SOCIAL CONSTRUCTION OF THE ALGEBRAIC STRUCTURES

A Model for its Analysis

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ABSTRACT

It is written the first part of the results of an investigation that characterizes the Social Construction of the Algebraic Structures using the methodology of an historical analysis that allowed to give account of the social context and as well as of the circumstances in which diverse books are published and articles that were written between 1870 and 1945 around the Algebraic Structures. A conceptual system was constructed to characterize the constituent elements of the social construction of the algebraic structures within the framework of some theoretical constructors on the social construction of the knowledge of P. Berger and. Chevallard; internalization, externalization, diffusion, representation and reproduction of the knowledge. Under this constructed frame we described three processes that comprise of the epistemological construction of the algebraic structures that we described by contextual phases they locate in the time. For each phase we emphasized the importance of action that certain agents that, in the history of algebra, they are marked by his incidence in the consideration of the importance of the formal scheme that prevails in the education of abstract algebra in the level superior.

Keywords: Keywords: Social Construction, internalization, externalization, validity, legitimation and institutionalization of knowledge

1 Introduction

The mathematical knowledge that is found proposed in the programs of study of the different educational levels have its origin, in principle, in the mathematical knowledge developed as knowledge used. This knowledge generally suffers a process of functional transformation to convert in teachable knowledge, processes that Yves Chevallard it has called, didactic transposition. However, the mathematical knowledge that will be transposed has passed already for at least three processes previous to those of the transposition; it has been validated by the academic community that produces, it has been institutionalized for this same community or a great that contains them and has been legitimated as a didactically significant knowledge.

The validity of the knowledge is tied to who to produces and it happens in group of agents that are integrant of one determined academic community. The legitimation as a didactically significant knowledge, on the other hand, is tied to whom, without having produced the knowledge, it qualify it

in order that this one is incorporate to the system of teaching, by distinguishing it and by separating those of its history and of the community that validated it to put it to disposition of who transposes it (Chevallard, 1998)

Chevallard consider that the social construction of the knowledge begins when is diffused, moment to the one that arrive after that this knowledge already has passed for a process of to clear the personality of its producer and begins to fulfill totally different functions; reproduction and representation of the knowledge. In the sociological ideas of Berger (1969), we can locate this moment with the one that he calls externalization through which the knowledge is label as human product. According to this author the externalization is the permanent overturning of the human being towards the world, so much of the physical activity as mental and mark the moment by means of which the material objects and the non-material created for the man reaches the objective reality. This material and symbolic objects enrich the totality of the existent objects and impose its logic to the individuals that they employ them. The cultural permanence of these objects (its epistemological valence, according to Chevallard), it will depend of a specific community and of the use that this give you.

The symbolic objects and material understood as product of the physical and intellectual activity of the man acquires meant for other through the internalization process, causing that the man becomes converted in product of the society according to Berger. The actions of the other also is internalize for the man and they begin to acquire for it a significant character by converting in a typical conduct it arrives even to normed their own actions. In the measure in which is given this internalization and in the measure of the intersubjective that they acquire the objects and actions in a community of individuals, it is constituted a process of institutionalization of the knowledge, reaching the maximum level as social construction, that is to say, once the actions is identified as one form to act and they are habit, that form to make the things in the frame of meanings associated to the objects symbolic and material they constitute a program that regulates the interaction of the integrant of the community and that they put into practice as prescribed execution manners that it enjoy an acceptance generalized and unconditional (Berger and Luckman, 1997). To other communities to correspond to transform the knowledge institutionalized as object of knowledge in object to teach by means of the process of legitimation, eliminating the context significant to put it in hands of the community that finally will convert it in object of teaching through the process of didactic transposition.

The diffused knowledge, either for oral media and/or writings it is put within reach of other individuals that are part of certain communities and for every one of these communities this knowledge acquires a different meaning; for the academic community, for example, this knowledge is a susceptible knowledge, in principle, of being questioned and afterwards validated. It is the academic community who finally will produce it as scientific knowledge, as product of the activity of the mathematicians. According to Chevallard is the community that transformed it in object of knowledge.

According to the model described so far, a study of social construction of knowledge accounts for the moments of externalization, internalization, the process of validation, legitimization and institutionalization, the communities where such moments and processes occurred, of the agents that make up these communities, the role they have in them, the actions performed and cultural conditions that frame these actions over time. Thus, each of these elements are considered constituents of the social construction of knowledge.

This article describes the framework that occurred the axiomatization, classification and systematization of particular algebraic structures as processes that are part of the development of tools and

creation of a language that shape and support the emergence of the concept of algebraic structure that we located as products of moments of externalization. We place the productions written and reported by some authors (Corrry, 2002 (d), Katz, 2007, Hernandez, 2009) as important by its incidence about the exposition of a way new to generate mathematical knowledge, as well as the influence that they had in the emergency, of the call, structural vision of the mathematical one that it consolidates as well conceptually to the algebraic structures.

2 Antecedents

In the educational mathematics a study of social construction of the knowledge can be given in two lines; 1) The context of the school, with the end of understanding how the students and teachers build the school mathematical knowledge and the factors that, of one or otherwise, they influence in this construction and 2) The social construction of the mathematical knowledge as knowledge produced in the academic community and what happen until becoming in teachable knowledge, for understand or give sense to its presence in the escolar system.

The search that we did and whose results we present immediately, they allowed, by a side, to notice the scarce investigation that there is in the plane of our interest (the second) and by other, identify the difficulties of the students in the apprenticeship of the algebraic structures for, in our investigation, follow the track that what is tried get the apprentice, taking those subjects to the classroom.

They are few the studies that, as those of Katz (2007), they describe the happen historical of the algebraic structures as mathematical knowledge that after passing by the processes of didactic validity, legitimation, institutionalization and transposition they are imprisoned of individual or collective ideological and philosophical conditioners that do it little or more attainable in a education-learning process. The article of Katz, present the history of the divided algebra in four stages that he calls "conceptual stages" ; the geometric, the static thing of solution of equations, those of function dynamics and the abstract. Describes how is what, the century XX, algebra had become converted in the search of common structures of diverse mathematical objects for establishment of an axiomatic system that it would systematize them. Katz asserts that before teaching the algebraic structures of groups, ring and fields it is necessary that the student know sufficient examples of these to give you a chance to consider them as a useful and necessary generalization.

In the investigations about the problematic of the apprenticeship of the abstract algebra, the area of the mathematics in which is located principally the study of the algebraic structures (groups, rings, modules, between others), we identify two aspects that they are related directly with the interest of our investigation. One of them is the emphasis that it makes in the problem they have the students to formalize concepts that if they obtain to construct with respect to different mathematical objects, they cannot symbolize because a language and a system of appropriate symbols lack that allow them, then, to manipulate them (Dubinsky, Dauthermann, Leron and Zazkis, 1994) and by other side, the emphasis that becomes in the incapacity which they present the students who have identified conducts or characteristics common in sets of mathematical objects but that they cannot organize nor systematize so that, he is useful for the construction of new mathematical knowledge (Simpson and Stehlílková, 2006).

Works as those of Hazzan (1999), they raise that the students of abstract algebra tend to reduce the level of abstraction of different forms to achieve understand the concepts and properties of this

subjects, converting them unconsciously in mentally more accessible concepts.

The structural vision; conception of the generated mathematics by David Hilbert (1862–1943) where the form to represent and to reproduce the mathematical knowledge, occurs as an abstract way that it unifies several existent theories until before 1930, principally in the theory of algebraic of the numbers and the theory of the polynomials; it seems to be the origin of the aspects problematic that interested us and that are described in the mentioned investigations, this agrees with some initial conclusions at that we arrived after a first closeness to the historical-epistemological evidence of the social construction of the algebraic structures. This preliminary analysis, carried out as part of the methodology that we employ, it showed these they emerge once particular structures, that it spring up in diverse fields of the mathematics, in moments and different stages, its axiomatic is constitutes, is classified and is systematized afterwards of its construction. More still the genesis of the actions that is organized in a systemic and that generates the emergency of the structural vision, occurs to satisfy a philosophical and ideological perspective that it proposes new forms in the construction of mathematical knowledge, actions that they are regulated by intentions also of philosophical and ideological character and propose new forms of construction of mathematical knowledge.

3 A Model for the Characterization of the Social Construction of the Algebraic Structures

We leave from the principle of that the reality is constructed socially, by understanding this reality, *"as quality of the phenomena that we recognize as independent of our volition"* (Berger y Luckmann, 2001, p. 13); this quality dominates our actions and our ideas in the measure in which we recognize. Each of the phenomena they compose to the reality have specific characteristics and each individual accumulates knowledge of this characteristics that they give meaning and they go constructing the reality at the same time as it regulates the actions that in it carries out. It establishes a *dialectic process* between the reality that its imposes as pre-existent and the actions that in her they are realized by means of which the society constitutes itself and consequently the social contexts in that those phenomena finish immersed.

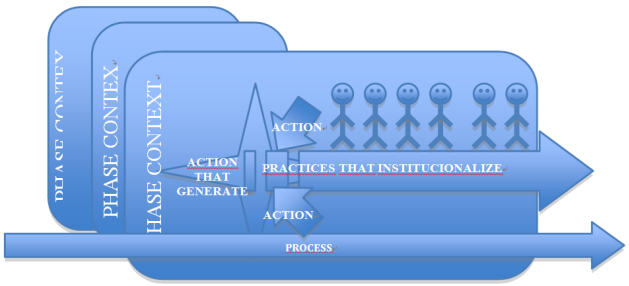
According to Berger (1969), the dialectic process mentioned happens for three moments; externalization, objectivation and internalization. The externalization moment it gives through actions driven by the accumulation of knowledge. Here they interest particularly; the diffusion; that it is understood as the moment in which a built knowledge in the individuality to give know other by some oral or written means, the representation; understood as the whole of material and symbolic objects next to the meaning that is associated to them and that give sense to certain knowledge produced for the individual, and the knowledge reproduction, that refers to the employment of a certain built knowledge by other in a specific context. The products of these actions that it can be articles, books and all kind of documents by means of which is reported results of an investigation, as well as, languages, symbols and any tools that does possible the objectivation of the knowledge.

The actions by means of which externalization is possible, they are oriented by an intentionality of double character: an individual character that obeys the need of the social being that identifies it or puts as part of certain environments and communities; and a social character that obeys the need, to next to others, producing the reality (Berger 1969). These actions they diversify while the reality becomes more complex and it is grouped, according to the social character of the intentionality with

which it is realized until comprising of processes by means of which a body of knowledge it is established as social reality. These processes are part of the validation, institutionalization and legitimation of the knowledge, that is to say, by means of them who the externalize knowledge that an individual, objectivized by its products, it is internalized by the integrant of the community to the one that belongs, community that prints you a level of acceptance that gives you the necessary solidity in order that, as knowledge validated, as knowledge validated, they was useful for the construction of new knowledge in the same community or puts it to disposition of other communities orienting and sustaining its development. The validation of knowledge, mark the begin process of institutionalization.

As the validity, the institutionalization and the legitimation of the knowledge occur in communities whose members internalizes the symbolic objects that are product of the actions, we give off that these occur by phases, that is to say, they occur in a community located in the time and they are realized by certain agents, for which, the products of the diffusion, representation and reproduction of the knowledge acquire a significant character by converting to the actions and its intention in a typical conduct that it gets to normed its own actions. To this typical conduct as system of actions constructed in the frame of social and individual intentions that it is constituted in a regulator of the interaction of the members of a community and that they carry out as execution manners prescribed that it enjoy an acceptance generalized and unconditional we will call it *practice that institutionalizes*.

We will call *context phase* to a system of practices that institutionalize that it is originated by an action, we will call, *action that generates* and that was carried out an individual that we will call *agent* of the context phase.



A schematic representation of the characterization of the social construction of the mathematical knowledge.

4 Methodology

We carry out an investigation of historical-epistemological character by following the methodology of a historical analysis considered in Sierra and González (2003) and consists in 5 stages: 1) initial statement of the investigation, 2) heuristic criticism stage, 3) analysis of the documentation, 4) hermeneutics stage, and 5) exposition stage.

In the stage of initials statements of the investigation, in addition of delimiting the subject of investigation, it is necessary to define the line that will follow the investigation and the general frame below which will be developed. In base to a first revision of the bibliographical material, one draws up a possible trajectory that it allows to decide the feasibility of the investigation according to the material whereupon it is counted and initial hypotheses are formulated.

In our case we delimit the subject of investigation and the general line of development by locating it as a study of social construction of the mathematical knowledge that fundamentally we will guide below the sociological ideas of Berger and Luckmann and from the anthropological perspective of the outlined education by Chevallard.

In the first bibliographical revision we find the book, Modern Algebra And The Rise Mathematical Structures of, written by Leo Corry that speak about the origin of the algebraic structures from a historical perspective. In this one, Corry explains the origin and development of the ideas of structure and the dialectic relation that exists between this and the structural vision. This text was determinant for the location of dates, actions, ideas and its agents that are part of the social construction of the algebraic structures. Also it relate about the influence of ideological cut that they occurred between these agents and that were fundamental in the publication of books, that Corry indicate, as determinants in the inclusion of diverse themes of the abstract algebra in the education.

In the heuristic-critic stage it is realize the search and selection of documentary sources that contribute with some element about the subject in discussion. It necessary to locate and classifies the sources of forms such that do not produce documentary vacuums. It elaborated a historical critic for verifying the authenticity of the source and that it does for reach of the objectives of the investigation. In this stage we select four publications that are part of our material of analysis; the Treatise Algebra on of George Peacock published in 1845, *Lehrbuch der Algebra* of Heinrich Martin Weber published in 1895, Modern Algebra of B. L. Van der Waerden, published in 1930 and the Elements of the Mathematics of Nicolas Bourbaki published in 1935. We established the context phases of the social construction of the algebraic structures and that will describe as part of the results at present article.

The stage of analysis of content consists in studying the material under historical criterions, design some instruments that will permit approach the objective that one has considered, same that the interpretation of the data via the analysis of data that it realized. We design tow instruments for analysis of contend; the first, permitted us, without carrying out properly the analysis of content, locate the documents as product of processes of externalization and internalization of certain agents. This information we found it in the prologue and/or introduction of the text as well as in the biography of the authors. The second instrument consisted in a second card in which, by each work analyzed, it locate some aspects that permitted us identify the other constituent elements of the social construction of the knowledge.

The hermeneutics stage the which that it give appropriate answer to the planted questions in the investigation and indicate the possible causes by those who it is produced the analyzed historical facts, This stage we are realizing it following the context phases that we established in the critical heuristic stage. In present article it corresponding to the first context phase; the phase of incipient formalizations of the algebra.

5 Results

5.1 Process and Context Phases

The algebraic structures in its actual form, it can say that it characterized fundamentally by three aspects; the formal treatment, its function that systematize and abstraction level. These aspects are those who finally generated the emergency as a treatment for the mathematics in general and of the algebra in particularly.

To speak of the formal aspect of the algebraic structures it is necessary to speak of theories and of the language. The formal language is built with words of the common language or of the common sense, undressing them of the meanings that have in these for using them explicitly as says the formal language constructed to speak of a theory. The taken words of the ordinary language are devoid of meaning, that eliminates, in the measure of the possible, double interpretations.

To formalize a mathematical theory this must be provided of an axiomatic system; the axioms are considerate as basic points, independents and consistent to each other and on which, a mathematical theory without internal contradictions can be constructed (Rañada, 2003).

The axiomatic method it fixed as initial posture in the writing of mathematical theories (in texts or articles) of form such that its formalization is easy to conceive. Thus, to provide with an axiomatic system to a theory, represents important advantages as for development, giving multiple contents to the undefined terms (elements of a group, for example) or dissociate the diverse aspects of a mathematical situation to study them with greater depth. For to formalize an theory that it has a axiomatic system, it select a first-rate language appropriate L for the theory it serves to speak about the elements of set and that have the function to avoid the different interpretations; this language has a precise syntax, in such a way that does it moreover, susceptible to study as a mathematical object. Through of the formalization of mathematical theories the basic processes of the thought are optimized.

By the described it in the previous paragraphs, we divide the social construction of the algebraic structures in three processes: The process of Formalization, The Process of Systematization and the Process of Didactic Transposition of the Algebra. Next we describe the elements that they influenced the determination of the context phases they integrate every one of these processes.

In the Lehrbuch der Algebra of Heinrich Martin Weber, published in 1895, it can be identified a faithful image of the algebraic knowledge and of the form in which the algebra was conceived in its time. Weber makes an abstract presentation of many concepts that until the moment had been treated in a particular way (Corry, 2002b). The work that Heinrich Weber and Dirichlet made together in some writings about the algebraic functions it seems to have been an important influence in order that Weber directed his efforts to the study of the algebra and the numbers theory. Dirichlet in addition influenced a strongly Richard (1831–1916) when this one carried out a stay in the university of Göttingen where Dirichlet was professor. Dedekind published a complement of the *Vorlesungen über Zahlentheorie* of Dirichlet as class notes (Reck, 2008). We consider the work of Weber as a product of externalization based on the internalization of the ideas and knowledge of Dirichlet and other mathematicians of his time. We locate to Weber as agent of the *context phase of incipient formalization of the algebra*, first phase of the *Formalization process*.

Heinrich Weber went professor of Hilbert to his entrance to the university of Königsberg, under its influence, of Lindermann and of Dedekind went that Hilbert was interested in the theory of invariants, which represented his first area of investigation (Rodríguez, 2000).

Hilbert was professor at the university of Gotinga since 1893, his work and personality it positioned as the center of mathematics and its teaching, more important of the time. He was protagonist of one of the more important discussions in the history of the mathematics, in which it is considered emerges, the structural vision of the mathematics. As a result of these discussions they arise, those that are known as philosophical currents the mathematics; formalism, logicism and intuitionism that they occurs between 1870 and 1910. The origin of these discussions happened by the propose the distinguish between the that exists by "construction," it's to say, that it exists because it estab-

lishes an explicit algorithm for construct, and that it exists because it is "fact", as Hilbert said, "*It can demonstrate that the attributes contain a notion, cannot conduct a contradiction, by the application of a finite number of logical deductions. I will say that it is demonstrated the mathematical existence of the notion under discussion*" (Ferreiros, 1999, p. 5). This proposition divided the options of the mathematicians, the way to represent and to reproduce mathematics that they were known it and its development since then. Meanwhile, the axiomatic method that was created by Euclid can be considered axiomatic concretely, and his geometry a constructive mathematics—given us two different points, A and B, it can be constructed a segment that unites them—. The axiomatic geometry that was formalized by Hilbert is considered as a formal and abstract mathematics, proposes an mathematics of the "fact";—given two points, A and B, exists a segment that the unites them— The contemporary mathematicians of Hilbert' s time, who was without a doubt one of the central personalities in mathematics in the eighteenth century, it distinguished between those who were with his ideas; Emil Artin (1898–1962) and Emmy Noether (1882–1935), for example, and those who were against his ideas; Luitzen Egbertus Jan Brouwer (1881–1966). Or those who simply demonstrated the impossibility of reaching important key goals proposals by Hilbert, as in the case of Kurt Gödel (1906–1978).

Hilbert conceived the idea of the formalism as the facts to reduce the mathematics to one finite game, with a finite number of formulates, defined of finite form. According to these ideas, the mathematics they became for Hilbert, in a general theory of the formalism and the axiomatic in the method of mathematics investigation. The mathematicians of the era (1870–1910) lived the preoccupation of called *"rigorization of the mathematics"*, that it consisted justly in establishing an axiomatic system for the mathematical theories existent, not by considering the axioms as absolute and evident truths, but as starting points that refer the fulfillment of certain properties for the elements of a set, that can include some elements no defined, from which they could be deduced one long list of theorems that will shape the theory.

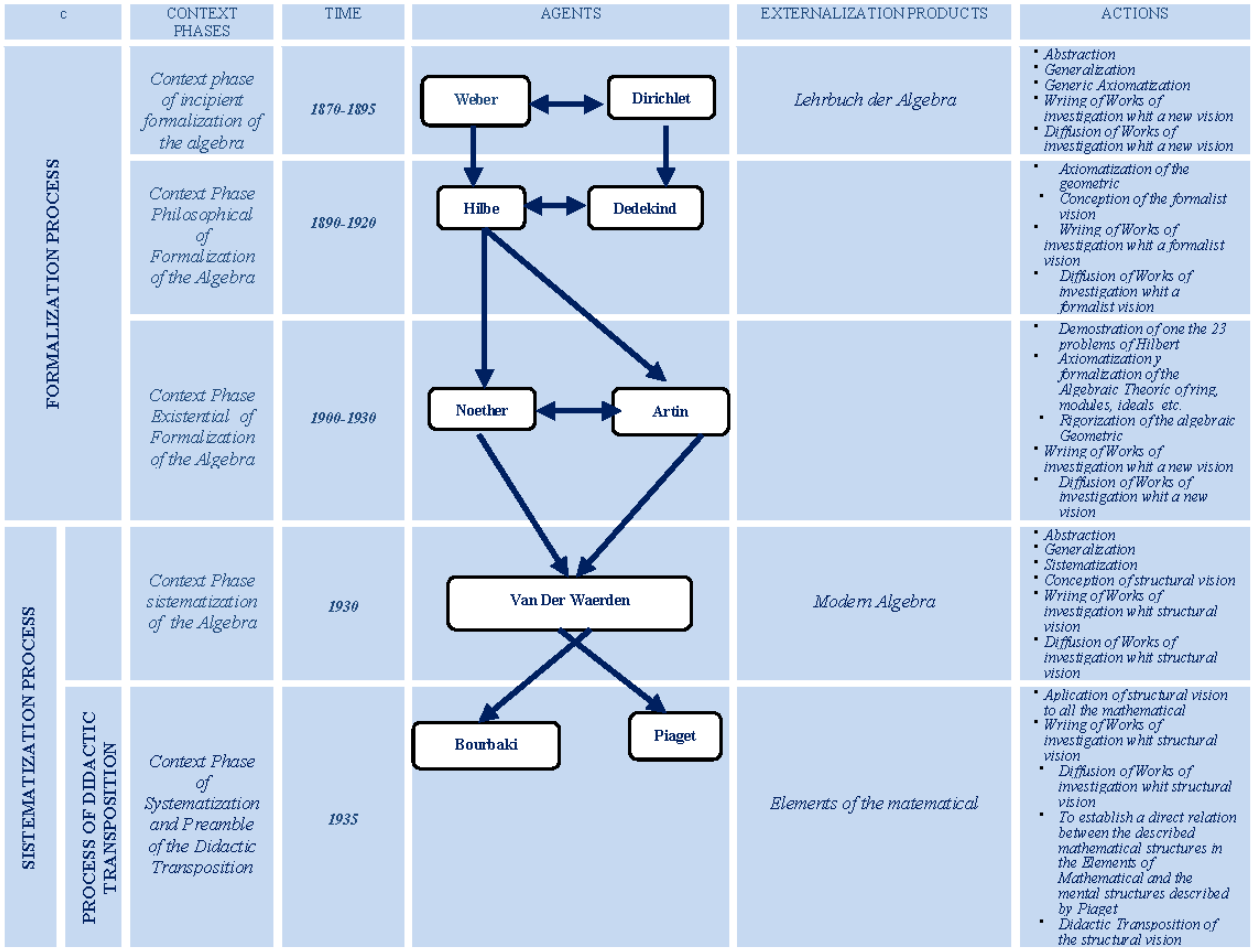
By all the described, we consider to Hilbert as agent of *Contextual Phase of Formalization of Algebra*, whose products of externalization represented a strong influence for the destiny of the mathematics.

The internalization that did Emmy Noether and Emil Artin, disciples of Hilbert, of their ideas, they were translated in products of externalization representing, reproducing and divulging his built knowledge. Van der Waerden declares in the *Modern Algebra; "Based in part on lectures by E. Artin and E. Noether"*, it find that go der Waerden went disciple of Noether during seven months in the university of Göttingen and knew to Artin in the university of Hamburg in where participated in a course and they programmed to write a book together. It says that after an initial revision of advance that present Waerden, Emil suggests you that continue writing and publish it but in an independent way (Rodríguez, 2000), the result of this work went, finally, the two volumes of *Modern Algebra.* This permitted us identify to Emmy and Emil as agents of the *Phase Existential of Formalization of the Algebra*, third context phase of the *Process of Formalization of the Algebra*. *Modern Algebra* is a product of the process of externalization of Van der Waerden through which diffused the built knowledge in base to the internalization of the ideas and knowledge of Noether and Artin. We locate to Waerden as agent of the Context Phase of Systematization of the Algebra.

In the Modern Algebra and in the axiomatic method of Hilbert is that a community of French mathematicians finds the inspiration for, as a product of the externalization of his ideas, written in the *Elements of the Mathematics* of Nicolas Bourbaki, the image that of the mathematics that it had internalization and that it clarified the fundamental goal of their publications; endow to the math-

ematics of a theoretical unification in the presence of one apparent dispersion that lived during the first third of the century XX (Hernández, 2009).

The Bourbaki text resulted from great impact in the insertion of diverse topics of the algebra in its structural vision in the escolar environment as product, between other things, to the declarations of Jean Piaget in those who it referred as structures naturals to the mother structures of Bourbaki. This insertion we considers it as a product of the internalization of those ideas in the escolar environment, and to the Bourbaki group as the agent of the context *Phase of Systematization and Preamble of the Didactic Transposition* of the corresponding algebra simultaneously to *the Process of Systematization and to the Process of Didactic Transposition of the Algebra*.



The processes and the phases of the social construction of the algebraic structures.

c	CONTEXT PHASES	TIME	AGENTS	EXTERNALIZATION PRODUCTS	ACTIONS

5.2 Formalization Incipient Context Phase

The beginning of the process of formalization of the algebra went marked in the history of the mathematics for the publication of the Heinrich Weber text for two reasons fundamental. From that Niels Abel demonstrate the insolubility by radicals of the equation of degree five in 1824, and until 1845

that it is known the works of Galois, the investigations that it is considered in the frame of the algebra were fundamentally those who it is dedicated to the study of the forms and conditions to find the roots of polynomials. It sprang up however, of 1854 to 1880, diverse investigations in different theories of the mathematics in those who introduced, by a side, the language gives the set theory and for the other, the application of results of the Galois theory, making evident its value as unifying factor. As examples we can refer to: the theory of algebraic curves of Dedekind and Weber, investigations on the theory of matrix and quaternions realized by Cayley, the theory of the regular polyhedrons worked by Hamilton in 1856 in the Treaty of Substitutions, the Camille Jordan works published in 1870, investigations about the classification of the geometries made by Kline in 1872 and in the algebraic theory of the numbers promoted by Dedekind in 1877. These investigations sustain the institutionalization of the ideas and forms to approach subject of the generated algebra for Galois, that was validated until after its death.

Heinrich Weber (1842–1913), conscious of the last advances in the algebra, molds the possibility to formulate new algebraic concepts in purely abstract terms. The action we consider as generating of the context phase of incipient formalization it happens in 1893; Weber went the first that gave abstract definitions of group and field in the frame of a single article named; *"Die allgemeinen Grundlagen der Galois"* where says *"Ein system von dingen irgend welcher Art in endlicher oder unendlicher Anzahl wird zur Gruppe, wenn folgende Voraussetzungen erfüllt sinde...* [A system of a finite or infinite number of things of any type are a group if the following conditions fulfill]. At once it enunciates three of the four properties of group know now as, associative and existence of the inverse and neuter element, it enunciates through its equivalent, called cancel laws by the right and by the left. Clear up moreover that the composition of two elements not always is commutative, a few paragraph after defines field "*Eine gruppe wird zum körper, wenn in ihr zwei Arten der Composition möglich sind, von denen die erste Addition, die zweite Multiplication genannt wird*" [A group is converted in a field, if possible, in its two types of composition, of those which the first his name is addition and the second multiplication]

The diffusion as generating action is product of alternate actions of observance of similitudes or common aspects, between two or more, theoretical subjects and its objects of study, that we considers as an abstraction action.

In the mentioned article of Weber observes us clearly an action of generalization and axiomatization, in which he present the conditions for consider that a set of " things of any type " below a specific operation, it is a group. These actions; axiomatization, abstraction and generalization, have the intention, that finally this described in the mentioned article and in *Lehenbuch der Algebra*; integrating the knowledge that during a period of time had developed in subject of the algebra. In your Weber writings it incorporates a complete body of new individual ideas and the developed techniques along the century XIX that are part of the necessary base for the formalization of the algebra. However the study of the groups appears subsidiary to the study of the polynomials equations and your solutions, even in the *Lehrbuch der Algebra* the general concept of group appears after 475 pages.

The community of mathematicians of which it comprised Weber (Dedekind, Cayley, Jordan and Kline) shared already some elements of the formalistic current proposed by Hilbert from 1870, in the investigations generated by their integrant molded already the incipient stage a new tendency to investigate in mathematics and particularly in algebra.

The declaratory thing of Weber in the prologue leaves clear his intentionality "*Es war meine Absicht, ein Lehrbuch zu geben, das, ohne viel Vorkenntnisse vorauszusetzen, den Leser in die moderne Algebra einführen*

und auch zu den höheren und schwierigeren" [It was my intention gives a textbook that not presupposes you previous knowledge to the reader and made a presentation of the modern algebra of the easy thing to the difficult thing]. It stands out the importance of the development of the algebra and the fundamental influence that has had the theory of groups above all in the numbers theory. "*die neueste Entwicklung der Algebra ganz besonders von Bedeutung geworden sind; das ist auf der einen Seite die immer mehr zur Herrschaft gelangend Gruppentheorie, deren ordnender und klärender Einfluss überall zu spüren ist, und sodann das Eingreifen der Zahlentheorie"* [the most recent development of Algebra has reached a very special importance; this is on the one hand the group theory that every time obtains more dominion and whose organizer influence are perceived by all sides, particularly in the theory of numbers]

Also it emphasizes the influence and participation of some of his colleagues so much in the writing of the book as in the development of his ideas that also deposes explicitly in the prologue:

"*Zuerst gilt dieser Dank meinem Freunde Dedekind für seine treue Hülfe bei der Correctur, und wenn er auch auf den Plan und die Ausführung meines Werkes keinen Einfiuss ausgeübt hat, so möchte ich doch nicht unerwähnt lassen*" (Weber, 1895, p.7) [This gratefulness firstly for my Dedekind friend, by its faithful aid in the correction, and even though has not exerted influence in the plan and the accomplishment of my work, would not want to omit to mention it].

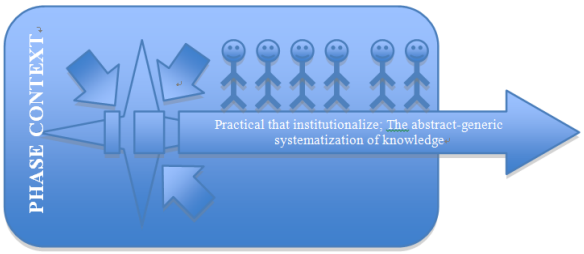
"*Auch der mannigfachen Anregung und Belehrung habe ich hier zu gedenken, die ich meinem Freund und-Collegen F. Klein verdanke, der das Fortschreiten der Arbeit mit regstem Interesse begleitet hat und dessen sachkundiger, stets bereitwilligst gegebener Rath in manchen Theilen des Buches von grossem Einfiuss gewesen ist*" (Weber, 1895, p. 7), [Here also I have to remember the multiple suggestions and instructions, that I must thank a friend and colleague F. Klein, who accompanied the advance the work with much interest and whose competent advice, gave always with good will, has had a great influence in some parts of the book].

For the context phase of the incipient formalization of the algebra we locate a practical that institutionalize; The abstract-generic systematization of knowledge integrated by actions of abstraction, generalization and generic axiomatization and the generating action of oriented diffusion below diverse intentions; the integration of generated knowledge until the moment, presentation of knowledge below a same general idea, it facilitates the application of these new ideas to other mathematical theories; with Dedekind and Weber as agents.

The scientific production of the era was not regulated absolutely, this depended of the relations and recognition that the authors had generated in a community of individuals that it would constitute as the group of specialists that they would administer the accumulated knowledge that it generated in the algebra under relevance and structure of meaning of them. This community it ended establishing the mediations for the recognition of the scientific results that were susceptible writings of diffusion.

6 Conclusions

The concept of algebraic structure emerges once particulars structures, that arise in diverse fields of the mathematics, in moments and different stages, they are equipped with an axiomatic system, classified and systematized afterwards to its construction. The structural vision as conception of the mathematics, it implies, represents and reproduces the mathematical knowledge, unifying several existent theories until before 1930 anchored initially to the algebraic theory of the numbers and the



Scheme of the Phase of Incipient Formalization of Algebra.

theory of the polynomials, has its roots in the formalistic current, proposal by Hilbert, that it prevailed in the community of mathematicians in front of the intuitionist currents and logicist.

Until before 1845 the representation and reproduction of the molded mathematical knowledge in objects of oriented diffusion, it made evident a null presence of symbolic material associated to ideas of abstract-generic representation of concepts of now called, Abstract Algebra. It until the publication of the *Lehrbuch der Algebra* of Henrich Weber it that appears, of way objectivized, an incipient abstract-generic reproduction of the concept of group and excel as product of the externalization of his ideas that as well represent the internalization of the externalizates concepts by Galois in his writings know in a posthumous way.

The phase of incipient formalization of the algebra represents for the structural vision the start-up of actions of systematization and generalization of the ideas of Galois, that together the ideas of Hilber, it profiles the axiomatization of the theory of the today known as abstract algebra and of the mathematics in general.

The practical that institutionalize of abstract-generic systematization as system of actions driven by the accumulation of knowledge and individual and social intentions, it is established in effect, as regulating of the interaction of the integrant of the community of mathematicians of the era, generating manners of representation and reproduction prescribed of the mathematical knowledge they begin to enjoy an acceptance generalized and unconditional originated by the publication of the *Lehrbuch der Algebra* of Henrich Weber.

The oriented diffusion in the scientific communication and the divulging of the works of investigation proposal, of incipient way, in the *Lehrbuch der Algebra* of Henrich Weber as a new form of making algebra and is more profitable given its Generality, offers the possibility of explaining diverse theoretical elements and give origin to other new. It profiles as action that it generates it shows one's profile as generating, non-single of the phase in this article, but of the other context phases of construction of the algebraic knowledge.

The community of mathematicians they identified as agents of the context phase of formalization incipient, impel, in an implicit way, new forms of representation and reproduction of the mathematical knowledge. Forms that are sustained and consolidated by the agents of the subsequent context phases (philosophical and existential of formalization), by complementing of this way the process of formalization of the Algebra.

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