History and Pedagogy of Mathematics 2012
16 July – 20 July, 2012, DCC, Daejeon, Korea.

SOCIAL STRUCTURES IN MATHEMATICS EDUCATION

Researching the History of Mathematics Education with Theories and Methods from Sociology of Education.

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Introduction

This talk is divided in two main parts. In the first part, I discuss external motives for studying the history of mathematics education. They are external in the sense that they do not primarily concern the production of knowledge to the field of history of mathematics education itself. In my view there are two external motives. The first concerns how research in the history of mathematics education can make contributions to the field of history of education in general. The second motive is to do with how research in mathematics education may benefit from research in history of mathematics education. In the discussions of both these motives, I touch upon a need to study the people and the groups of people behind concepts, arguments and stories connected to mathematics education. In the second part of the talk, I discuss in more detail why and how I have included a sociological perspective. This second part takes off in a discussion on contemporary research about the history of Swedish mathematics instruction. My criticism is, however, of rather general nature and I am inclined to believe that also an international audience can appreciate the discussion. The second part also contains a section where examples of my results are presented; this section serves as a demonstration of how theories and methods from sociology of education can be applied. The second part, but also the talk, ends with an outline of my future research.

Part I – Motives for studying the history of mathematics education

I believe that research like mine, i.e. projects that include comprehensive studies of the goals, contents and methods of mathematics education, can make significant contributions to educational history in general; significant in the sense that historians in other specialties (e.g. politics, economics, ideas, science or education in general) cannot attain the same result. Lövheim (2006) and Prytz (2007) constitute a good example for this discussion. They treat the same type of school (secondary schools), the same time period (about 1900-1960) and the same country (Sweden), but with different focus and quite different sources. Lövheim, with a background in history of ideas, focus on more public curriculum debates about science and mathematics, while Prytz makes a comprehensive study of the contents and methods of mathematics education. To some extent, they do treat the same sources: curriculum documents and teacher periodicals. But were as Lövheim studies also debates in for instance the parliament and the public press, Prytz turns to textbooks, exams and literature specialized in mathematics education. Thus, two types of educational debates are studied: public and professional.

In Lövheim's treatise, two main ideals, with accompanying lines of argumentation, stand out: on one side the classical or humanistic ideal and on the other the realistic ideal. To keep it short, the most important goal of the humanists was to cultivate the students' knowledge about man and his culture. Important school subjects in that perspective were languages[1] , history and religion. The main goal of the realists' was to promote knowledge about things and nature. Important school subjects in that perspective were for instance science, economy and geography. Lövheim analyses how the arguments changed over time, in different debates and in different media, and to what extent they had impact on curriculum documents. However, if we turn to Prytz study of the professional debate on geometry instruction[2] in the lower secondary schools, it was not a reflection of the debates studied by Lövheim. First of all, geometry instruction of that time contained a large proportion of theoretical geometry with focus on propositions, proofs and purely geometrical problems. This applies in particular to the later years of the secondary schools: the axiomatic method was introduced in the equivalent of year 7 in the Swedish school system of today. Applications in science, work life or everyday life were not prominent in the geometry textbooks. However, this orientation of geometry instruction, despite the ongoing battle between realists and humanists in the public debate, was never really put into question in the professional debate; the need to include more applications was not an issue. The more or less unquestioned overarching goal of geometry instruction of lower secondary school was to foster a general formal education, in this case logical thinking and a critical attitude regarding language and spatial intuition. Rather, the main disagreements concerned the teaching of the subject and the design of textbooks, e.g. the order of propositions and the design of proofs. Moreover, Prytz shows that this by no means was a superficial debate; textbooks were produced in accordance with the different viewpoints.

The apparent focus on the theoretical side of mathematics rather than its applications, but also the explicit goal about general formal education, implies that the debaters in the professional debate advocated a classical ideal. During the 19th century, the argument about general formal training was often used by classicists as they defended the position of classical languages. However, these debaters, all of them secondary school teachers and textbook authors by the way, cannot have been negative to a realistic education. All of them were science teachers and taught physics in secondary schools, i.e. they had a solid background in science. Hence, the public debate and the professional debate were different in character in the sense that the former concerned general goals, while the latter mainly concerned methodological issues. This is perhaps not that surprising. The interesting thing is that the disagreements regarding goals in the public debate did not turn up in the professional debate. Moreover, the conflict between proponents of classic and realistic ideals, so fundamental in the public debates if we follow Lövheim, was not a major element in the professional debate.

The point I want to set forth is that these differences are possible to discern only by a thorough investigation of the goals, contents and methods of school mathematics that were debated in teacher periodicals and realized in textbooks. And it is interesting to know about such differences; they tell us something about not only school mathematics, but also something about how the Swedish school

[1]In the beginning of the period 1900-1960, the position of classical languages was still an issue. But the focus of the debate was shifting towards just modern languages.

[2]This debate took place in the leading teacher periodical on mathematics and science instruction of this time. To a reader of today, major debates about geometry instruction might seem a bit odd; geometry is a rather small topic in the Swedish syllabus. However, during the period 1900-19600 geometry constituted a much greater part of school mathematics than it does today. The big change in this respect took place in the early 1960's.

system functioned. That may in turn provide a better understanding of how and why school mathematics changed in a certain way.

However, the differences in the debates just accounted for might perhaps be accidental or a result of my selection of sources. Moreover, how do we know that the professional debate mattered? The debaters might have been a group of people without influence? These questions touch upon a more general problem. My impression is that much of the research on the history of mathematics education is based on textual analysis. The problem is that purely textual analysis cannot answer questions about the importance or influence of the investigated texts. In the next part, I develop my thoughts about this problem and how sociological analysis can help us overcome it.

Before that, I turn to my second motive for studying the history of mathematics education. This motive is in linked to the fact that educational researchers in Sweden quite often make statements about the history of mathematics education. This applies for researchers from mathematics education as well as education. These statements are not part of the empirical investigations, but occur in some sort of background chapters. As I see it, these historical accounts serve as a motivation for posing the actual research questions; a motivation alongside arguments regarding scientific and practical needs.[3]

The main problem with these historical backgrounds is not the accuracy of factual claims, but their bias. A common feature is that the researchers pay considerable attention to the great school reforms of the 1960's, i.e. the introduction of Grundskolan[4] (a new school type that includes the former primary and lower secondary schools) and Gymnasieskolan (a new school type that comprises the former upper secondary school and different higher vocational schools). In these backgrounds, the authors point out that the reforms were accompanied by radical changes in mathematics education, e.g. the abandonment of classical geometry, new teaching methods, and later on the introduction of the New Math. Another common feature of these backgrounds is that they contain explanations of why mathematics instruction was reformed during the 1950's and the 60's. Three types of reformist arguments regarding mathematics are put forward.

- School mathematics was considered old-fashioned from a scientific point of view. Therefore, the courses were changed.
- School mathematics was considered old fashioned in relation to the needs of a modern society. These needs could be in science or technology, but also so-called everyday situations. Therefore, in order to provide knowledge more suitable for such areas, the courses in mathematics were changed.
- Pedagogical and psychological research had brought a new awareness about mathematics instruction and learning, which resulted in a reformation of the courses and new teaching methods.

By putting forward these arguments as a source of change, the authors of the historical backgrounds links the changes in school mathematics to changes in society and science. This relationship is underscored with even greater emphasis by Selander (2001) in an essay on Swedish school mathematics of the 18th century:

[3]A more comprehensive account of the backgrounds together with references to the works is given in Prytz (2007).

[4]Note that Grundskolan (year 1-9) and Gymnasieskolan (year 10-12) are the basic types of schools in Sweden today.

> For centuries school mathematics has evolved from a formal education and a deductive method to today's functional orientation with elements of an inductive method. Early on, one considered mathematics, along with Latin, important for the students' ability to think and make judgments, and mathematics provided useful training in systematic working procedures and clarity of thought. ⋯During the 18th century, a long mathematical tradition based on Euclid's geometry was complemented by arithmetic; thereby, mathematics was more adapted to the new needs of the era that followed upon the expansion of shipping and trade. ⋯During the late 19th century, with the breakthrough of industrialism and the formation of modern schools, with industrial chemistry and new sources of energy, with railroads and national time standards, with the organization of labor and capital and the formation of the modern national state, school mathematics was renewed once again, as the practical relevance of school mathematics was accentuated even more (Selander 2001), pp. 41-42)[5]

In the historical backgrounds, the first half of the 20th century is treated very briefly in comparison with the concern about the 1950's and 60's, most times not at all. In cases where mathematics instruction of this period is given a slightly longer treatment, it is often described in terms of its traditionalism, isolation, and stagnation. In a report, on theories on common education[6] and mathematics instruction in Sweden, Magne (1986) claims that mathematics instruction at the common level was completely unaffected by the reform movements in other Western countries before 1950. However, he does suggest that mathematics instruction at secondary level was influenced by international movements.[7] In the same report, though, Magne points out that the international debates on geometry instruction around 1900 never reached Sweden, nor did the international debates on algebra instruction during the 1920's. Håstad (1978) takes it even further in his doctoral thesis on mathematics education:

> If we must mention the force that has played the leading role in the development of mathematics instruction, the answer is simple: *tradition*. However, there are a number of other "persons in power" that have possessed important supporting roles. The study of their influence is crucial. And how important is tradition? In order to come to grip with its role [the role of tradition], I make the following simplification. *The mathematics instruction that took place up to the 1950's should be considered tradition*. The plausibility of such an assumption is compellingly vindicated by the fact that mathematics instruction has been relatively static during a long period of time and that only minor modifications have taken place during the previous decades (Håstad 1978, p. 134).[8]

In an essay on school mathematics, Unenge (1999) summarizes his experiences of Swedish mathematics instruction before 1960 in a similar way.

> Well until the late 1950's, mathematics instruction was more or less unchanged. The way I was taught as a student in Realskolan was the way I taught my students in Realskolan 15 years later (Unenge 1999, pp. 24-25).

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[5] The quote is originally in Swedish. The translation is done by Prytz (2007).

[6] Common education is a translation of the Swedish word folkundervisning. In Sweden this concept concerns education for the masses or the commons. Other translations of folkundervisning would be public education or peoples' education.

[7] Magne (1986), p. 6

[8] The underlining as well as quotation marks are original.

These historical accounts are by no means entirely wrong. I do not question the correctness of the factual claims about the 1950's and 60's, but we have to be aware of what aspects of the situation they describe. I have no doubts about the descriptions of the reformist arguments and the changes in the curricula; undeniably, these arguments were a part of the debate and the changes in the curriculum did take place. The problematic aspect is that only the arguments of the 'winning team' are mentioned – the arguments of those whose wishes came true in the reforms of the 1960's. The critics of the reforms and their counter arguments are not considered at all. In fact, from the historical backgrounds you cannot tell whether there were any critics. It is almost as if the changes that took place followed some kind of natural order related to the industrialization of Swedish society.

Another problematic aspect is the descriptions of the time before 1950 as being traditional and static. Prytz (2007) shows that changes in mathematics education did take place, in this case geometry. In the leading teacher periodical, teaching methods and contents were discussed and textbooks, e.g. Euclid's *Elements*, were criticized. New alternative textbooks were indeed produced; for instance, some authors began to use the symmetry concept and propositions about symmetry replaced traditional propositions about congruence. The latter are essential in traditional editions of Euclid's *Elements* and should not be considered a "minor modification" as Håstad puts it. Therefore, I think that Magne's and Håstad's characterizations of the time before 1950 as being static and traditional are misleading: mathematics education did change. However, attempts to reform mathematics education in other countries might have been more radical than in Sweden. Moreover, since the changes of Swedish school mathematics in the late 60's were so fundamental, e.g. the introduction of New Math, less fundamental changes in the past may stand out as insignificant, especially to people like Magne and Håstad, who in the 60's belonged to various expert groups that worked for the reformation of mathematics education.[9] Håstad was for instance one of the experts on New Math in Sweden. Thus, if we consider the historical accounts of Magne and Håstad, it is fair to say that the view of the 'winning team' has been transformed to historical facts.

I think it is important to detect this type of biases, not just because historical accounts in educational research ought to be unbiased, but also because researchers, teacher educators or administrators may benefit from a less biased history as they pose questions about mathematics education. In order to pose innovative questions it may be fruitful to question things that we today take for granted. If we consider the historical backgrounds mentioned above, they all include an idea of rapid and fundamental changes of mathematics education during the 60's and early 70's. However, if we compare these changes with the previous changes during the period 1900-1960, which indeed took place, we find people in leading position whose arguments about how to develop mathematics education that appears to be similar (Prytz 2007, 2012). Even though they considered different concepts, e.g. symmetry in the 1920's and sets in the 1960's, they relied on the same educational assumption: mathematical concepts and the teaching ought to match the students' spatial intuition.[10] Thus, the choice of concepts and representations of concepts were considered important in the development of mathematics education. It would be interesting to investigate further what happened with this assumption after 1970. My impression though, after having taught at teacher education for the last five years, is that it

[9]Kilborn et al (1977), *Hej Läroplan. Hur man bestämmer vad våra barn ska lära sig i matematik.*

[10]Before 1950, the debaters used the Swedish word "åskådning". The concept is quite similar to the German concept anschauung. In the 1960' s, the debaters used the word "konkretisering", a word that was used also in the methodological part of the curriculum documents. I usually translate konkretisering by visualization. Despite the different word, if we compare the reasoning about "åskådning" and "konkretisering", there are considerable similarities.

still is an important assumption. For example, textbooks about mathematics education often contain a chapter about how concepts may be represented and explained by different types of materials or pictures (cf Löwing & Kilborn 2002). These books put an emphasis on why different materials and pictures ought to be used and they provide examples of how materials and pictures can be used to explain different concepts, e.g. division and percentage. Yet there is a lack of concepts and general principles about how to design materials and pictures for different mathematical topics. Moreover, in most cases there are no scientific references attached to central propositions and examples regarding these matters. In what respect these missing parts reflect a shortage of research results or an inability to include such research results in teacher education I cannot tell. Nor do I pass any judgments about this missing part. I just want to draw attention to that these missing parts in teacher education, and possibly also in research, do not match some of the historical accounts mentioned above; the latter establish that pedagogical and psychological research became a part of Swedish mathematics education after 1960. My point is that biased historical accounts may conceal continuities or discontinuities in the development of mathematics education that might deserve serious questioning.

So as to detect biases in historical accounts, I think it is helpful to know about the authors' involvement in reformations of mathematics education. In that perspective, the relation between position and standpoints becomes interesting. Knowledge about a person's positions may reveal motives as well as influence. Håstad and Magne, mentioned above, is a good example of that. In the next part of this paper I discuss how this relation may be studied by means of a sociological analysis.

Part II - On the use of a sociological perspective in research on the history of mathematics education

Why a sociological perspective?

If we by the phrase 'using a sociological perspective' mean that we consider relations between people and groups of people as we study the history of mathematics education, the question might seem a bit trivial. However, it is not, at least if we consider contemporary research about the history of mathematics education in Sweden (cf Kilpatrick & Johansson 1994, Bjerneby-Häll 2002, Hatami 2007, Prytz 2007, Lundin 2008). The main material in these works is educational texts, e.g. textbooks, policy documents and teacher periodicals and the main method is pure textual analysis. The latter makes it difficult to explain why content and methods of school mathematics changes, even though the researchers put changes in school mathematics in connection with general scientific, economic, political and social changes. My main objection is that these researchers cannot explain why some arguments, textbooks, authors and debaters were more influential than others. General societal changes (economic, political or social) do not, per se, produce a certain kinds of school mathematics. Nor do changes in mathematics as a science. You may understand such general changes as possible incentives for change in a school system, but you cannot derive from them, in more detail, the contents and methods of school mathematics during a specific time period. You would then ignore that an educational system most likely comprise different groups of people who interpret, not necessarily in the same way, what societal change is, but most importantly, what its consequences for school mathematics are. Moreover, these groups may have had different amounts and types of influence over school mathematics. The previous comparison between public and professional debates about mathematics education indicates that this was the case during the period 1900-1960. My point is that if we want to understand why mathematics education change in a certain direction it is necessary to integrate textual analysis with a sociological analysis that focuses on groups with influence over school mathematics.[11]

My approach in this matter has been to think of different arenas of decisions. There is a central political arena (including for instance a parliament and a government) and a central administrative arena (e.g. a national school board) where groups of people make decisions about education. Another type of arena comprises all the teachers that make decisions as they plan and carry out the teaching. A question that interests me is what other types of arenas of decisions are there and who inhabits these arenas? An important research task in this perspective is to discover such arenas, i.e. to identify and describe groups of people and positions that had influence over mathematics education during certain periods of time. If we know about these arenas it is possible to investigate influence and why mathematics educations change in a certain direction. More precisely, to follow the transmission and reception of ideas about school mathematics between arenas in both directions: which ideas that were recognized as important and which that were ignored; which ideas became influential, which did not.

For some years I have grappled with questions about the influence of ideas, arguments or textbooks in mathematics education. Obviously, these questions concern power in a school system. My underlying idea has been to study course plans, textbooks, teacher periodicals and exams; educational texts that have affected teachers as they plan, carry out and evaluate their teaching. Consequently, the people in control over the design and production of these educational texts have power over the content and methods of teaching. However, I am by no means the first to stress the significance of educational texts in studies of how school systems work. For example Lindensjö & Lundgren (2000) explain why the production of educational texts should be an integral part in studies of how educational reforms are received in different contexts. The important aspect from my point of view is that if we can determine groups of people involved in the production of educational texts, we can also determine one or more arenas in the decision-making process. Moreover, by studying relationships between people within a group, but also relationships between groups, we may say something about influence.

How to study social structures of mathematics education—some general notes

In my studies of the social structures I have applied a prosopographic method combined with Bourdieu's theory of field and capital (cf Bourdieu 2000, Broady 2002a). A prosopography can be considered a collective biography (cf Broady 2002b). The collective in this case are the authors or editors of educational texts. An important part of the analysis is the identification and description of these peoples' properties (or capital or assets) and how people and properties are linked to each other. Examples of properties are teacher experience, educations and authorship in different areas or genres, but also positions in school administration or science and positions such as editor of teacher periodicals.

[11] For a thorough discussion about social and institutional aspects regarding the formation of school mathematics, see for instance Popkewitz (1988). Regarding the need to study social contexts in research about the history of mathematics education it has been emphasized by for instance Schubring (2006) and Howson, Keitel & Kilpatrick (1981). But, also researchers who focus on school governance emphasize this need; for example Lindensjö & Lundgren (2000) underscore that social relationship and power relationships should be considered in studies of educational processes.

Here, standpoints about mathematics and education are also seen as properties. Another important part of the analysis is to identify those assets that were valued by people in the investigated collective. This can be done by investigations of explicit statements, grading systems or symbols of recognition, but also by comparing assets of people in certain positions with the assets of those who was not in these positions. Note that position may function as an asset. For instance, headmaster of a school is a position, which can be an asset if you want to make a career in the school administration. By identifying patterns in peoples' properties and positions, it is possible to discern value and reward systems (systems of recognition) and how they are linked to such things as standpoints regarding education and mathematics, but also the production of educational texts. Hence, an important part of the analysis is to discern the logic between assets, positions and standpoints. For that end there are specific concepts and techniques, see Broady (2002a). This is, however, not the place to go into the details.

Regarding the choice of sources, my basic criterion has been that the texts have been produced for teachers and used by teachers in connection to teaching, including for instance curriculum documents, textbooks, books about teaching, teacher periodicals and exams. But, teacher periodicals are particularly interesting since they contain debate articles in which it is possible to discern demarcations between different groups of people as contradictory opinions become visible. However, the method outlined above has its limitations in historical studies; limitations related to the existence of educational texts and to what extent practices within an educational system (national, regional or local) relied on such texts. So far, I have mainly studied the history of mathematics education in Sweden from about 1860 and onwards and for that period the approach works. But for periods further back in time, I think that the supply of educational texts was different. This is to do with the professionalization of teachers and the occurrence of professional periodicals and specialized literature on teaching, but also changing systems of degrees and national exams and less detailed central policy documents. Yet, these reservations should not immediately imply that the supply of educational texts was smaller and the approach should be abandoned. A first step should perhaps be to rethink the idea of educational texts. It might happen that types of texts that we today do not associate with usage in schools, may have been used for educational purposes in past times. The Bible or why not Euclid's *Elements* are examples of that.

Some results

In this section I outline some of the main findings and results from my research in the history of Swedish mathematics education during the 20th century.[12] In connection to this I discuss two sets of questions that have been touched upon earlier on: 1) How do we know that differences in arguments, debates or textbooks are not accidental? Or are they a result of the selection of sources rather than a reflection of actual differences of opinion? 2) How do we know that a certain educational texts mattered? Where some authors more influential than others?

As mentioned before, the public debate and the professional debate about mathematics education

[12] Thorough descriptions and analyses of debates and arguments about content and methods of geometry instruction during the period 1905-1962 are accounted for in Prytz (2007). In the same treatise you find also thorough descriptions and analyses of textbooks together with statistics on textbooks. Facts about the debaters during the period 1905-1962 and the sociological analysis are accounted for in Prytz (2009). The analysis of changes connected to the major school reforms in the 1960's is about to be published in 2012. The paper was presented at the Second International Conference on the History of Mathematics Education held in Lisbon, Portugal, in October 2011.

during the period 1900-1960 were different. One difference not yet discussed is the small number of people engaged in the professional debates. If we consider the professional debate on geometry instruction, it involved a total of seven peoples during the period 1905-1962. All of them were authors or editors of textbooks on geometry and other topics. Apart from these seven, there were about five more authors of geometry textbooks during this period of time. You might think that the small number constitutes a historiographical nuisance as the standpoints of these people cannot be considered representative for all teachers. This is indeed true, but we must remember that we are considering the production of educational texts. In that perspective 7 people out of 12 is a rather large proportion. Moreover, an interesting fact is that the production of educational texts about geometry was concentrated to a small group of people. I will return to this fact later on in this chapter.

First, let us look at the differences in contents of the public and professional debates. These differences appear not to have been a coincidence if we consider the professional debate in a sociological perspective. There is evidence of that the group of people involved in the production of educational texts about geometry instruction was independent in relation to other institutions in society. Prytz (2009) shows that the professional debate on geometry instruction, during this period, functioned as a field. This means that there was a certain pattern in the debaters' background, their way of reasoning and their positions. One important aspect of a field is that the people in it tend to value, but also fight about, standpoints, actions, objects and symbols that are common to this group of people. A second important aspect is that the same people tend to disregard other types of standpoints, actions, objects and symbols that are valued by other groups of people or institutions. The people who debated geometry instruction fit into this pattern.

The peoples who reached leading positions in school mathematics, i.e. editor of the major teacher periodical on mathematics and science education and advisor at the central school board regarding mathematics education, shared some properties. These properties separate them from people who did not reach these positions. If we consider the standpoints of the peoples who reached leading positions in school mathematics, they all made a clear distinction between scientific and educational standards. School geometry had to apply to both, but in cases in which a conflict arose, the latter should prevail. This distinction was not made by people that did not occupy this type of positions. Instead, they meant that scientific standards had an educational value. In fact, much of the debates were about the relevance of various scientific and educational standards, but also which of the two types of standards that would be superior to the other.

This difference in standpoints divides the debaters in the same way as differences in the debaters' backgrounds do, which is also typical of a field. One part of the analysis of backgrounds has been to consider different types of recognized skills. Here we can see that skills related to being a prominent research mathematician was not a common property of the people in leading positions. Indeed, all the debaters had a background in science since they had a Ph D in mathematics, but only one of them made some kind of scientific career in mathematics. However, he did not reach a leading position in connection to school mathematics. Instead, the property of the people in leading positions that was common to them and only to them was success as textbook author. They were among the most successful of their time, not only in geometry but in other mathematical topics as well. Moreover, I would say that a symbol of recognition was linked to textbooks, a recognition of the skill to write and edit textbooks. I think it is reasonable to say that recognition was given by teachers as they chose to purchase a textbook. But, we cannot understand recognition in the sense that teachers chose the textbook that was optimal in relation to their idea of teaching. However, it is fair to say that if a teacher[13] chose a textbook, he or she also valued that same textbook on the basis of some, at least, elementary standards; especially when there were always at least two different textbooks to choose from that were regularly republished during the whole period 1900-1960. In this sense, I think it is reasonable to talk about teachers giving recognition of skills to author and edit textbooks. Another argument to why a symbol of recognition was linked to textbooks is that textbook design was something the debaters fought about: it was a central subject in the professional debate about geometry instruction. Moreover, the discussions about textbook design were initiated by non-established textbook authors, who happened to be the same people that did not reach leading positions. The critical articles were also comprehensive and so were the replies of the criticized authors. My point is that textbooks and textbook design triggered action and reaction. When the contenders in the professional debate attacked the established, they focused on things with high symbolic value among professionals. Issues that were important in the public debate about mathematics education came in the background.

Here, I want to pick up the question about whether the differences between the public and the professional debate should be considered accidental or not. I would say no. As just mentioned, Prytz (2009) has showed that the people involved in the professional debate about geometry instruction functioned as a field. An important aspect of a field is independence. The differences between the public and professional debates reflect the independence of people involved in mathematics education vis-à-vis other groups in society, e.g. scientists and politicians.

I also want to return to the questions about the importance of the professional debate and influence of debaters and the texts. The sociological analysis puts us in the position to answer such questions. I think it is fair to say that some of the debaters did have more influence or power than others. The group of people who reached leading positions in school mathematics not only had these positions; they also controlled a large part of the supply of educational texts regarding geometry as they were editors of the major periodical that treated the subject and they authored the most used textbooks. On this point it is important to know that central curriculum documents during the period 1900-1950 were very brief and general on the contents and methods of school mathematics and all other school subjects. Moreover, there were no specific books about mathematics instruction in secondary school until the early 1950's. There were however national exams at the end of lower secondary level (Realskolan) and the end of upper secondary level (Gymnasiet), which may have influenced the teachers. Still, this influence was restricted to a small number of exercises on a couple of occasions every year. Moreover, not all students took the exam at lower secondary level; it was not a prerequisite for entrance to upper secondary level. My point is that textbooks and articles in the leading teacher periodical on mathematics and science education constituted the bulk of educational texts that treated content and methods of mathematics instruction in a comprehensive way. Hence, the production of the main part of the educational texts regarding geometry was controlled by half a dozen people. So I think it is fair to say that these people had influence and that the professional debate mattered. Moreover, if we want to understand why mathematics education changes in a certain way, why some ideas and arguments gain influence or not, this is an important group to study.

[13] Or the teaching staff at a school.

Further research

In the coming five years, I will be researching to what extent general reforms of the Swedish school system, but also specific attempts to change school mathematics, affected the content and methods of school mathematics. I will also try to answer questions about why some attempts to change were successful and others were not. The theories and methods that have been previously described will then be applied. A first step is to study the period 1950-1975. This was a period when the Swedish school system went through great changes; a system with parallel school types was replaced by a system with one compulsory school type (year 1-9) and a voluntary upper school type (year 10-12). Also mathematics education changed in connection to this. One of the major specific attempts to change school mathematics was the new course plan in mathematics, influenced by New Math, which was introduced in 1969. In order to discern changes, comparisons with the first half of the 20th century will be necessary. On that point, it will be possible to use the results from my previous studies (Prytz 2007, 2009) on geometry instruction during the period 1905-1962.

One aspect that interests me in particular is how changes of structures that are to do with the organization of the school system affect the content and methods of school mathematics. That is, I am also interested in causes other than those that are related to explicit attempts to changes the contents and methods of school mathematics.

A structural change of this kind is the forms of course development. In the 1960's, the state tried to get greater control over the courses. One means was to issue comprehensive course plans. In the secondary school statutes of 1933, descriptions of the contents of courses are brief. For instance, the contents of the mathematics courses of Realskolan (lower secondary school, year 5 to 9) were described in approximately 240 words. Moreover, the secondary school statute as a whole did not contain sections on teaching methods, neither general ones nor specific ones for each subject. In the curriculum of 1962, the description of the purely mathematical content of the corresponding courses of Grundskolan (year 5-9)[14] comprises about 1000 words. In addition to that there are comments and directives regarding planning and teaching methods in mathematics that comprises approximately 1550 words. And in addition to that there is a final section on how, in what order and to what extent each mathematical topic ought to be treated. This final section comprises about 3500 words (Prytz 2012). Apart from that, there was a general section on teaching methods. Considering this difference in the national curriculum documents, the textbook authors[15] before 1950 had greater freedom, but also more power over the content of the mathematics courses. Thus, the forms for the development of the mathematics were different before and after 1950.

A preliminary study (Prytz 2012) indicates that the professional discourse in the leading professional periodical about mathematics education changed in connection to this; more precisely the leading peoples' attitudes towards teachers and authorities. Before 1950, the leading person presented important assumptions and research results as possible to discuss; their certitude was not presented as evident. In the 1960's the attitude was different. The leading person presented important assump-

[14]Year 7-9 of Grundskolan comprised two types of courses in mathematics: general and special. The latter corresponded to the mathematics courses of Realskolan. The chapter of the 1962 curriculum that treats the mathematics courses contains common sections for both types of courses and specific sections for each type of course. I have counted the words of both the common sections and the sections that treat the special course specifically.

[15]Unless otherwise indicated, the term textbook author also refers to the publishers involved in the production of the textbook.

tions and research results more as non-discussable facts. There were also other changes in the investigated articles that reflect this change in attitude. The argumentation got shorter. The meaning of the propositions became less distinct and explicit references to scientific treatises disappeared. Moreover, this change in attitude can be tied to changes of the forms of course development. It is about how dependencies between leading people and the teachers changed as course plan authors got more power on the behalf of textbook authors. Textbook authors are dependent on the teachers, the buyers of textbooks, in a way that the course plan authors are not.

A second structural change that interests me is the abandonment of a parallel school system and the introduction of one mandatory school type. This change also included the decision to have a heterogeneous classes in the whole of Grundskolan rather than specialized classes during the last years, for instance academic and vocational classes. Thus, the conditions for differentiation became quite different. In Sweden, the concept individualization was central in the discussion about how to deal with heterogeneous classes. The Central school board even initiated a project that was supposed to develop material and teaching methods for school mathematics so that the teaching could be individualized (Prytz 2012).

In order to discern to what extent organizational changes affected the contents and methods of school mathematics, the investigations will be repeated in Germany. Germany is interesting since Swedish and German school systems, on a general level, were organized in similar ways before World War II. In the 60's, however, the Swedish school system changed in a more comprehensive way. In West Germany they kept, for instance, a parallel school system. By using the German case as a kind of reference point, it is possible to study to what extent the Swedish organizational model with heterogeneous classes affected the contents and methods of school mathematics. Or rather, it is then possible to avoid pure counterfactual speculations if you want to answer such questions.

Of course, this type of comparative studies can be applied on other countries. The more countries and school systems that are investigated, the more comparisons can be made. As I see it, more countries also mean better opportunities to discern global similarities and local characteristics. So, I want to end this talk with an offer. If you are interested to carry out the kind of studies that is described in this talk, please let me know and I will provide relevant references. I also want to invite you to the Third International conference on the History of Mathematics education in Uppsala next summer. In connection, to this conference it will be possible to arrange a short introductory course about prosopography, Bourdieu's theory on field and capital and how they can be applied in historical research.

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