AFTER THE GÖSTA MITTAG-LEFFLER (1846-1927) AND JULES HOUËL (1823–86) CORRESPONDENCE : THEIR GENERAL AND PARTICULAR THOUGHTS ABOUT MATHEMATICS TEACHING

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ABSTRACT

Jules Houël (1823–86) taught real and complex analysis at the Bordeaux' faculty of science from 1859 to 1884. In 1872, the young Gösta Mittag-Leffler (1846–1927) wrote him about complex analysis. Then they corresponded until 1883 about the theory of functions and complex analysis, how to teach it and also the mathematics teaching organization in Europe (especially France and Germany).

INTRODUCTION

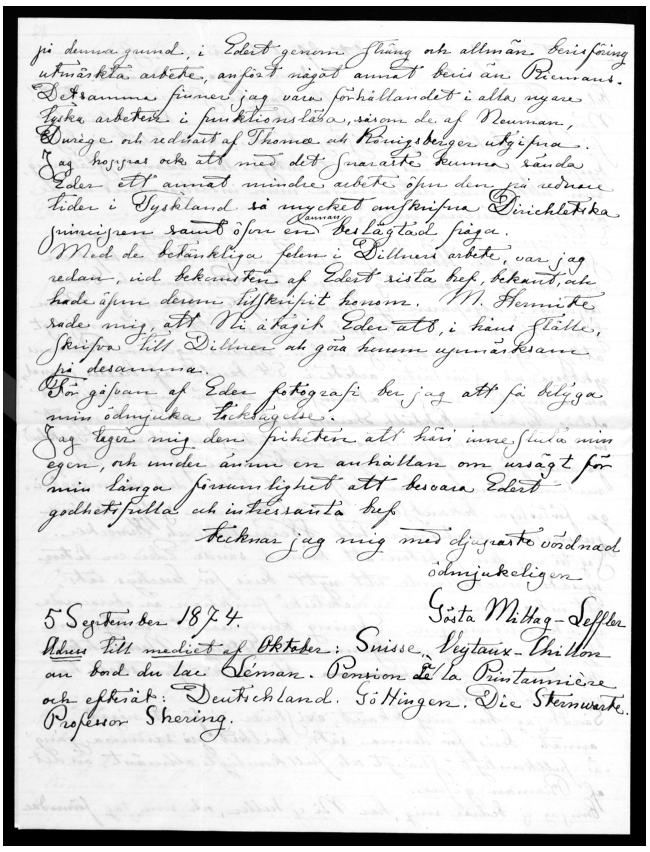
Nowadays, the name of Mittag-Leffler is well-known in the mathematics studies because of the theorem of factorization of a meromorphic function[1], although Houël's name is anonymous. In 1872, Mittag-Leffler was 26 years old and docent at Uppsala University. At that period, Houël was 49 years old and was well recognized by many European mathematicians. In his first letter, Mittag-Leffler wrote to Houël on the 15th July 1872 :

> Mister distinguished Professor,
>
> As I know you are one of the few French mathematicians that master my mother language, I took the liberty, encouraged by Mister the Professor Dillner, of sending you my work titled «Separation of roots of synectic functions of one variable» I just published. First of all, please allow me to offer my humble and respectful thanks for the knowledge of the complex quantities that I have gained from reading your great and exhaustive handbook «Théorie élémentaire des quantités complexes».

That was the beginning of the correspondence between the two mathematicians; it lasted until May 1883. Their correspondence is made of seventy letters: thirty-five written in Swedish by Mittag-

[1] See (Rudin, 1975) for instance.

Leffler and thirty-five written by Houël in French.[2] One can find the Mittag-Leffler's letters[3] in the Caen-la-mer library in Caen, France and the Houël's letters[4] are in the Kungliga Vetenskaps Akademien in Stockholm.[5] Eric Lehman, professor emeritus of mathematics at the Caen university translated the Mittag-Leffler's letters into French.[6] The correspondence between Mittag-Leffler and Houël is really interesting because Houël is the first non Scandinavian mathematician whom Mittag-Leffler wrote to so we can follow the genesis of his ideas and the many issues are covered. Obviously, the starting point is mathematics and more precisely complex analysis. Repeatedly, Mittag-Leffler and Houël discuss the theory of functions of one complex variable and elliptic functions; they discussed also the ways of teaching them. The organization of mathematics teaching in Europe and especially in France and Germany is a recurrent topic. Finally, the mathematics journals are omnipresent in the correspondence: Houël's *Bulletin*, Dillner's Tidskrift, Weyr's Archiv, Mittag-Leffler's Acta, . . . In our paper we'll present the exchanged ideas on complex analysis teaching in connection with the state of the theory of functions at that period, the organization of mathematics teaching in France and Germany according to Houël and Mittag-Leffler. But first of all, we have to present both Mittag-Leffler and Houël then the historical, political and mathematical contexts necessary to appreciate their points of view.



The last page of the letter dated of 5th September 1874 written by Mittag-Leffler to Houël

[2]Mittag-Leffler learned French in school and university and studied in Paris six months in 1873. So he understood well French.

[3]Certain letters are slightly damaged because the paper used was quite thin.

[4]They are in good condition.

[5]There are copies in the Mittag-Leffler Institute, Djursholm, Sweden.

[6]Let me thank him and his mother.

1 ABOUT GÖSTA MITTAG-LEFFLER

Gösta Mittag-Leffler[7](1846–1927) was issued of an old Swedish family with German origins and was made of artists, pastors, politicians, entrepreneurs. He studied French, German, Latin, mathematics, physics in Uppsala university and supported his Ph.D.'s thesis in 1872—dir. Göran Dillner—in complex analysis on «applications of the argument principle». Mittag-Leffler wanted to specialize in elliptic functions ; at that period, Dillner was connected to French mathematicians as Hermite, Briot, Bouquet, Houël. Mittag-Leffler, docent in 1872 at Uppsala university, decided to study in Paris during a year. He spent six months in 1873 in Paris studying with the most well-known French mathematicians ; but Hermite advised[8]him to study in Germany—Göttingen or Berlin. Finally, Mittag-Leffler studied a couple a months in Göttingen with Schering and then in Berlin with Kronecker and Weierstrass. He was one of the best Weierstrass' students and became his friend.

In 1876, Mittag-Leffler taught at Helsingfors[9]university for five years and then came back to Stockholm to teach in Stockholm college, founded by the new king Oskar II of Sweden in order to promote Scandinavian mathematics. In 1882, he married Signe af Lindfors issued of a rich and powerful Finnish family. Mittag-Leffler became an entrepreneur too. In Stocholm college in 1884, he recruited the brillant S. Kowaleskaja recommended by Weierstrass. The Sthockholm college became an important centre of mathematics, competing with the best French and German universities : the school of Scandinavian mathematics was born.

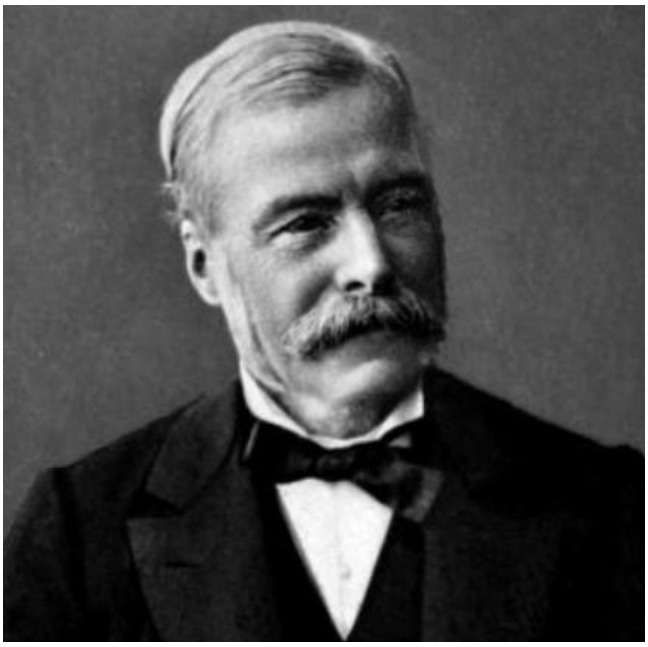
Gösta Mittag-Leffler about 1880

[7]See (Stubhaug, 1999) for a deailed biography of Mittag-Leffler.
[8]«Weierstrass : he is a master of all» would have said Hermite to Mittag-Leffler.
[9]Helsinki now

Mittag-Leffler founded in 1882 the mathematical journal Acta Mathematica from a Sophus Lie's idea ; still in order to promote Scandinavian mathematical school. The Acta Mathematica is still an important mathematical journal edited by the Mittag-Leffler Institute in Djursholm.[10] He was connected to many mathematicians : Hermite, Poincaré, Picard, Painlevé, Weierstrass, Houël, Markov, Moore, . . . Some say that his correspondences total about 12 000 letters. He continued his mathematical research on complex analysis and elliptic functions throughout his life. His most famous theorem is about factorization of meromorphic functions ; his masters Weierstrass and Hermite gave another proof of that theorem. He was connected to many important European politicians. So we can assume that he was a key figure of the mathematical world about 1880. He received many honours from the mathematical community.

2 ABOUT JULES HOUËL

Jules Houël (1823–86), issued of an old protestant Normand family, studied mathematics in Ecole Normale Supérieure from 1843 to 1846 ; he failed the « agrégation» in 1846—which he passed in 1847—, Houël began with teaching in different high schools. He worked in the high schools in Bourges (1847), in Pau (1848–49) , in Bordeaux (1850) and Alençon (1851–52). In 1852, he left teaching for mathematics and astronomy research, which led in 1855, to the defence of two theses—one in mechanics and one in astronomy (Houël 1855)—. Cauchy, member of the jury thesis, showed a real interest about them. Houël wanted to work on astronomy but couldn't enter[11]the Observatoire de Paris. After all, Houël took over V.A. Le Besgue at the «chaire de mathématiques pures» in Faculté des Sciences de Bordeaux in 1859 where he taught until 1884—when he retired because of health troubles.



Jules Houël about 1880

[10]Mittag-Leffler had no child and bequeathed his beatiful house in order to create a mathematical Scandinavian center.

[11]After Houël's descendants, U. Le Verrier director of the «Observatoire de Paris» would have presented him from entering it. That information is still to control.

He published the *Théorie élémentaire des quantités complexes* (four volumes) and the *Cours de calcul infinitésimal* (four volumes), fruits of his teachings from 1860 s until 1880 s. Those courses are really interesting because they are quite exhaustive, rigorous[12]; some pedagogic examples can help the reader and a history of the theories taught too. The *Théorie élémentaire* and the *Cours de calcul infinitésimal* had a certain success in France and Europe.

Houël enlivened the Société des sciences physiques et naturelles de Bordeaux. He promoted it so much that in 1867[13], «almost all the Bordeaux' mathematicians were enrolled in it». Houël published many mathematics papers, historic papers or translations in the *Mémoires* of the «Société», especially about complex analysis and non Euclidean geometries.[14] From 1864 to 1872, Houël was archivist of the «|Société» and developed substantially the activity and the connections. In 1872, the «Société des sciences physiques et naturelles» de Bordeaux counted more than a hundred connections among the learned societies in the whole world.

Houël was a polyglot : after the French mathematician P. Barbarin (Barbarin 1927), he knew at the end of his life «all the European languages», although he did never travel out of France. He translated, for instance : from German, papers written by Lejeune-Dirichlet, Riemann, Balzer, Lobatchevski[15], Lipschitz ; from Swedish, papers written by Mittag-Leffler about elliptic functions ; from Hungarian, the opuscul written by J. Boliay about non euclidean geometry; from Russian, books written by Lobatchevski and papers by Imschenetski and Bougaïev; from Norwegian, *La vie d'Abel* by C.A. Bjerknes, from Italian la *Théorie des équipollences* by Bellavitis.

Houël corresponded with many European mathematicians from the 50 s until the 80 s. here are additional examples : in France, Darboux, Laisant, Lefoy ; in Italy, Cremona, Forti ; in Germany, Balzer, Lejeune-Dirichlet, Borchard, Klein, Ohrtmann, ; in Scandinavy, Bjerknes, Dilllner, Lie ; in Bohemia, Durege, Emil and Eduard Weyr.

Houël was from 1870 to 1882, co-editor with Darboux of the *Bulletin des sciences mathématiques et astronomiques*, journal founded under the direction of the «Commission des hautes études»—presided by Chasles—with the support of the «Ministère de l'instruction publique» in order to diffuse[16] the new mathematics from Germany into France.[17] The polyglottism and opening mathematical mind of Houël were principal motivations of that choice. The *Bulletin* lasted until about World War II.

[12]See (Zerner 2008).

[13]As Houël wrote it to Charles Berger, in his letter dated of 12th January 1867 almost summoned in the introduction.

[14]On the life and work of N. Lobatchevski, for instance.

[15]The most of Lobatchevski's papers were written in German.

[16]After the 1870 French-Prussian war and the success of the French analytical School in the beginning of XIXth century, the French mathematicians didn't use to study German sciences, which were really important during the second part of the XIXth

[17]See (Gispert 1987).

3 ELEMENTS OF CONTEXTUALIZAION: HISTORIC LANDMARKS, TEACHING POLICY IN FRANCE AND MATHEMATICS (ANALYSIS) TOPICS DURING THE XIXTH CENTURY

3.1 Historic landmarks in the period of the correspondence between Mittag-Leffler and Houël in Europe

The period of correspondence (1872–83) is really interesting in Europe: it stands just after the 1870 French-Prussian war which ended by the foundation of the Third Republic in France, the Second Reich in Germany led by the conservative and nationalist Otto von Bismarck.

The fact that victorious Germany took Alsace and Lorraine—almost—fanned the hatred of the French against the Germans. The beginnings of Third Republic in France are marked by an largely royalist National Assembly—legitimist and pro Orléans, until 1876, when Republicans became majoritary because the Chambord's Count[18] alias Henri V, refused the Crown of France for ideological[19] reasons. Sweden had a new king since 1872, Oskar II who was enlightened conservative and created in 1878 Stockholm College. Finland was under Russian rule since Napoleonian wars after seven centuries of Swedish occupation.

3.2 Teaching policy in France during the XIXth century

Before the French Revolution, most of teachings in France were the fact of religious people—for instance, Jesuits. In Year II of the French Revolution, the Convention voted a text[20] telling that teaching would be layman, free and necessary and the Daunou[21] law. That text claimed actually teaching was not necessary, organized an primary and secondary teachings. According the French Revolution, public teaching had to give culture to citizens—Enlightment influence—and legitimate/assure the survival of democracy. Universities had been replaced by professional schools in medicine and law at that time and «grandes écoles» such as Ecole Polytechnique and Arts et Métiers.

Under the «Consulat», the first high schools were created in 1802. In 1806-8, Napoleon Ist founded the «baccalauréat»—exam at the end of high schools, the imperial university[22] and forced the teaching[23] of philosophy in high schools. Actually, teaching monopoly by the state was organized by those laws: the different degrees of teaching would be the faculties—medicine, law, literature, sciences, the high schools, the colleges, the institutions, boarding schools and primary schools. Under the «Restauration», it was decided[24]that all communes should propose primary teaching to all children without any resources condition. In 1833, the Guizot law encouraged the foundation of primary superior schools for poor children, who could access neither college nor high school.

The Falloux[25]law of 1850 authorized the creation of catholic schools and confered to Catholic Church the control of their programms and teachers. There was no more important change before the

[18]The last Bourbon.
[19]One important point is that he wanted the white flag for France back, which was impossible
[20]5thNivôse, Year II.
[21]3rd Brumaire, Year IV.
[22]10thMay, 1806.
[23]17thMarch 1808.
[24]29thFebruary 1816.
[25]15thMarch.

His Grace Dupanloup about 1870

beginnings of the Third Republic. In the 1870's, the power of the clerical party, leaded by His Grace Dupanloup, was clear : the «Assemblée nationale» was in majority made of Royalists—Orleanists and legitimists. The Comte Jaubert proposed, in 31th July 1871, a new text in order to finish the state monopoly on superior teaching : there was three votes deliberations[26]about it, which went to the freedom on superior teaching law, in 12th July 1875—316 votes for and 266 against. At the end of 1870's, the Republicans were majoritary in the «Assemblée» and were capable of decide of a new policy in public instruction, which leaded to the famous Ferry laws, in 1880-2. They limited the power of Catholic Church in superior teaching, made the teaching layman, necessary and free for children between 6 and 13 years old. The Goblet law, in 1886, forbid to religious people to teach in public schools. Public instruction became the key of Third Republic.

3.3 Mathematical topics (analysis) in Europe during the first half of XIXth century

In the first half of XIXth century, some mathematicians tried to found rigorously the notions of real analysis[27], new analytical theories appeared and some ancient theories took a new turn.

In his *Cours d'analyse de l'Ecole Polytechnique*, Cauchy defined rigorously about 1820 the notion of convergence of a series, worked on continuity and derivability too. Bolzano worked also on continuity and prooved the intermediate values theorem in 1817 But continuity was not precisely defined enough. Weierstrass gave the modern way of its definition at the end of the century.

The theory of complex functions began with Cauchy's works; from 1815 to 1845, he worked on «Cauchy-Riemann equation», integration of a complex function along a contour, the residues, complex derivation. In the 1850 s, Riemann exposed the theory of multiform functions and the applica-

[26]3-4-5thdecember 1874, beginning June 1875, 8-12 July 1875.
[27]Infinitesimal calculus.

tions to abelian functions.

The theory of elliptic functions comes from the idea of rectification of ellipses: it began from the Greek antiquity and took another form with the analytical geometry of Descartes in XVIIth century. The elliptic curves were «transcendental» curves: they were studied first by the Bernoullis and Leibniz. Then Mac Laurin, D'Alembert, Euler studied those curves and at the end of the XVIIIth century, Gauss expressed certain elliptic integrals with the lemniscatic sinus and cosinus. In 1825, Legendre classified the elliptic integrals in three types and in 1826, Abel had the genial idea of inversing the elliptic integrals and considering them as complex functions. That was the rebirth of the elliptic functions theory : Jacobi, Gudermann[28], Liouville, Riemann, Hermite, Briot, Bouquet, Clebsch, Kronecker, Weierstrass, Mittag-Leffler ... continued the Abel's work. All along the XIXth century, the elliptic functions theory was a main topic especially in Germany and France.

4 ABOUT TEACHING COMPLEX FUNCTIONS ACCORDING TO HOUËL AND MITTAG-LEFFLER

The beginning of the correspondence between Mittag-Leffler and Houël deals with complex analysis and the way of teaching it. In 1872, Mittag-Leffler was docent in Uppsala university and taught already complex analysis ; Houël just published the first three parts of the *Théorie élémentaire des quantités complexes* and taught since several years the complex analysis in «licence ès sciences mathématiques». In his letter answering to the Mittag-Leffler's first letter, Houël wrote in July 1872 :

> Here's how I understand that we must expose this theory in elementary education $f(x)$. We start by taking a function f(x), well know for real values of the variable. One replaces x by a complex variable z and one verifies that—except for singular cases that I do not throw away for, and which I reserve for later discussion—the ratio $[f(z+dz)-f(z)]/dz$ approaches certain value, function of z, and independent of the argument of dz.

Mittag-Leffler answered in August 1872 :

> Define a function of complex quantities as a function obtained by replacing in the form of a real function $z=f(x)$ the real variable by a complex variable, has, in my opinion, several disadvantages.
>
> The first drawback is that you build then the theory of complex quantities on a previous theory of real quantities, while , in my opinion, the correct approach should be the opposite. The theory of real quantities should be a special case of the theory of complex quantities.
>
> A second disadvantage is that it is not sure at all that the fundamental relation $df/dx+idf/dy=0$ is a consequence of what we have for x real : lim $f(x+h)-f(x)/h = limf(x-h)-f(x)/h$. It will happen to the forms of elementary functions, but this can not be a property of functions in full generality .
>
> Moreover, if one relies on the definition of a function of a real variable , one must develop it in detail, and might one not then be facing the same problem as when you want directly define what is a complex function ?

[28] Weierstrass' master.

Finally, the problem of teaching complex functions lies in the inadequate state of the theory; in 1870, the notion of a general function is not clear neither the notion of a complex function, as Houël wrote in July 1872 :

> If we call any function of x quantity whose value depends on that of x in a certain way, then we must abandon establish a general theory of functions, because there is not, i believe, a single proposal for which one can not imagine a function that puts it in default. Some functions, such that $f(n) = 1 + 1/2 + \cdots + 1/n$, are essentially discontinuous. Others, though continue, have no differential, even for the case of real variables. Hankel[29]has shown than can be expressed by the signs of analysis—definite integrals or infinite series—functions with a finite range of singularities that make inapplicable to those functions all the rules of analysis. After showing none of the definitions proposed so far for the idea of function, take in its generality was satisfactory, he concludes that we must adopt to the function definition has given Riemann , which is nothing else that the definition of a monogenic function by Cauchy. This means that a non continuous and non monogenic function can not give rise to any general theory , and that in a theory that wants to be comprehensive , it must dismiss the functions that would put all the theorems in default.

In the first letter written by Mittag-Leffler, it is explicit that the different properties of complex functions are not well connected :

> As you see, I started my work by specifying the definitions of continuity, monogenity, monodromy. I wrote the definitions a little differently from what is usually done. In particular, the definition of monogeneity is of a different nature of this concept as it has over recent times. I do not think one is entitled, in an overview of the theory of synectic functions to evacuate the monogeneity from the building of synecticity.

And Houël in his first letter to Mittag-Leffler :

> Sometimes a function is synectic for the generality of the points in the plan and it ceases to be in some singular points ; and it can cease to be continue without ceasing being monogenic, as is the case for the points I have called, according to Neumann, the first kind of infinite, or it may lose both continuity and monogenity, as in the case of $e^{1/z}$ for $z = 0$ (second kind of infinite).

Both points of view on complex analysis teaching are interesting : Houël proposing a inductive way to make the students «feel» the principle although Mittag-Leffler would think of a deductive way in order to avoid to build «complex intuition» from «real intuition». Houël's point of view is the one of a pedagog but the Mittag-Leffler's one is entirely theoretical. In both points of view, it appears that the notion of a general function and a complex function is not clarified and the different properties - continuity, monodromy, synecticity, monogenity - of complex functions are not in the early 1870s well connected.

Another topic in the correspondence between Mittag-Leffler and Houël connected to complex analysis teaching is elliptic functions teaching. As soon as 1877, Houël asked Mittag-leffler in order to

[29]See (Hankel, 1870).

build a course and a chapter of his *Cours de calcul infinitésimal* reachable by his students of «licence ès sciences mathématiques» : that exchange of letters on that topic is quite technical, so we can't include it in our paper. We present the asking of Houël about it, in his letter dated march 1877:

> I am currently in a very great embarrassment, about a chapter on elliptic functions that I would like to insert in my *Cours de calcul infinitésimal*, and that was not in the original plan of the book. But a ministerial decree which introduced the elliptic functions in the program of the «licence ès sciences mathématiques» will require that I devote some pages. Secondly, the program of the exam is already very busy, I can hardly think of anything else to give an overview presentation was with rigor and at the same time in a manner quite elementary. There is a considerable number of approaches to this theory ; but the old methods of Jacobi, Gudermann, etc do not seem quite rigorously establish the double periodicity. There is the second method of Jacobi , adopted by Schellbach, based on the properties of the functions. But this method does not seem very easy for the beginners. You would do me the biggest favor by giving me some pointers on how I should follow. Is it possible, in thirty or forty pages, to share a simple an rigorous way to a beginner, possessing the first principles of the theory of functions of a complex variable, until the notion of the integral taken around a point—the residue of Cauchy ? Is it, I say, possible to teach seriously periodicity of elliptic functions, notions of integrals of second and third species and especially the expressions of these functions from functions θ?

That part of the correspondence about elliptic functions teaching for beginners is contained in eight letters.

5 THE ORGANIZATION OF MATHEMATICS TEACHING IN FRANCE, GERMANY ACCORDING HOUËL AND MITTAG-LEFFLER

The French mathematician Justin Bourget—father of the novelist Paul Bourget—and friend of Jules Houël thought that Houël was probably the person who knew the most about teaching organization in Europe, in the letter dated 17th April 1876 :

> What are we going to do for universities? You have to take advantage of it and study this topic thoroughly in a newspaper like *Le temps* ou *La république française*. You have spent a lot of time thinking about the subject, you are familiar with the way things are organized in other countries, you are able to say a million of intelligent things on the topic.

Jules Houël was interested for a long time by the organization of teaching generally and mathematics teaching ; he used to ask his correspondents how it worked in their homelands, as for instance to Mittag-Leffler, in his letter dated 13th September 1874,

> If you can spare the time I would appreciate if you could give me details about the way universities in Sweden, Finland and Russia are organized.

At that period of the 1870 s, which is the beginning of the Third Republic in France, Houël was very critical about the organization of teaching: according to him, there were too many vacations, too

many exams, not enough work from both students and professors. The following quotations are in the letter dated 13th September 1874 written by Houël:

> We nearly spend half the year doing exams or being in holyday. With the time we have left we only teach 2 lessons a week (I teach 5 of them as I can't accept the title of professor for nothing). Are there still professors who think teaching is tiring !
>
> You must know that there is a lot to say about further education in Paris. So what are you going to say once you have seen how it works in the rest of France.
>
> We don't have universities but only faculties which are part of universities but are not linked together. Divide ut imperes, the motto of dictators, to which the famous emperor has conformed.
>
> In France, public instruction is not better organized than in Sweden. I even think we should copy your institutions, especially those of your important universities.

Houël pointed out the fact that French faculties were too small and had too few students ; Houël had probably between 2 and 8 students[30]each year in his courses. The superior mathematics teaching was centralized in Paris where the «Ecole Normale Supérieure», the «Ecole Polytechnique» and the «Sorbonne University» drained the majority of the students. Many professors worked in both of those schools.

On the contrary, the organization of mathematics teaching in Germany and Russia appeared as an example to Houël :

> I don't know when France has universities as Germany or Russia (who have 7 without Helsingfors and Dorprat).

According Houël—Mittag-Leffler agreed—the origins of bad oragnization of education in France came from domination of catholicism and from Napoleon Ith' organization.

> I am entirely of your opinion: times have changed since the days when Latin et Greek in Europe have produced this great intellectual revolution as the Renaissance ! [...] We suffer the fate of all countries where Catholicism dominates. If governments of François Ist and Louis XIV had not paralyzed by their persecutions, as absurd as cruel, the beneficent influence of the Reformation, France would not have experienced all these setbacks,, and would have continued to hold its rank among the nations that lead the progress. [...] But here it is the bishops who interfere with that, and hardly wise minister had the time to abolish the absurd practice of Latin verse in high schools, His Grace Dupanloup restored it by his credit.

And

> . . . Void where our higher languishes, with the absurd organization bequeathed to us by evil genius of France, this evil being that insists on naming the great Napoleon! This scourge of God harmed in any possible way, but I think he did more fatal is its organization of public education, where he demonstrated both ignorance of a corporal and a Jesuit obscurantism.

[30]See (Zerner, 2008).

CONCLUSION

We have now to resume the information given about the correspondence between Mittag-Leffler and Houël in regard with their context. First, the mathematical considerations, which are the starting point of their epistolary exchange, are totally in the era of time: complex functions and elliptic functions. It is true that Houël had more a pedagogic position although Mittag-Leffler was a theorist and specialist of complex analysis—Houël was an astronomer. They felt that those theories were not achieved but Mittag-Leffler was more modern in his points of view in particular in the idea of making a theory of continuous functions.

About public instruction and its organization, they agreed—that's why we present only the Houël's point of view: about it, Houël's ideas were revolutionary in the 1870 s. Houël promoted the model of German universities against the French organization. Houël wasn't content of remarking it : it analyzed the failure on the one hand by the centralized organization and the baccalaureate weight decided by Napoleon Ist and on the other hand by the influence of the clerical Party in France. Mittag-Leffler who studied in France and Germany confirmed the views of Houël.

To conclude, we would like to compare these topics with those of other correspondences of Houël or Mittag-Leffler to show its originality. The main correspondences with Mittag-Leffler already studied are with Poincaré and Hermite. The correspondence between Mittag-Leffler and Poincaré deals with the participation of Poincaré to Acta Mathematica; the one between Mittag-Leffler and Hermite is more diverse: it deals with different mathematicians, new mathematics—set theory of Cantor for instance—and mathematics in general.

The main correspondences with Houël already studied are with Beltrami and De Tilly. Both have as a great topic the non Euclidian geometries; with Beltrami, it is more about differential geometry and geodesy and with De Tilly about generalities and foundations of those geometries. In regard of those different correspondences, the correspondence between Mittag-Leffler and Houël is really interesting because it is in tune with the mathematical times but really personal too in their analyses on public instruction, which is a great topic of the Third Republic in France.

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