

EUCLID'S PROPOSITION II.5: A VIEW THROUGH THE CENTURIES

Geometry, Algebra and Teaching

Leo CORRY

Tel Aviv University, Israel
corry@post.tau.ac.il

ABSTRACT

Book II of Euclid's *Elements* has played an important role in the historiography of Greek mathematics. The main reason for this is that its propositions express geometrical results that are easily reformulated in modern algebraic symbolism. This has given rise to a well-known historiographical debate on whether Euclid's original conception behind Book II was purely geometrical (and hence interpreting it in algebraic terms is anachronistic) or if, rather, it was algebra written in the language of geometry (and hence it can be seen as be characterized as "geometric algebra").

Beyond the historiographical debate, a look at the ways in which Book II was presented in the various editions of the *Elements* starting from the Middle Ages and up to the late 19th century shows that not only historians, but also mathematicians trying to come to terms with its contents, looked at this book in different ways, concerning the roles of algebra and geometry in the results presented in it. Thus, Book II of the *Elements* offers a unique point of view from which to consider in historical perspective the changing relationship between geometry and algebra.

More specifically, one illuminating way to understand this process is to focus on a specific result of Book II, Proposition II.5, and to analyze the metamorphoses underwent by it, since the time of classical Greek mathematics to the early twentieth century. In this talk we take a guided tour that highlights the ways in which changing views about the interrelations between algebra and geometry in different mathematical cultures may influence the multifarious interpretation given to one and the same result. Particular attention is paid to the ways in which the uses of the *Elements* as a textbook for the study of elementary mathematics affected this issue. By analyzing selected texts produced in changing historical contexts, it is shown that, while symbolic manipulation and other mathematical ideas that we typically associate with algebra were incorporated in various ways to proofs of II.5 already beginning with the Greek commentators of Euclid, none of these additions or their combination did ever imply a definite change of orientation that all subsequent authors felt compelled to follow. At various times and up until the nineteenth century, one can still find mathematicians who preferred, for different reasons and in changing circumstances, to move back and forth from a purely geometrical to a more algebraically-oriented approach to Book II of the *Elements*, and particularly to II.5.

Among the mathematicians whose versions of II.5 we explore are Heron, Ibn-Qurra, Gersonides, Clavius, Barrow and Wallis.