A COLORFUL CASE OF MISTAKEN IDENTITIES

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ABSTRACT

In 1594, the Medici Press printed an Arabic version of Euclid as part of its overall publication program. It was one of the earliest European attempts to print Arabic from moveable type and the result was elegant indeed. But the book has long been the victim of a mistaken identity. Not only has the book itself been misidentified, but the entire history of the treatise (which we can now trace for more than four centuries) has been repeatedly influenced by mistaken identities. My paper aims to clear up at least most of these mistaken identities. This historical research lays the groundwork for informed use of this historical episode in our teaching of mathematics.

Keywords: Pseudo-Ṭūsī, Medici Press, Euclid's *Elements*

1 Introduction

In 1594, the Typographica Medicea, which had recently been established by Ferdinand de Medici in Rome, issued an elegant edition of an Arabic treatise on Euclidean geometry with the title *Kitā Taḥrīr Uṣūl li-Uqlīdis* [*Redaction of the Elements of Euclid*]. The publication, one of the earliest attempts to typeset Arabic mathematics in Europe, represented the highest standards of workmanship, expertly blending type and diagrams. Because of the clarity of its typeface and ease of access (as compared to manuscripts) the treatise quickly became a standard source for scholarship on the history of Euclidean geometry in the classical Islamic period. Although the motivation for this publishing experiment is still somewhat unclear, the press directors obviously had several potential markets in mind, for the treatise was issued with different title pages—some were in Arabic only and some were bilingual Arabic and Latin (Cassinet 1993, 20-21). They apparently also expected a fairly high demand for the edition because they printed 3000 copies. These expectations were clearly incorrect—nearly a century later, almost two thirds of these copies were still in the storerooms of the press (Jones 1994, 108). In this paper we survey the later history of this remarkable edition and its influence in mathematical circles—a history long clouded by a mistaken identity.

2 Mistaken identity

The title pages (both Arabic and Arabic-Latin) of this remarkable edition have produced one of the more long-lasting errors of historical studies of mathematics because they attribute the authorship of the treatise to Naṣīr al-Dīn al-Ṭūsī (597 / 1204 – 674 / 1274), one of the best-known and most influential of the mathematicians of the medieval Islamic world.[1] His redactions of the Arabic translations of important Greek mathematical treatises, beginning with Euclid's *Elements*, progressing through several smaller Greek tracts (known collectively in Arabic as the *Mutawasiṭāt*, or intermediate books) and culminating with Ptolemy's *Almagest* laid a new foundation for mathematical studies that continued to form the core of the curriculum in mathematics education until the nineteenth century.

Despite the bold title page statement that the Rome 1594 treatise was none other than the Taḥrīr Kitāb Uqlīdis by Naṣīr al-Dīn al-Ṭūsī, the most influential Euclidean text of the Arabic / Islamic world, it was a different treatise — different in style and diction, different even in mathematical content (at least as far as its "demonstration" of Euclid's parallel lines postulate is concerned). Since the printed text differs in important features from the many surviving manuscripts of the *Taḥrīr Kitāb Uqlīdis* that carry al-Ṭūsī's name, it was often initially assumed to be only a re-editing of the text by the author. This hypothesis seemed plausible initially but became untenable when scholars discovered a note in a manuscript copy of the treatise, manuscript Or. 50 in the Laurenziana Library in Florence, stating the date of completion as 698 / 1298 (Sabra 1969, 18). Since al-Ṭūsī died in 1274, to continue to ascribe the treatise to his authorship would require us to elevate "ghost-writing" to an entirely new level!

It was early recognized that text the Rome 1594 edition differed in some ways from that found the genuine redaction by al-Ṭūsī. For example, the Rome edition contains only 13 books, while that of al-Ṭūsī contains in addition the two apocryphal books (numbered fourteen and fifteen) that are ascribed to Hypsicles in the Arabic Euclidean tradition.[2] The erroneous ascription on the title page, however, inclined many historians to believe that it represented only a re-editing of the original text. This hypothesis becomes considerably less likely when one looks at the texts in some detail. For example, the redaction of al-Ṭūsī contained nearly 200 notes added by the author. Some discuss mathematical questions raised by Euclid's text, others substitute direct proofs in place of Euclid's indirect demonstrations, still others offer alternative demonstrations for various propositions.[3] These added notes are not found in the Rome edition, except for a few editorial notes describing differences between the Arabic translations of al-Ḥajjāj and Isḥāq ibn Ḥunayn. Furthermore, the Rome edition incorporates into its text many explicit references to definitions and earlier propositions as justification for points in the mathematical argument. Neither Euclid himself nor al-Ṭūsī included such references. There are many differences in diction and in style as well. For example, for statement of the problem in proposition I, 2 (I translate from the Arabic):

T = We want to extend from a given point a line equal to a bounded line.

PT = We are to add to any specified point a straight line equal to a bounded straight line on condition of the two of them being in a single plane.

[1]The origins of this title page ascription are a mystery. Al-Ṭūsī's name is not mentioned in any of the surviving manuscripts, and certainly not in the manuscript from which the Rome edition was typeset.

[2]Modern scholarship accepts the attribution of book XIV to Hypsicles (1st century BCE), perhaps based on an earlier lost treatise by Apollonius. Book XV is now considered to be a compilation from several sources, one of which appears to be Isidorus of Miletus (6th century CE), who is reputed to have written commentaries on the *Elements* that no longer survive (Vitrac & Djebbar 2011, 31-32).

[3]The majority of these notes were borrowed from the *Kitāb fī Ḥall Shukūk Kitāb Uqlīdis* [On the resolution of doubts raised by Euclid's treatise] written by noted mathematician Ibn al-Haytham (died 432 / 1031) although Ibn al-Haytham's name is not mentioned (De Young 2009).

Such extensive variations in formulation, not to mention the many differences in diagram patterns make it more difficult to assume that the Rome edition is merely a re-editing of al-Ṭūsī's treatise.

Although we can now be sure that the author of the Rome redaction was not al-Ṭūsī, we do not know who to assign as author. Some scholars (Rosenfeld & Ihsanoğlu 2003, 211–219)have argued that it must be the son of al-Ṭūsī, Ṣadr al-Dīn, who took over his father's position as head of the research institute at Marāgha. But until now we have no contemporary documentary evidence concerning the author's identity, so many scholars prefer to designate him as Pseudo-Ṭūī. I shall follow this designation as well.

3 Typesetting the Rome Euclid

It had long been known that there were two manuscript copies of the Pseudo-Ṭūsī in the Biblioteca Medicea Laurenziana in Florence. These manuscripts were misidentified, however, in one of the most widely used reference works in the history of science — the *Dictionary of Scientific Biography*. In his article outlining the complex transmission of the *Elements*, John Murdoch, identified these manuscripts as Bibl. Laur. or. 2 and or. 51. Neither number seems to be correct if one checks the online catalog from the library. Only after expending more than a thousand euros of my university's research budget to purchase scans of every work identified with al-Ṭūsī in the orientali collection did I discover that the correct identification (or. 20 and or. 50) had been published years earlier in a footnote to one of Sabra's studies (1968, 15) of parallel lines in the Arabic tradition. Needless to say, I quietly glossed over this minor point when making my report to the university administration.

In my investigation of this well-known impostor and based on these expensive scans, I can now state with confidence that the Rome edition was typeset from manuscript orientali 20. Typesetter notes in Italian and other markings in the margin of the manuscript make this conclusion certain. I can also state with certainty that the text of orientali 20 was copied from orientali 50. The copying is evident when we examine passages that were canceled out in the text of Or. 20 and rewritten in different form in the margins. In one case, the copyist accidentally turned two leaves. When he discovered the error, he canceled the entire section and rewrote it correctly in the margins of Or. 20. We can see that the lacuna left by the copyist extended from the last word on folio 46a until the first word on folio 47b of Or. 50 (De Young 2012).

It is curious that the diagrams of manuscript Or. 20 were not copied from Or. 50. This unusual situation is clearly evident when we look at diagrams for books VII–IX, Euclid's discussion of numbers and their characteristics. Traditionally, diagrams found in these book used line segments to represent numbers. Or. 50 follows this convention, but Or. 20 adopts a new technique – using columns of dots to represent numbers. Although this new style may be more consistent with the spirit of Euclid's work, it leaves a mystery — where did this innovation come from? So far, the technique has not been observed in any earlier manuscripts, so perhaps it was the copyist himself who made this change in the diagrams.

4 How did the press get its Arabic manuscripts?

Although the story is still somewhat murky, it appears highly probable that the manuscripts used by the press in its publishing efforts were brought to Italy by Ignatius Niʿmatallāh, former Patriarch of the Syrian Orthodox communion (1557-1576), who had been forced into exile through another kind of mistaken identity. Ignatius, a typical medieval polymath, was also a practicing physician. Because of his skills, he had become personal physician to the local Muslim governor. The local Muslim intelligentia were not happy to have a high-profile Christian in such a powerful position and frequently stirred up trouble at the court opposing the Patriarch. Perhaps to diffuse some of this tension, the governor, during one of his evening salons, "honored" the Patriarch by placing his own turban on his head and declaring him a "convert" to Islam. Although there is no indication that the Patriarch wished or intended to renounce Christianity, the governor's "honor" forced Niʿmatallāh (through a kind of mistaken identity — his fellow Christians now regarded him as a traitor to his faith) to abdicate his position. He fled to Italy, taking with him his collection of manuscripts. In Italy, he continued to pass himself off as "Patriarch" (perpetuating yet another mistaken identity) and was generally received with great honor in the halls of power. While traveling from Venice to Rome, he was apparently introduced to Ferdinand de Medici, who was contemplating a new business venture — a publishing house to produce Arabic texts. Ferdinand had money but needed Arabic manuscripts while Niʿmatallāh had Arabic manuscripts and needed money. A deal was finally struck and the "Patriarch" joined the board of the infant publishing house, putting his manuscripts at its disposal and setting in motion one of the great mistaken identities in the history of mathematics.[4]

5 Influence of the Pseudo-Ṭūsī edition in Arabic

In Arabic, the Pseudo-Ṭūsī redaction was far less influential than the treatise actually authored by Naṣīr al-Dīn — at least if we consider number of surviving manuscripts to be any indication of popularity and influence. Nevertheless, it is now possible to trace instances of influence from the Rome 1594 edition for several centuries in the Arabic-speaking world. For example, a century after the publication of the treatise, a manuscript copy (Tehran, Sipahsalar 540) was written out from the typescript. Since manuscripts do not have title pages, the treatise is not explicitly assigned to al-Ṭūsī. Nevertheless, this manuscript has also been the victim of a mistaken identity. Someone — almost certainly not the copyist himself — wrote a note on the flyleaf identifying the manuscript as the *Iṣlāḥ* [*Uṣūl Uqlīdis*] [Correction of Euclid's Elements] by Athīr al-Dīn al-Abharī (died 663 / 1265). The text makes clear, however, that it is actually a copy of the Pseudo-Ṭūsī *Taḥrīr* and not al-Abharī's *Iṣlāḥ*.

Initially, we might be surprised that someone would want to copy a printed book by hand. But the conclusion that this is what happened follows from a small note inserted in the margin beside the demonstration of proposition VI, 1: "Apparently a diagram should exist here." The note makes sense when we look at the Rome edition, page 134. There is a blank space, apparently left for a diagram that was never inserted. Moreover, the diagrams in Sipahsalar 540 preserve the same distinctive features shown in the Rome edition. Although we don't know where it was copied, the manuscript is dated 1101 / 1670.

Some two centuries later, the Pseudo-Ṭūsī version surfaced once again, this time in the form a two-volume lithograph edition published in Fez (1293 / 1876). Like the earlier Rome edition, the title page proudly proclaims it the work of Naṣīr al-Dīn al-Ṭūsī. So even three centuries after the first print edition, the mistaken identity of the author continued to live on. This new lithograph version

[4] Saliba (2008, 199–212) gives one of the most complete summaries of the colorful Patriarch's life. Toomer (1996, 22–24) also provides a short biographical sketch of his role in transmitting Arabic learning to Europe.

has not yet been fully studied, but preliminary surveys indicate that its diagrams exhibit distinctive features exactly like those found in the Rome edition. The text is identical to that of the Rome edition, although written now in Maghribi Arabic script and many of the obvious typographical errors in the Rome edition have been corrected. The observation that proposition III, 11 is incorrectly numbered III, 12 in each printed version is very strong evidence for a direct genetic connection between the two. The probability that the incorrect number should appear independently seems incredibly small.

It seems clear that even though there are few surviving manuscript copies, the text continued to be read and copied from time to time. But at the same time, there is no evidence of its influence in the broader tradition. No commentary on the text has yet been identified, and the surviving copies have little or no marginalia apart from corrections to scribal errors. The lack of influence is somewhat puzzling because the treatise seems ideal for mathematical education, especially for younger students just beginning their journey in mathematics. Al-Ṭūsī's treatise was aimed at more mature students who already had acquired some basic knowledge of philosophy and logic. The Pseudo-Ṭūsī treatise assumes very little from the learner. The extensive inclusion of references to earlier propositions, similar to the system one finds in many modern textbooks of Euclidean geometry help novices to find their way through the logic of the arguments. Only the existence of the Fez lithograph, though, hints toward any educational use of the treatise. And this hint is indirect and incomplete at the moment because there seem to be only a few copies in existence and those that have been digitized on line show no evidence of use by students. Still, why would anyone print a treatise on Euclidean geometry unless it was expected that there would be some market to repay the investment. And the most logical market to assume is from the madrasa. More study will be needed on the educational system in North Africa in the 19th century in order to resolve some of these puzzles.

6 Influence of the Pseudo-Ṭūsī in Europe

From the time of its publication, the treatise had been of considerable interest to European mathematicians. Their focus had been almost entirely on one small section of the treatise, though — the demonstration of Euclid's parallel lines postulate. This demonstration follows proposition I, 28.[5] It is built on three lemmas:

1. Any two straight lines [being] placed in a plane, if there fall upon them lines, each one of which is perpendicular to one of the two and cutting the other in acute and obtuse angles such that the all the acute angles are toward one end of the lines and the obtuse angles toward the other end, I say that the lines are getting shorter the closer one moves toward the side facing the acute angles and that they are getting longer the closer one moves toward the side facing the obtuse angles.
2. Two straight lines extended perpendicularly from the endpoints of a straight line being equal to one another and we connect their two endpoints with a straight line, each of the angles formed between the two perpendiculars and the straight line connecting their endpoints is right.
3. For any triangle whose sides are straight lines, its three angles are equal to two right angles.

[5] A modern French translation of the entire demonstration has been given by Jaouiche (1986, 233-241).

Having established these lemmas, the author turns to Euclid's statement of the parallel lines postulate: "If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles (Heath 1956, I, 155)." There are several possibilities to consider. The angles formed by the line falling on the two lines will be either (a) one right and the other acute, (b) both acute, or (c) one obtuse and the other acute. In each case, the postulate claims that the two lines, if extended indefinitely, will meet on that side. The author gives a demonstration for each of these possible cases, thus "demonstrating" Euclid's postulate.

The existence of this "demonstration" had apparently been traveling as a rumor among mathematicians for some time before the Rome edition was published. The Jesuit mathematician, Christoph Clavius (1538-1612) wrote in the introduction of his 1589 edition of Euclid (Knobloch 2002, 419): "We learned long ago that the Arabs demonstrated the same principle. Though I diligently looked for the demonstration for a long time, I could not see it, because it is not yet translated from the Arabic into Latin." Clearly, some rumors were spreading concerning the existence of an Arabic demonstration of Euclid's postulate during the years before the Rome edition was printed. And in the last edition of his work, Clavius lamented (Knobloch 2002, 419): "I never got the permission to read it though I continually asked for it [from] the owner of the Arabic Euclid."

Who was this rather selfish owner of Arabic manuscripts who would not allow Clavius to view his sources? Saliba (2004, 211-212) argues that it was most probably the Patriarch himself. He had, at considerable difficulty, carried along from Antioch a considerable number of manuscripts. He probably had a good idea of their worth in the European market, and, as an exile, he knew he would need to support himself in a foreign country. Eventually, he negotiated an agreement with Ferdinand to donate his manuscripts to the Press. And in return, he received an annual stipend and was guaranteed access to his books as long as he lived. But during the period that Clavius was requesting permission to see the manuscripts, the negotiations were still taking place, and the Patriarch was probably unwilling to jeopardize their outcome by allowing access to his collection. As a result, Clavius had to figure out the mathematics alone. His results, however, are in some ways remarkably similar to those in the printed edition (Clavius 1612, 48-53).

Clavius had complained that the Arabic text of the demonstration was not available in Latin. So far as can be ascertained at present, the treatise was never translated in its entirety.[6] The first Latin translation of the demonstration of the parallel lines postulate was made by Edward Pococke (1604-1691). It seems not to have been published at that time, but was quoted by John Wallis (1616-1703), Savillian Professor of geometry at Oxford, in a lecture on the parallel lines postulate (11 July 1663). Wallis (1693, II, 665–678) included the translation in his magnum opus. He critiqued the Pseudo-Ṭūsī demonstration in order to set the stage for his own approach to the problematic postulate.

Wallis's critique was studied by Saccheri (1667–1733), one of the founders of non-Euclidean geometry. In Scholion III of proposition XXI in his classic treatise, *Euclides ab omni nævo vindicatus* [Euclid vindicated from every blemish] (1733 / 1920, 100-109) he discussed the approach of Pseudo-Ṭūsī, citing Wallis as his source. His own investigation of the "Saccheri quadrilateral" has many similarities

[6] Youschkevitch (1976, 183) claimed the existence of a complete translation, published in Rome in 1657. No one has been able to verify the existence of this translation (Mercier 1994, 213).

with the second lemma developed in the earlier Arabic proof.[7] And so an obscure and still unnamed Arabic mathematician came to leave his mark on the history of non-Euclidean geometry, while the much more famous mathematician, Naṣīr al-Dīn, who also explored the parallel lines postulate in his own writings, left almost no direct impression on the development of the subject.

7 What can we learn?

We now know much more about the history of this particular Arabic discussion of Euclid's *Elements*. We can trace both the antecedents of the printed Rome 1594 edition and its influence over the next three centuries in both the Islamic and European civilizations. Given the paucity of documentary evidence at our disposal, this is a rather surprising result.

In some respects, it seems surprising as well that this particular treatise seems to have been so completely overshadowed by the genuine redaction of Euclid written by al-Ṭūsī. The Pseudo-Ṭūsī version seems in some ways better adapted to serve as an introductory textbook. It makes the logical arguments of geometry more explicit, and it provides copious references to earlier propositions and premises that support Euclid's arguments and conclusions. At the same time, we might speculate that it was precisely these pedagogical conveniences that might have helped to keep the treatise alive over the centuries although there is at present no documentary evidence to offer support for the hypothesis. Aware of the many mistaken identities that have developed over the centuries, we can always hope that as we examine more manuscripts we may find additional copies that of the Pseudo-Ṭūsī treatise that might help to answer some of our questions.

8 Summing up

Trying to sort out the intricacies of this curious history of errors for the past year has been great fun for me personally. I have always liked jigsaw puzzles. For me as a historian, collecting pieces of information and relating them together to make a coherent historical picture is not unlike working a jigsaw puzzle. And when the puzzle is complete, there is a kind of personal satisfaction and a sense of accomplishment.

But, as my colleague, Glen van Brummelen (2010, 2) wrote recently, we historians are caught up in an important practical dilemma: "On the one hand the proper scholarly treatment of history, especially in the past few decades, increasingly demands good contextual awareness and a resistance to glib answers. On the other hand, our main clients, school teachers and educational associations, demand easily digested 'sound bites' that may be inserted with little fuss into an existing curriculum."

I think very few of us would take refuge behind the oft-quoted sentiment of G. H. Hardy (2005, 49): "I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world." We believe mathematics is useful and many of us believe that history of mathematics is also useful. But useful in what specific ways? I can only suggest a few general ideas.

On the most mundane level, whether we produce or use history of mathematics we should obviously make every effort to insure that our facts and interpretations are historically correct when

[7] Although Saccheri's work languished in obscurity for almost a century, it was revived in the nineteenth century and is now recognized as a classic.

we bring historical vignettes into our classrooms. Unfortunately, many writers of recent histories of mathematics geared toward non-professional audiences, have often simply repeat what older sources have said and so perpetuate the myths that have crept into our discipline. Our students also need to be aware of the multi-cultural fabric from which modern mathematics has been cut. Few discussions of Saccheri, for example, mention the importance that concepts developed in the remote mountains of medieval Persia centuries earlier played in the development of his ideas. This despite the fact that Saccheri explicitly cites Wallis who had explicitly quoted the demonstration of the supposed Naṣīr al-Dīn which he acquired in Latin translation from Pococke. A great deal of work has been done over the past fifty years to explore the multi-cultural dimensions of mathematics. If we can begin to incorporate some of that work into our classrooms, we will certainly enrich our students understanding of mathematics as a human enterprise as well as increase their sensitivity to and appreciation for other cultures.

Although I must leave development of lesson plans or learning modules to mathematics educators, I suggest that one possible use for this historical episode, might be to introduce the subject of non-Euclidean geometry. One might do so by providing students with a translation of the lemmas and demonstrations of the Pseudo-Ṭūsī and ask them to critique the arguments from a Euclidean standpoint — can students find where the proof assumes an equivalent of the parallel lines postulate? (Even though no English translation from the Arabic is now available in print, there is a high-quality French translation by Jaouiche (1986, 233-241) that could be used to generate an English text. Jaouiche also gives an informative discussion of the mathematical and philosophical implications of the argumentation, both by the Pseudo-Ṭūsī and other Islamic mathematicians which might serve as useful background for the teacher who feels unsure of his abilities.) Those researching the application of ethno-mathematics in mathematics education can no doubt suggest other fruitful ways to use this material.

I would only say in conclusion that I believe we educators should resist the attempt to integrate history of mathematics into mathematics education. Although this may initially seem an odd statement, allow me to explain. Integration implies the forced merger of two essentially disparate entities — Americans might think of think of integrating the segregated school systems of the 1960s civil rights movement. The result can be a half-hearted token representation of history of mathematics. Rather than force a few bits or "sound-bites" from history of mathematics into an alien mathematics curriculum in the name of integration, I think we should aim to make history of mathematics integral to mathematics education. Rather than continue to perpetuate the long-standing disciplinary compartmentalization of mathematics and history and literature, our educational systems and our students can be enriched by interdisciplinary initiatives that emphasize the web of inter-connections among the traditional disciplines. Such interdisciplinary perspectives on mathematics can be especially useful to students who are not focusing on mathematical disciplines. Of course, as individual teachers we can often be overwhelmed by the demands of our day-to-day activities. We may feel we have no time or energy to develop interdisciplinary approaches. But organizations like HPM that regularly bring together historians and mathematics educators in fruitful exchanges can play a key role in supporting and furthering our efforts.

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