THE ROLE OF THE FRENCH ASSOCIATION OF MATHEMATICS TEACHERS APMEP IN THE INTRODUCTION OF MODERN MATHEMATICS IN FRANCE (1956–1972)

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ABSTRACT

The French Association APMEP (Association of Professors of Mathematics in Public Education) played a major role in the implementation of the Reform of Modern Mathematics. As soon as 1956, in an APMEP meeting, the mathematician Gustave Choquet compared the mathematics teachers to members of a Museum, who show dusty and useless objects. In the years 1958–1960, teachers began to introduce some elements of Modern Mathematics in classrooms, like sets and relations. From 1960, there had many papers in the *Bulletin* of the Association to explain Modern mathematics to teachers and to exhibit experiments in classrooms with these mathematics. In 1964, it was the beginning of local courses and national sequences for the Television organized by the Association.

We propose to come back to this rich period with numerous books and papers to understand the interest of the Association APMEP for the Modern Mathematics and the interrelations between teachers, mathematicians and others. For the teachers, Modern mathematics will permit to develop mathematical activities, to democratize teaching and to adapt the mathematical teaching to the modernity of the society. But it appears clear rapidly that the new notions present difficulties in regards of the ancient contents of teaching, for instance in geometry. We will focus on this aspect to analyze the first failures of the Reform. We will also examine how the teaching of mathematics became quickly a teaching of a complicate language. In its "Charter of Chambéry" of 1967, the Association APMEP asked for a Reform of mathematics teaching, but also for the creation of IREM (Institute of Researches on Mathematical Education). At this period, the IREM had the task to teach Modern Mathematics to all the French teachers, but some years after, many of their members strongly criticized the Reform.

1 The French context of the Reform

The reading of the Official Instructions of Ministry of Education of January 1957 gives an idea of the French context of the Reform of modern Mathematics. It was emphasized on the serious danger that France meets, on the intellectual level and on the economical level, because of the lack of increasing number of engineers, researchers and technicians and also on the urgent necessity to turn a increasing number of young people towards the scientific careers.

1.1 The economical and scholar context

The period corresponds to the "Cold War" with a major event in 1957, which is the launching of the Spoutnik by the URSS. The European Organization of Economical Cooperation (OECE created in 1948 and became OCDE in 1961) created in 1958 a Committee to make more efficient the teaching of sciences and of technics and organised a Colloquium in 1959 in Royaumont (France) to promote a reform of the contains and of the methods of the mathematical teaching (Barbin 1989).

On this occasion the French mathematician Gustave Choquet presented a program of teaching for secondary schools and the French Bourbakist Jean Dieudonné exclaimed "A bas Euclide!" (Down with Euclid!).

Many reforms of the scholar system occurred in the years 1960. The Reform Berthoin extended the schooling of the pupils until sixteen years old (1959). Last year, the examination to enter in the Lycées (upper secondary schools for sixteen to eighteen pupils old) was deleted. In 1963, the important Reform Fouchet created the "Collèges d'Enseignement Secondaire" (CES), they replaced "little classrooms of Lycées" to welcome all the pupils in the same institutions.

1.2 The social context and the modern mathematics

In the end of the years 1960, some famous works of social researchers are edited in France. In his paper "Teaching systems and Thought systems" (1967), Pierre Bourdieu opposed "schools of thought" and "culture of class". He explained that as the School is put in charge to communicate the social system, it has been organized to answer to this function. For him, to realize this programme, named "culture", the School has "to programm a culture" in such a way to facilitate a methodological learning. Four years after, the sociologists Christian Baudelot and Roger Establet wrote *L'école capitaliste en France* (1971), which showed in detail the links between the social system and the scholar system.

The idea that the modern mathematics was the mathematics needed for a democratic School was expressed often in the years 1960–1970. For instance, the French mathematician André Lichnerowicz, which was in charge to organize the Reform, wrote in a union journal of January 1973 that the new mathematical teaching was necessary, not to form professional mathematicians, but above all and at first to form the future citizens. For him, the future citizens had not to be passively subject to the various frames, which will be imposed or proposed to them. They must have the power to say no to the too much clever manipulators of computers and they don't have to capitulate in front of a pseudo-scientific terrorism.

There was also the idea that the pupils of all the social class will be in an equal situation because all the parents will be in an equal position towards of a new mathematic that they did not know. The French mathematician Jean Frenkel wrote in a *Bulletin of APMEP* in 1972 that for some years, parents of all the social class would be equally disarmed to help their children to learn. It was naïve, because special books for parents were edited and private schools were created at soon as the Reform was in the air.

2 The "leaven" of the Reform : the French Association APMEP (1956 – 1968)

In a meeting in Melun in 1952 (a town in the South of Paris), the mathematicians Dieudonné, Choquet and Lichnérowicz met the Swiss psychologist Jean Piaget. Four years later, in a meeting of APMEP in Sèvres near Paris, Choquet compared the teachers of mathematics to guards of Museum, which show dusty objects without interest for most of them (Bareil, 1992). From the years 1958–1960, teachers and schoolbooks began to introduce basic notions of modern mathematics, like sets and relations. The French Association des Professeurs de Mathématiques de l'Enseignement Public (APMEP) is an Association created in 1909, which gathers all the mathematical teachers from the Nursery School to the Universities.

2.1 New mathematics in the *Bulletin* of APMEP (1960–1963)

First studies on the theory of sets and on experiments in classrooms appeared in the *Bulletin* of APMEP in the years 1960–1961. In particular, two important papers appeared on the theory of the modern mathematics: Choquet wrote a paper entitled "Research on a easy axiomatic for the first teaching of geometry" and an other French mathematician Jean Colmez wrote a paper "The structure of modern mathematics". In this paper of 1961, Colmez explained that this structure of mathematics can contain every rational theory relative to any human activity. For him, the Mathematics are the science of reasoning and propose plans of reasoning to the other sciences, which are in a way prefabricated.

Claude Pair was a young mathematics teacher of the Lycée Poincaré in Nancy when he wrote a paper on "The affine geometry in the first grade of the Lycées", (1961) that means for pupils aged fifteen or sixteen. Some months later, he related "An experience of teaching of modern notions", where he emphasized that we have to keep the great geometrical ideas for later to serve as examples, because they will enter in wider theories.

The *Bulletin* of May-June of 1963 contained papers about the relations between mathematics and the reality. Inside, Gilbert Walusinsky, a famous leader of APMEP, wrote a paper entitled "The rule of three will not held". The title is a joke using the two sentences: "La Guerre de Troie n'aura pas lieu" (a drama of Jean Giraudoux) and "La règle de trois n'aura pas lieu". He considered that the automatism of the mathematical "rule of three" is not without danger and he showed to avoid it by using modern mathematics in the lower grade of Collèges, with pupils eleven or twelve aged.

Always in this *Bulletin*, Walusinsky wrote a paper on "Spatial studies and mathematical teaching", where Spoutnik I and Exporer I are taken as examples. While, Jean Kuntzmann wrote about the needs of men working on applied mathematics and on automatisms. But he made clear that these careers were also opened to the young women. Remember, that in this period, French young women began to consider that it is normal for them to have a work.

In the annual commentaries of votes in 1963, we can read that some teachers of the Association APMEP judged that it was necessary to adapt the curricula to the "possibilities" of the pupils and to take in account the "pedagogical realities of now". Indeed, in the beginning of the years 1960, a new population of pupils acceded to the Collèges (see above) and it seems that the teachers thought that the difficulties met by these pupils could be solved by the introduction of a modern teaching.

2.2 The "Chantiers" of APMEP (1964) and the Charter of Chambéry (1968)

On Walunsky's initiative, the Association APMEP created in the years 1964 "Les chantiers mathématiques", which were a television serie on modern mathematics and prepared the "Grande Commission".The "Commission Lichnérowicz" began to work three years later, and many of its 18 members belong to APMEP.

In January 1968, the Association APMEP organized a Colloquium with around forty participants, where a first text on "the steps and the perspectives of a Reform of mathematical teaching" was elaborated. This text will be completed and adopted by the General Assembly under the name of "Charte de Chambéry" The text answered to three questions: Why the mathematical teaching has to be reformed from the Nursery Schools to the Universities? Why the reform is possible? How to realize the Reform? The answers were accompanied of many concrete proposals, in particular to determine the different steps of the Reform.

The first main goal was to teach to all pupils a mathematic useful for the modern world. Indeed, there was a true democratic willingness to render an abstract level more accessible, and not reserved to only some privileged persons as in the past. The purpose is to teach a contemporary mathematic, which had to be a part of the culture for everybody, in the period of computers and automatisms.

The second goal was to teach the mathematic with an active pedagogy. The Charter emphasized that the introduction of a new contain in the mathematical teaching will be ineffective, and may be harmful, if it is not accompanied by an appropriate pedagogy: active, open, the less dogmatic as possible, organising the work of the pupils by groups and appealing to their imagination.

The Charter of Chambéry stressed on the necessity to create Academic Institutes on Research on Mathematical Education (IREM), because the effectiveness of the Reforms had to begin by a serious experimentation and by an increased effort to develop the in-service teacher training.

3 The great lines of the Reform and the special case of Geometry

The official new curricula appeared steps by steps from July 1968 to June 1971. The first IREM began to be created in Main text in 1969, which means that the Reform will be installed before the works in the IREM really was always effective. It is possible that the social movement of May 1968 explains the rapidity of the decisions.

3.1 The introduction of modern mathematical notions

From the Nursery School to the Universities, the new teaching was symbolized by the notion of set represented by the drawing of a "patate" (potato) with the drawings of union and intersection of sets. The relation of equivalence, with the properties of reflexivity, etc., and the properties of the operations, like commutativity, associativity, etc. were considered as basics notions. They were presented in a naïve manner from the lower grades.

In the two first years of Collèges, with pupils from eleven to thirteen aged old, there was an introduction of the "language" of sets and relations. In the two next years, the fields Q of the rational numbers and the field R of real numbers were "constructed". The initiation to the geometrical proof disappeared of the Curricula in the same time than the three "cases of equalities of triangles" of the "old" geometry. In place, Curricula offered an affine geometry turned towards the linear algebra.

In the Lycées, the "vocabulary" of the sets and the relations was linked with a teaching of logic. The affine and Euclidean geometries were based on linear algebra. The Curricula gave a good place to calculus. The differential calculus contained the notions of continuity of a function, limits, derivability of numerical functions of one real variable in the upper classroom. The integral calculus was based on the sums of Riemann, on differential equations. A part concerned probabilities on a finite set.

3.2 The special case offered by the geometrical teaching

The geometry offered a special case for the members of APMEP. In the years 1960, it seems that they could not imagine to not teach geometry. Indeed, the geometry was considered as the field for initiation of the reasoning from many centuries. But, what kind of geometry?

The members of APMEP wrote in the *Chantiers mathématiques* of 1964–1965 that, as it was necessary to teach geometry, the true problem was to improve its presentation (Daubelcour 2004, p.174). Moreover, they considered that it was more or less obvious that the study of vector spaces would permit to give to the geometry his place in modern mathematical teaching, in such a way the geometry would be adapted to the nowadays conditions of the science and of the world.

Here it is interesting to mention the two books of Choquet and Dieudonné of 1964, which were intended to teachers and to students. Choquet wrote in his *The teaching of Geometry* that we have to prefer the methods leading to the fundamental notions that twenty centuries of mathematics finished to bring out: the notions of set, the relations of order and of equivalence, the algebraic laws, the vector spaces, symmetry, and transformations. Always in 1964, Dieudonné explained in his *Linear Algebra and Elementary Geometry* that, in the time of a great proliferation in all the sciences, all the things that can condense and lead to unification has a virtue that we could not over-estimate.

Frenkel edited a book entitled *Geometry for future teacher* (1973), where he developed the ideas expressed ten years before by Dieudonné. He considered that, mathematically speaking, elementary geometry has not to be distinguished from linear algebra, but only by artificial boundaries on the dimension and sometimes by the field concerned. As consequence, we find in the schoolbook for the last grade of the Lycées this definition: in an affine space E associated to a vector space e, we named affine variety through a point A and with direction e', a subspace of e, the set of the points M of E, such that the vector AM is an element of e'. We noted (A, e') this affine variety (Daubelcour 2004, p.184).

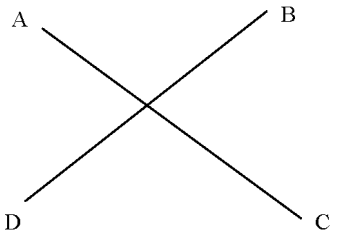
3.3 The geometry in the "retraining" of teachers in the IREM

The first IREM created in France, Paris, Strasbourg, Bordeaux and then Rouen, etc. were generally managed by militants of the Reform, and many trainers are teachers which belonged to APMEP, because they had study modern mathematics already. As a young mathematical assistant of the University, I entered in the IREM of Rouen in 1974 because the director, which was also my teacher in the University, initiated me to the new geometry.

To explain that the difficulties with the Reform were reinforced by the contain of the schoolbooks, I would like to tell an experience that I met in a training session in 1975–1976. A group of teachers of Collèges came to show to me an exercise of a schoolbook, because they were not able to solve it. It was a book for the pupils thirteen or fourteen aged, where the geometry was presented from the axioms

of connection, in the same manner that Hilbert in his *Foundations of the Geometry* of 1899. For instance, "two distinct points determine a straight line" is an axiom.

The exercise shown by the teachers considered a finite geometry with four points A, B, C, D. The lines were noted $\{A, B\}$, $\{A, C\}$, etc. The first question asked to prove that A, B, C, D are two parallel straight lines. The teachers explained to me that it was not true and they point out this drawing, where two straight lines cut.



That means, that the idea that two points not only define, but that they "are" a straight line was too difficult to understand, because of the vividness of the old geometry. But $\{A, B\}$, $\{C, D\}$ has not a common point, so they are parallels.

4 APMEP and the reactions against the Reform of Modern Mathematics

The Association APMEP decided to issue a statement of the Reform in 1971 and to publish a new charter, entitled *Charter of Caen* (1972). The purpose was to make concrete proposals in favour of the Reform, about the necessity to create an IREM in each Academy and the manner to organize in-service teachers training, but also to examine some criticisms, which are pronounced from 1971.

4.1 Against the new teaching of Geometry in Collèges

From June 1971, the APMP expressed a doubt about the conditions in which the new Curricula has to be applied in the two last grades of Collèges. The modern mathematics already applied in the two first grades met a success, but the next step seemed compromised. In a letter published in the Bulletin of APMEP of the Autumn of 1970, some members of the Institutes of IREM asked that the Curricula of the two last years of Collèges did not contain a complete deductive theory of geometry and a systematic construction numerical structure.

In despite of that, new Curricula appeared in 1971 for the last two classrooms of the Collèges. One mathematical definition crystallized the critics: the one of an affine straight line, which was intended to pupils thirteen or fourteen aged.

By definition, an affine straight line D is a set E equipped with a family F of bijections from E to R such that:

a) For every element f of F and for every element (a, b) of $R^* \times R$, the application defined by $g(M) = af(M) + b$ also behold to F

b) Reciprocally, if f_1 and f_2 are two any elements of F, there exists (a, b) belonging to $R^* \times R$ such that $f_2(M) = af_1(M) + b$

The set F is called the support of the affine straight line D, an element M of E is called a point of the affine straight line D.

This definition provoked a little scandal. Jean Leray, French mathematician and member of the French Academy of Sciences, red it to two successive Ministers of Education of France, and they were not able to understand it. A funny paper appeared in the famous French satirical journal *Le Canard Enchaîné* (Bareil, 1992). In this period, the two promoters of the Reform, Choquet and Dieudonné, also protested. The first one wrote that, like it is putted, the Reform was an attack against the Geometry and against the recourse to the intuition. While the second one explained that it was a new scholastic, a more aggressive and more stupid scholastic placed under the banner of the modernism.

4.2 Against an elitist and dogmatic mathematical teaching

From 1975, there were more general critics, which considered that the generous ideas of the promoters of the Reform had not been reached, and moreover, that they completely failed.

In a paper entitled "Retro mathematics or modern mathematics" (1975), the French Physicist and thinker François Lurçat wrote that the formalism is tolerable for the Youth coming from the Upper Class, in despite it is bored and it discourage a part of them. But, for him, it constituted an almost insupportable obstacle for a young pupil coming from the Low Class or the Popular Class of the Society.

In the Institutes IREM, some researchers presented alternative proposals opposed to the Curricula of the Reform, like the teaching by themes, the teaching by projects, etc. In the middle of the year 1975, a National Committee "Epistemology and History of Mathematics" was created by the IREM, where the idea emerged rapidly that History of Mathematics could be a therapeutic against dogmatism (IREM, 1982). The Institutes IREM of the North and the West of France organised three National Colloquiums on "Mathematics and Society". They emphasized the elitist character of the Curricula of the Reform, which replaced the Latin by the Mathematics as a scholar discipline to select pupils.

5 Conclusion: on the "pernicious effects" of the Reform

By "pernicious effects" of a Reform, we mean effects which are not foreseen by the promoters, but which finally go against their ideas (Barbin, 1989). For any Reform, there are "pernicious effects" often linked to the change of a pedagogical innovation into an institutional renovation. But there are also particular effects, which are interesting to analyze in the purpose to understand more about the conditions of changing in Curricula, specially for scientific teaching.

Here we will focus on the epistemological conceptions, which was under the Reform of modern mathematics, because we would like to emphasize that a Reform not only needs mathematical trainings for teachers, but also epistemological trainings which can render more clear the purposes and the possibilities.

5.1 The mathematic as a study of structures

One of the main idea of the Reform of modern mathematics was the necessity to teach the notions of sets and relations, the fundamental structures of the algebra, the basic notions of topology. But the question was: how it could be possible to reconcile this mathematical contains to an active pedagogy?

For instance, in the third grade of the Collèges, the pupils are asked to start with objects or drawings to "bring out" the axioms of Geometry. That means that the mathematical foundations and struc-

tures could be spontaneously go out from the heads of the pupils, which are putted in situation to "recognize" them. But the mathematical theories had been constructed by the humanity in a long history where the problems and situations were more complicated than those proposed to the pupils.

This led "to manipulate" the pupils: after to have "bring out" the axioms they were asked to prove the theorems, which concern formal objects, by axiomatic. In particular, pupils had to prove theorems without using figures, which could help their reasoning. At this period, the situation of the pupils, which are supposed to construct themselves all the notions, appeared in all the scholar disciplines. It was one of the subjects of "Alert the babies!", a film produced by Jean-Michel Carré (1978).

5.2 The mathematic as an universal and abstract language

This conception led to a formal discourse, a "manipulation" of symbols, an excess of vocabulary and of sophisticated notions. The idea that "the mathematic is a language" came from the conceptions of some Bourbakists, like Dieudonné, and it was widely adopted by mathematicians, politicians and educationalists. It is clear that such a conception isfavourable to select pupils who are accustomed to handle language without references to any problem, which could bring a meaning.

Moreover, the notion of "abstract language" reinforced the opinion that there exist two kinds of students: "abstract pupils" and "concrete pupils". Indeed, it was the only manner to explicate the failure of many pupils, and it was also a good way to explain that it is not possible to make something with the "concrete pupils".

The idea of a universality of the mathematical language was also a good manner to justify that the mathematics was the discipline of selection "par excellence". Indeed, this selection did not depend of the social origin of the pupils, but only expressed the necessity to learn the modern rationality.

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