TEACHING AND LEARNING OF FUNCTIONS IN MODERN JAPAN

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ABSTRACT

The concept of functions was introduced into Japan around 1860, in learning differential and integral calculus. At first, a function was considered as a synonym of expression. At the Imperial College of Engineering in Tokyo, Professors Ayrton and Perry taught students engineering and mathematics unified together. They used squared papers extensively from 1876. Functions were treated extensively there.

The ideas of the movement for reform of mathematical education, advocated by Perry and Klein, were introduced into Japan early in the twentieth century. Functions and graphs were gradually introduced into school mathematics. In 1942, a drastic change of the curricula of the mathematics and natural sciences for secondary schools was made. Functions played a central role in the new syllabus of mathematics. It was a reception of the ideas of reform of mathematics education as well as the adoption of the traditional way of learning in Japan. Though the syllabus was not carried out completely due to the war, functions have played an important role in school mathematics since then.

1 Introduction

This paper deals mainly with teaching and learning of functions in Japan from 1860s to the middle of the twentieth century. In the following, names of Japanese and Chinese are written in the order of surname and given name, according to the customs in Japan and China.

First we mention briefly about learning of Western mathematics in Japan up to 1870s. Japan closed the country to outsiders from the seventeenth century to the middle of the nineteenth century. Only very limited trade with China and Holland was permitted. Dutch and Chinese vessels were allowed to trade at Nagasaki. Christianity was strictly forbidden. Even Chinese translations of Western books on natural science, unrelated to Christianity, if these books were translated by missionaries, were prohibited to import to Japan until 1720. As a result, only a very small number of Japanese had some knowledge of Western mathematics at the period of national isolation. Most of them had only some knowledge of Western elementary mathematics: for instance, arithmetic only, or arithmetic and elementary algebra. Only a few of them learned some topics of higher mathematics.

Until early 1870s, the ways of learning Western mathematics were as follows:

(1) learning mathematics under a Western teacher,

(2) learning mathematics under a Japanese teacher having some knowledge of Western mathematics,
(3) learning mathematics from Western books,
(4) learning mathematics from Chinese translation of Western books,
(5) learning mathematics from books on Western mathematics written in Japanese.

Among these, (1) was realizable only for very limited ones. Others learned Western mathematics either under a Japanese teacher at first, then by themselves from books, or by themselves from books from the beginning. Western mathematics was learned mainly from Western books, written in Dutch in the early days, later in English or in French, sometimes in German. Western mathematics was also learned from Chinese translations. For Japanese at that time, books written in Chinese were easier to read than those written in Western language. Publication of books on Western mathematics written in Japanese began in 1857, only elementary mathematics at first. For learning higher mathematics, (5) is excluded, as there were no books on higher mathematics written in Japanese at that time.

After the Meiji Restoration (1867–1868), Japanese Government intended to modernize Japan by introducing Western civilization, especially Western technology, into Japan. Modern educational system was introduced in 1872. As to mathematics, the Department of Education decided to teach only Western mathematics at all school levels—teaching of traditional Japanese mathematics was abolished. It was too radical to be carried out, however. Many teachers were unfamiliar with Western mathematics. So the curriculum of arithmetic of elementary schools was revised within a few months. Since then, mathematics has been taught in Japan in Western style, with some consideration on the traditional way of calculation, the use of *soroban* in elementary arithmetic.

2 The concept of functions introduced into Japan in the nineteenth century

The concept of functions was introduced into Japan around 1860 in learning differential and integral calculus. Until 1870s, the number of Japanese who learned elements of differential and integral calculus was very small, and most of them learned calculus from books, written in Western language or by Chinese translation of Western books. Among these, the following books played important roles in learning differential and integral calculus at that time.

(1) "Elements of the Differential and Integral Calculus" by Albert Ensign Church, revised edition,
(2) "A Treatise on the Differential Calculus", "A Treatise on the Integral Calculus" by Isaac Todhunter,
(3) "Daiweiji Shiji" by Alexander Wylie and Li Shanlan (Chinese translation of "Elements of Analytical Geometry and of the Differential and Integral Calculus" by Elias Loomis); Japanese call this book "Daibiseki Shūkyū",
(4) "Weiji Suyuan" by John Fryer and Hua Hengfang (Chinese translation of an article by Wallace); Japanese call this book "Biseki Sogen".

In most books on differential and integral calculus which were introduced into Japan in the nineteenth century, a function is defined as a variable quantity connected with another variable quantity (or other variable quantities). For instance, Church defines a function as follows (Church 1876, p. 1):

> One variable quantity is a function of another, when it is so connected with it, that any change of value in the latter necessarily produces a corresponding change in the former.

We quote also here from Todhunter's Differential Calculus. Todhunter gives some examples of functions (Todhunter 1871, p. 1):

> Suppose two quantities which are susceptible of change so connected that if we alter one of them there is a consequent alteration in the other, this second quantity is called a *function* of the first. Thus if x be a symbol to which we can assign different numerical values, such expressions as x^2, 3^x , $\log x$, and $\sin x$, are all *functions* of x. If a function of x is supposed equal to another quantity, as for example $\sin x = y$, then both quantities are called *variables*, one of them being the *independent variable* and the other *dependent variable*. An *independent variable* is a quantity to which we may suppose any value arbitrarily assigned; a *dependent* variable is a quantity the value of which is determined as soon as that of some independent variable is known.

In "Daiweiji Shiji", a function is defined as an algebraic or analytic expression of another variable quantity (or other variable quantities).

In "Weiji Suyuan", a function is defined like in Church, but actually a function is regarded as an algebraic or analytic expression of the independent variable(s).

Simple examples of functions and most functions treated in the textbooks of calculus are those represented by algebraic or analytic expressions. At that time, an algebraic function was defined as a function represented by an algebraic expression of the independent variable. This is the old usage of the term "algebraic function". Therefore, even if the students regarded functions as algebraic or analytic expressions of independent variables, setting aside the definition of a function in the textbook—just like the case of "Weiji Suyuan", they wouldn't have much difficulty and trouble in learning elements of calculus. In this way, "functions" were actually considered as a synonym of "expressions" at that time.

Graphs of functions are treated simply in the textbooks of differential calculus at that time. Textbooks at that time laid emphasis on geometric properties of various plane curves and tracing of curves as applications of differential calculus to geometry, and not on the graph of a function as a geometric representation of the function.

3 Functions taught at Tokyo Kaisei Gakkō

We mention in this and the next sections briefly about educational institutions in Japan where elements of calculus (and consequently the concept of functions) were taught in 1870s and in the early 1880s.

Tokyo Kaisei Gakkō, a predecessor of the University of Tokyo, was established in 1873 as an institution of higher education giving professional education in various fields, and the origin of this institution was "Yōgakusho" (Institution of Western Studies), established by Tokugawa Government in 1856. In 1877, the University of Tokyo was established by amalgamating Tokyo Kaisei Gakkō and Tokyo Igakkō (Tokyo Medical School). At Tokyo Kaisei Gakkō, five departments of special and technical learning were intended at first: Law, Chemical Technology, Engineering, Polytechnical Science, and Mining. Among these five departments, the first three were planned to be taught in English, the fourth in French, and the fifth in German. Professors were invited from Western countries. Mainly for

financial reasons, however, Mining was abolished, and Polytechnical Science was reduced to Physics (in French) at first, and then was also abolished in 1880.

Engineering course started in 1874 by appointing Robert Henry Smith, a graduate of the University of Edinburgh, to Professor of Mechanical Engineering. He taught mechanical engineering and higher mathematics in the academic year 1874–1875, and in the subject "Higher Mathematics" he taught quaternions and differential and integral calculus. From examination papers of Differential Calculus and those of Integral Calculus recorded in the "Calendar" of Tokyo Kaisei Gakkō, we find that an outline of differential and integral calculus was taught in that academic year. Textbook of differential and integral calculus was unknown, but, judging from the examination papers, the textbook was easier one than Todhunter's.

In the next academic year, "Higher Mathematics" was taught by James R. Wasson, Professor of Civil Engineering. He was a graduate of West Point Military Academy in the United States, and he taught calculus using Church as the textbook. Church's books were used as textbooks at West Point for a long time, and Wasson taught calculus based on the method at West Point.

Mathematics and Physics taught in Physics in French course were, until its end in 1880, of the highest level in Japan at that time. According to the curriculum of this course, the final topic in mathematics was the mathematical theory of heat, but the details of the actual teaching of each topic are unknown. The concept of functions and an introduction to differential calculus were taught in a subject "Complementary Algebra" before learning "Differential and Integral Calculus", following the curriculum of mathematics in lycée in France. Judging from the outline of the contents and order of "Complementary Algebra" cited in (Ogura 1948, p. 189), the author considers that the main textbook of this subject was Briot's one (Briot 1874).

In the University of Tokyo, the curriculum of mathematics at the Department of Science was drawn up in 1880. It was modeled after that of Great Britain. Textbooks of Differential Calculus and Integral Calculus were Todhunter's ones. As to functions, the first teaching of the theory of functions of a complex variable and elliptic functions was carried out in the academic year 1883–1884 by Terao Hisashi (1855–1923), Professor of Astronomy. Though the details are unknown, the author considers that the topics were treated in French style. For, Terao was a graduate in Physics (in French) of the University of Tokyo, and he studied astronomy in Paris.

4 Teaching of functions and graphs at Kōbu Daigakkō

Kōbu Daigakkō, or the Imperial College of Engineering, is a predecessor of the College of Engineering of the University of Tokyo. The college was planned in 1871 by Kōbushō, the Department of Public Works of the Government, as a college to train students to be engineers serving as government officials in that Department. Main factories in Japan were under government management at that time. The actual start of the college under the name of Kōgakuryō was in 1873. All professors were invited from the United Kingdom, most of them from Scotland. The principal of the college was Henry Dyer (1848–1918), Professor of Civil and Mechanical Engineering. He was a graduate of the University of Glasgow. He stayed at this College until 1882. Professor of Mathematics (from 1873 to 1878) was David H. Marshall, a graduate of the University of Edinburgh. Among the professors were William Edward Ayrton (1847–1908), Professor of Natural Philosophy and Telegraphic Engineering from 1873 to 1878 and John Perry (1850–1920), Professor of Civil and Mechanical Engineering from 1875 to 1879.

It was a six-year college of technical education. The whole course was divided into three: (1) the general and scientific course, the first two years, (2) the technical course, the next two years, and (3) the practical course, the final two years. Theory and applications, teaching and learning in school and practical training outside school were unified together. As to mathematics, a subject "Elementary Mathematics" was taught firmly in the general and scientific course. It was a standard course of elementary mathematics with some applications to practical problems, and not a merely application-oriented one. The contents were geometry, algebra, plane trigonometry, mathematical tables, spherical trigonometry and geometrical conics. Coordinate geometry and calculus were treated in "Higher Mathematics" in the technical course.

In the technical course, Ayrton and Perry taught subjects of engineering by unifying technology and mathematics together. They taught science and engineering by applying mathematics, and also topics of higher mathematics through solving practical problems in science and engineering. They used squared papers extensively since 1876. Functions were treated extensively, not only functions represented by analytic expressions but also functions represented by tables or graphs. They taught also finding of functional relation between two kinds of quantities from experimental data by plotting these on a squared paper. It was an epoch-making event in the history of the teaching of the concept of functions. Perry's experiences in Japan and United Kingdom resulted in his idea of "Practical Mathematics" and that for reform of mathematical education setting "utility" as the core.

In 1886, this College merged with the University of Tokyo, and the Imperial University was established. At the College of Engineering of the Imperial University, mathematics was taught by developing the methods in Kōbu Daigakkō. In 1890s, mathematics was taught by Inokuty Ariya (1856–1923), Professor of Mechanical Engineering. He was a graduate of Kōbu Daigakkō and was taught from Perry there. Inokuty taught calculus emphasizing graphical methods.

5 A step to the spread of the concept of the functions

First we consider books on calculus written in Japanese in the early Meiji era. Publication of Japanese translation of books on higher mathematics began 1872, and the number of publications treating some topics of higher mathematics began to increase from the middle of 1880s, though slowly.

Fukuda Han, who was a mathematician and a military engineer captain, intended to translate "Daiweiji Shiji" into Japanese. The Japanese translation "Daibiseki Shūkyū Yakukai" was planned to be published in ten volumes altogether, but actually only the first volume was published in 1872, after reviewed by his father Fukuda Riken (Fukuda Izumi), a mathematician who studied Wasan at first and wrote books on Western elementary mathematics in 1870s. The contents of "Daibiseki Shūkyū Yakukai" were an abridged translation of the first four books of "Daiweiji Shiji", that is, an introduction to coordinate geometry in plane, and no functions in this volume. In Japanese translation, formulas were written as in English original, and not in Chinese style.

Eight years after, Fukuda Han edited and, after reviewing by Fukuda Riken, published a book on differential and integral calculus, "Hissan Biseki Nyūmon" (Introduction to Differential and Integral Calculus by Written Calculation) in two volumes in 1880. This is the first book on differential and integral calculus written in Japanese. This book was edited by referring several books on calculus written in English and those written in Chinese. Among these, "Daiweiji Shiji" and Loomis' original text, and "Weiji Suyuan" played important roles. Formulas are written in Western style. A function is defined

as in “Weiji Suyuan”. “Hissan Biseki Nyūmon” was a book on differential and integral calculus in the transition period from Wasan to Western mathematics. Fukuda Riken and Han comprehended differential and integral calculus relating with Wasan, especially “Enri” (literal translation: theory of circles, a branch of higher mathematics in Wasan). For instance, we see in the exercise problems of “Hissan Biseki Nyūmon” mensuration of various figures as are seen in Wasan books.

Japanese translation of Todhunter’s “Differential Calculus” was published in 1881, and that of his “Integral calculus” in 1882. The translator was Nagasawa Kamenosuke (1860–1927).

Okamoto Norifumi intended to translate Church’s book on calculus into Japanese. Only the first part, differential calculus was published in 1883. In translating into Japanese, he made some enlargement to the original text; he considered it better to Japanese audience at that time.

In this way, textbooks on differential and integral calculus were translated into Japanese. At higher educational institutions, however, the textbooks of calculus were Western books written in English such as Todhunter of Williamson and not the ones written in Japanese until 1920s. For, the styles of Japanese translations was rather stiff, and, for a person who had a fair knowledge of English, reading original English text would be better for learning calculus.

As mentioned in previous sections, Japanese in the 1870s and in the early 1880s learned the concept of functions in learning calculus, a branch of higher mathematics. Therefore, only a very small number of Japanese had some knowledge of functions. In Western countries at that time, textbooks of elementary algebra which treated functions and graphs and books on mathematics for general audience containing some topics from higher mathematics were already published.

For instance, “Common Sense of Exact Sciences”, a posthumous publication of William Kingdon Clifford (1845–1879), edited by Karl Pearson, treated several topics from “higher mathematics”. It was a new departure in a book on mathematics for general audience. Functions and graphs were treated in this book. This book was translated into Japanese by Kikuchi Dairoku (1855–1917), Professor of Mathematics at the Imperial University, and published as “Sūri Shakugi” (it means: Explanation of Mathematical Sciences) in 1886.

Japanese translations of the textbooks of algebra which treated functions and graphs were published from the late 1890s: for instance, H. Bos’s textbook of algebra in French and George Chrystal’s “Textbook of Algebra” in 2 volumes and “Introduction to Algerbra”. Some books on algebra written by Japanese authors also treated functions and graphs. In this way, the concept of functions were, though very slowly at first, spreading in Japan since the last decade of the nineteenth century.

6 Introduction of functions and graphs into school mathematics

In 1902, the syllabus of middle schools (in Japanese: *chugakkō*), boys’ five-year schools for general education, was officially announced. As to mathematics, mathematics is divided into four subjects: arithmetic, algebra, geometry and trigonometry, and each subject should be taught rigorously by its own method. There are no functions except the term “circular functions” in the syllabus of trigonometry.

The movement for reform of mathematics education from the beginning of the twentieth century, advocated by John Perry and Felix Klein (1849–1925), was intended to reform school mathematics into modern and practical one by introducing the concept of functions into school mathematics. Perry stressed radical reform of mathematics education by setting “utility” as the core, and by getting rid

of Euclid. Klein regarded the concept of functions in geometric form as the central idea in school mathematics.

The idea of the movement for reform was introduced into Japan early in the twentieth century, first by Hayashi Tsuruichi (1873–1935), Professor of Mathematics at Tohoku Imperial University, Kuroda Minoru (1878–1922), Professor of Mathematics at Tokyo Higher Normal School, and Inokuty Ariya, Professor of Mechanical Engineering at the Imperial University of Tokyo. Kuroda studied mathematics education under Klein at Göttingen, and Inokuty was greatly influenced by Perry at Kōbu Daigakkō. Hayashi and Kuroda worked eagerly for the spread of the new trends of mathematics education, especially Klein's idea for reform. Inokuty introduced Perry's idea, especially that of "Practical Mathematics", and worked actively for the spread of the idea of practical mathematics and Perry's idea for reform of mathematics education into Japan. He was the first advocate of Perry's idea in Japan. But his activity was limited within the technical schools and not to teachers of mathematics at schools for general education. Meanwhile, some of the ideas for reform, for instance, the use of graphs as the visualization of functions, were gradually known to mathematics educators and teachers in the first two decades of the twentieth century.

The syllabus was revised in 1911. The new syllabus of mathematics begins with a remark:

> Though the syllabus of mathematics is written by dividing into four subjects: arithmetic, algebra, geometry and trigonometry, mathematics should be taught always considering mutual relations among (contents of each year and) subjects.

The syllabus had no mention of functions. Responding the remark cited above, however, "teaching of mathematics with consideration for the connection of algebra and geometry" became a topic to be considered in teaching of mathematics in middle schools in Japan, and it resulted in the introduction of the concept of functions into mathematics in secondary education.

Publication of textbooks of algebra for middle schools treating functions and graphs began in 1913. Namely, a textbook of algebra by Hayashi and that by Kuniyeda Motoji (1873–1954), Professor of Tokyo Higher Normal School, both published in 1913, treated functions and graphs. These textbooks were in use since 1914.

In Kuniyeda's book a function is introduced in the fourth year class, in the last chapter after learning ratio and proportion.Coordinates, graphs and functions are introduced starting from proportional relations

By arranging a chapter on functions and graphs in the end, previously learned topics may be summarized by using the idea of functions, and also pupils can see a new view of mathematics from the standpoint of "functions". Moreover, even omission of the chapter is possible: for instance, in case of shortage of school hours, and for a teacher who is passive to introduce functions in school mathematics.

On the other hand, Hayashi treats functions from the beginning. Functions are introduced in the second year class, the first year of learning algebra. After learning an algebraic expression and its values, graphical representation of the variation of the values of an algebraic expression is introduced. Then, coordinates, functions and graphs are introduced. Linear equations and linear functions are treated simultaneously. Hayashi's plan is based on Klein's idea: "The concept of functions in geometric form should be the central idea in school mathematics".

In this way, the idea of "introduction of the concept of functions into school mathematics" had been gradually spreading among teachers of mathematics in Japan. But, in the early stage, teachers of mathematics often understood "teaching of functions in geometric form" merely as "teaching of graphs" or as "teaching elements of coordinate geometry as a topic of connection between algebra and geometry".

By the way, the year 1913 was also the year of the first publication of a book on the theory of functions by Japanese author. Namely, "Kansūron" (Theory of Functions) by Yoshikawa Jitsuo (1878–1915), Professor of Mathematics at Kyoto Imperial University, was published in that year. Yoshikawa was a graduate of the Imperial University of Tokyo and studied analysis under David Hilbert in Göttingen. In this book, elements of complex analysis and some advanced topics were explained clearly, and the idea of Riemann surfaces were introduced early.

The Mathematical Association of Japan for Secondary Education, the predecessor of the present Japan Society of Mathematical Education, was established in 1919 to improve mathematical education in Japan. In 1924, both Ogura Kinnosuke (1885–1962) and Sato Ryoitiro (1891–1992) stressed reform of mathematics education in middle schools in Japan by taking the concept of functions as the central idea, and they also stressed that the idea of differential and integral calculus should be introduced as a final step of teaching the concept of functions. Ogura argued the necessity of radical reform of mathematics education by getting rid of Euclid and following the idea of Perry. According Ogura, "the essence of mathematics education is development of scientific mind of pupils" and "the core of mathematics education is cultivation of the concept of functions". Ogura's book (Ogura 1924) had influence on teachers and educators, especially those of elementary schools, and it resulted in a sweeping revision of the textbook of arithmetic in elementary schools in the thirties. Sato explained in his book of 1929 teaching of mathematics in middle schools more in detail by giving teaching plans that he had already practiced at the Middle School Attached to Tokyo Higher Normal School.

The curriculum of middle schools was revised in 1931 to cope with the spread of secondary education and to meet the demand of the times. The new syllabus of mathematics was very simple and indicated only main contents of the each year, without dividing into subjects such as algebra, geometry and so on. The syllabus also indicated that,

> In teaching mathematics, cultivation of the concept of functions should always be kept in mind.

This was the first mention of functions in the syllabus of mathematics of middle schools. Functions were not the central topics in the curriculum, however.

The national textbook of arithmetic for elementary schools was renewed entirely in the thirties. The new textbook, "Jinjō Shōgaku Sanjutsu" (Arithmetic for Elementary Schools), was edited and published by the Ministry of Education every year for one grade, beginning 1934, and the new textbook came into use from the first grade children of the school year 1935. It was edited following the ideas for reform of mathematics education from the beginning of the twentieth century and intended to develop mathematical thinking of school children through their various activities. Graphs and topics related to the concept of functions were treated extensively.

7 A drastic change

In 1942, the curricula of mathematics and science for middle schools and those for girls' schools were revised thoroughly and drastically. The revision was to cope with a time of crisis for the nation and to meet the pressing demand for the improvement of science and mathematics education at that time. Until that time, teaching of science in secondary education was rather old-fashioned. The new curricula were intended to cultivate scientific minds and creativity of pupils so as to cope with various difficult situations which they will encounter and to meet the national demand of the times.

The aim of the new syllabus of mathematics was the reorganization of school mathematics by getting rid of the "traditional system" of mathematics education, to cope with the national movement for the new order at that time. The new syllabus was intended to develop pupils' mathematical thinking and creativity through their various activities. Functions played a central role in the new syllabus of mathematics. Utility and applications of mathematics were emphasized. Many new topics were introduced in mathematics of middle schools: elements of analytic geometry, nomography, descriptive geometry, elementary probability and statistics, and the idea of differential and integral calculus. On the other hand, Euclidean geometry was not treated in the traditional way.

The syllabus indicated in an item of the remarks that

> Thoughout the syllabus, attention should be paid to cultivate the concept of relations.

The concept of relations is far more broad than that of the functions. The author considers that it was a step forward to introduce fundamental ideas in modern mathematics into school mathematics.

Emphasis was laid on learning by doing and heuristic methods: to let pupils discover mathematical facts through their various activities such as observations, experiments and considerations, and to let them synthesize and systematize these mathematical facts with appropriate suggestions and advices of the teacher. To acquire knowledge, doing was required. This was just the traditional way of learning in Japan: learning through training and self-study—acquisition of knowledge through training by themselves.

Emphasis was laid also on utility and applications. Knowledge without accompanied by practice was regarded as worthless at that time, and "integration of knowledge and practice" was advocated. This was influenced by learning through training, the traditional way of learning in Japan and the doctrine of inseparability of knowledge and practice by Wann Yann Ming (1472–1528), a Chinese philosopher of early sixteenth century. By the emphasis on utility and applications, many ideas which Perry had proposed and had stressed from the late nineteenth century were realized in the curriculum. It was a reception of the ideas of reform of mathematics education since the beginning of the twentieth century as well as the adoption of the traditional way of learning in Japan.

This curriculum was not carried out completely due to the World War II. This curriculum is essentially a curriculum in the peace time and not the one in the wartime, and careful preparations were essential for the carrying out the curriculum successfully, as Ogura and Nabeshima wrote in their book (Ogura & Nabeshima 1957). Though the curriculum was not carried out completely, functions have played an important role in school mathematics since then.

8 After the World War II

In this section we mention briefly about the teaching of functions after the World War II. After the war, the educational system in Japan was reformed entirely. The new system came into operation in 1947. Since then the curricula for elementary, lower-secondary and upper-secondary schools were revised several times, about every ten years.

Mathematics in upper-secondary schools has been taught systematically. Functions have been playing important roles. Elements of differential and integral calculus were taught systematically, not like the one in the syllabus of 1942, in an elective subject of mathematics, from the start of upper-secondary schools.

The revision of the Course of Study in 1960 was intended to develop students' basic knowledge and skills and to improve scientific and technological education so as to meet the demands of the society. Calculus and analytic geometry were enriched, and some new topics such as vectors and the idea of sets were introduced. On the other hand, to cope with the spread of upper secondary education, subjects whose contents were only basic ones were prepared.

Introduction of calculus into school mathematics was, at first, to introduce the ideas of calculus into school mathematics as a final topic of teaching the concept of functions. Since the 1960s, calculus in upper secondary schools has been changed to lay stress on students' acquisition of skills to cope with the rapid development of science and technology.

The New Math Movement, influenced by the works of Bourbaki group, spread worldwide from 1960s to 1970s. As a result, upper secondary mathematics leaned towards formal and abstract one and not practical one. Mathematics education from 1960s to 1970s was going in the direction opposite to the direction in which it had proceeded since the beginning of the twentieth century.

Functions are playing an important role in school mathematics since 1940s, and the introduction of elements of differential and integral calculus into secondary education has been made. In this way, popularization of calculus has been made.

The spread of upper-secondary education, emphasis on skills in school mathematics, introduction of new topics of abstract nature such as the idea of sets, and popularization of calculus caused new problems. For instance, students learn calculus of simple polynomial functions at first. Polynomial functions, however, can differentiated and integrated formally without using the concept of limits. Therefore, students can solve routine problems concerning differentiation and integration of polynomial functions just as solving problems of calculations in algebra, without understanding the concepts of functions and limits. We should make efforts to improve the situation and to develop teaching of mathematics.

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