitudes of the Mathematics Teacher. *Journal of Education for Teaching: International research and pedagogy*. 15,1. (13-33)

Ernest, P. (1991) *The Philosophy of Mathematics Education*, London. Falmer Press.

Evans, Jeff. Morgan, Candia. & Tsatsaroni, Anna. 2006. Discursive Positioning and Emotion in School Mathematics Practices. *Educational Studies in Mathematics*, 63 (2), 209-226.

Fauvel, J. and van Maanen (eds.) (2000) History in Mathematics Education: The ICMI Study. Dordrecht. Kluwer.

Fowler, David H., & Eleanor Robson, 1998. “Square Root Approximations in Old Babylonian Mathematics: YBC 7289 in Context”. *Historia Mathematica* **25**, 366-378.

Freeman, R. and R. Lewis. 1998. *Planning and implementing assessment*. London: Kogan Page.

Furinghetti, F. & Pehkonen, E. (2002). Rethinking characterizations of beliefs. In G. C. Leder et al. Eds. *Beliefs: A Hidden Variable in Mathematics Education?* Dordrecht: Kluwer. (39 – 58).

Gibson, J.J. (1977) *Perceiving, Acting, and Knowing: Toward an Ecological Psychology*. Hillsdale N.J. Lawrence Erlbaum.

Goldin, G. A. 2002. Affect, Meta-affect, and mathematical belief structures. In: G.C.Leder, Pehkonen, E. & Torner, G. (eds.) *Beliefs: A hidden Variable in Mathematics Education?* The Netherlands: Kluwer Academic Publishers.

Gresalfi, M.S., Barnes, J., & Cross, D. (2012) When does an opportunity become an opportunity? Unpacking classroom practice through the lens of ecological psychology *Educational Studies in Mathematics*, 80 (1-2), pp. 249-267.

Hannula, M.S. 2012. Exploring new dimensions of mathematics-related affect: embodied and social theories. *Research in Mathematics Education*, 14 (2), 137-161.

Kadijević, D. 2002. TIMSS 2003 mathematics cognitive domains: examination, operationalization and implications. *Zbornik Instituta za Pedagoŝka Instraživanja* 34 (2): 96-102.

Kilpatrick, J. 1993. The chain and the arrow: From the history of mathematics assessment. In Investigations into mathematics education: An *ICMI study*, ed. M. Niss, 31-46. Dordrecht: Kluwer.

Krathwohl, D.R., Bloom, B.S., & Masia, B.B. (1964). *Taxonomy of educational objectives: The classification of educational goals. Handbook 11: The affective domain.* New York: David McKay.

Krathwohl. David, R. (2002). A Revision of Bloom's Taxonomy: An Overview

Theory Into Practice, 41, 4, Autumn 2002. College of Education, The Ohio State University

Leder, Gila C. Pehkonen, E. & Torner, G. (eds.) (2002) *Beliefs: A hidden Variable in Mathematics Education?* The Netherlands: Kluwer.

Levinas, Emmanuel. (1985) *Ethics and Infinity: Conversations with Phillipe Nemo*, (tr.) Richard Cohen. Pittsburgh: Duquesne University Press. This elegant little book is very helpful in making Levinas' thought approachable to the uninitiated.

Levinas, E. (2010). Totalité et infini: essai sur l'extériorité. (*Totality and Infinity*)Paris : Librairie Générale Française

Levinas, E. (2011). Le temps et l'autre. (*Time and the Other*) Paris: Presses Universitaires de France.

Pólya, G. 1945. *How to solve it*. New York: Anchor Books.

Rogers, L. (2014). History of Mathematics in and for the curriculum. In *Deabates in Mathematics Education*. (Leslie, Dawn. & Mendick, Heather. eds.) (106-122) Abingdon, Oxford. Routledge.

Selden, J., A. Selden and A. Mason. 1994. Even good calculus students can't solve non-routine problems. *Research Issues in Undergraduate Mathematics Learning*, MAA Notes 33(1): 19-26.

Sfard, A. (2008/2010) *Thinking As Communicating: Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing.* Cambridge, UK C.U.P.

Skemp, Richard, R. (1971). *The psychology of learning mathematics*. Harmondsworth, UK: Penguin.

Skemp, Richard, R. (1976). Relational Understanding and Instrumental Understanding. *Mathematics Teaching*, 77, (20-26)

Smith, G., L. Wood, M. Coupland, B. Stephenson, K. Crawford and G. Ball. (1996). Constructing mathematical examinations to assess a range of knowledge and skills. *International Journal of Mathematical Education in Science and Technology* 27 (1): 65-77.

Topping, K. (1998) Peer Assessment between Students in Colleges and Universities. *Review of Educational Research*. 68, 3 (249-276) American Educational Research Association.

Wertsch, J.V.: 1991, Voices of the Mind: A Sociocultural Approach to Mediated Action, Harvard University Press, Cambridge, MA.