THE TOPOLOGICAL INTUITION OF LEONARDO DA VINCI

Leonardo VENEGAS

Universidad de los Andes, K 1 No. 18A-12, Bogotá, Colombia
lvenegas@uniandes.edu.co

ABSTRACT

In the early 16th Century was not strange at all that artists were interested in mathematics, which most notable area was Euclid's geometry. The purpose of the painters to simulate the depth of the space on flat surfaces—be it table, wall or canvas—required to explore resources quite unusual in this craft. It was not possible that scenery, characters, buildings and other painted figures compose an harmonious and credible work, if each one of them had not the appropriate proportion, as they were close or distant from the main level, and according to the angular variation under they were represented related to the front view. The technique required to get this was patiently developed in multiples empirical workshops over three Centuries. Each finding was highly significant and artists were proud of their hability to compete for contracts. Leon Battista Alberti, Piero della Francesca, Albrecht Dürer and Jean Pélerin—only pointing out some of the most significant contributors—wrote treatises of this emerging science, then called *Prospettiva* or perspective, in which a new type of geometry, not yet systematized in its principles, began to appear.

By that time, Leonardo da Vinci finished two decades of a successful work at the Duchy of Milan. In the last three years, he had the company and friendship of the mathematician Luca Pacioli. Leonardo collaborated with Pacioli, drawing the carefully elaborated illustrations the mathematician used in his treatise about the golden ratio, entitled *De Divina proportione*. In the years of working out the treatise, Leonardo studied not only the foundations of both the two-dimensional and the spatial geometry, as reflected in his manuscripts, but studied also the *Physics* of Aristotle, at least the first book, which explores the notions of *continuity* and *divisibility to infinity*. In fact, in the *Elements of geometry* of Euclid, *continuity* was only named once. Leonardo believed that, as well as the sculptor deals with discrete quantities, since each piece carved is separated from the rest, the work of the painter is related to the continuity of space, in which representation should appear all the things made by the Creator, with no space for vacuum.

So, the *Prospettiva* studied by Leonardo is not only the one that two centuries later would be known as Projective Geometry, but also he incorporated the alterations on the texture of the object observed, suffered by the density of the air which is interposed, as well as the changes of tonality suffered by the color in the distance. That is because he could say, in analogy to Dedekind, when he state his principle of continuity, that the surface, despite constituting "the limit of the body", "is not part of the same body", since it is also "the beginning of another body" (Br. M. 132a). In addition to analyze this sensitive issue, Leonardo made it a hallmark of his painting, known as *sfumato*.

At some moment of clairvoyance, between the years 1487 and 1490, Leonardo da Vinci wrote: "A point is not part of a line."[1] It must have been a singular observation among all those that illuminated his mind, for he made the annotation in a small pad, identified today as the *Codice Trivulziano*, whose 60 sheets, tight of profuse lists of terms extracted from literary works, give the impression of configuring a sort of private dictionary, to be judged by the way in which many of the 10,000 words that appear there are accompanied by an attempt of personal definition.

Of what source would the idea come to Leonardo that "a point is not part of a line"? By that time, the artist and inventor already stood out in the Duchy of Milan, and his personal library came close to the important figure of 40 volumes, among which was the *Physics* by Aristotle. Nevertheless, it is not there where Aristotle displays his interest for affairs as this one and others related (whether the surface forms part of the body it limits, for example), but properly in the *Metaphysics*.[2] And, true, we do not know for sure that Leonardo ever got to study this work,[3] but, even if he had, by other notes from his hand, that we will examine hereafter, we can state that, in what relates to this topic, Leonardo would take a direction different from that of the Greek sage, whom he so admired. To the pointed out library, soon enough would be added the *Elements* by Euclid, in whose study Leonardo would occupy a considerable time in the following years, especially under the impulse of the friar Luca Pacioli, with whom Leonardo would closely work on the creation and publication of the book *De divina proportione (About the divine proportions)*. But, as is known, already from the third definition of his First Book, Euclid leaves established a principle contrary to Leonardo's observation: "The extremes of a line arepoints."

It would not be in excess to point out that Leonardo knew to distinguish between the "mathematical" point and that other denominated "natural" in that time, equivalent to the minimum lump of dye the finest pen could leave over a board. After stopping on an anonymous manuscript, located today at the Laurentian Library, Leonardo makes the following reasoning: "The smallest natural point is larger than all mathematical points, and this is proved because the natural point has continuity, and any thing that is continuous is infinitely divisible; but the mathematical point is indivisible because it has no size."[4] As is fit to expect, Leonardo's attention was not circumscribed to the end points of a line but, in the same sense, it extended to the lines by which a figure is demarcated. Well now, Leonardo was not interested only in the conceptual understanding of these topics, for above all he saw himself as artist. And, besides an exquisite draftsman, Leonardo was a fast draftsman, as is appreciated in his tens of drawings about the birds in full flight, or in the ones of the turbulences formed by the fall of a water jet, or in the ones of the faces of protuberant features, caught in the brief passing of a tavern. Therefore, it must have resulted astonishing, to the fine and exact strokes draftsman, the discovery that lines do not exist in the world. "The line has in itself neither matter nor substance," he writes, with his specular calligraphy, on a sheet that isin the Royal Library of Windsor, in London, "and may rather be called an imaginary idea than a real object; and this being its nature it occupies no space."[5]

[1] *Codice Trivulziano*, 35r.

[2] Aristotle, *Metaphysics*, book III, 1001b–1002b.

[3] In fact, the discussion about the authorship of Aristotle initiated on that time, and the doubts would persist for several centuries. To cite a significant case, the young and brilliant contemporary of Leonardo, Giovanni Pico della Mirandola, considered the work apocryphal as he published it in his writing *Examinatio vanitatis doctrinae gentis* (IV, 5); and Pico was assiduous to the Neoplatonic Academy of Florence, institution that had exerted, for better and for worse, an important influence on Leonardo's formation.

[4] Laurentian Library, 27b.

[5] Royal Library, Windsor, 19151v.

In fact, what the careful investigator had observed is that lines, as it happens with numbers, belong to the exclusive field of thought and for that cannot be perceived with the senses. They can, indeed, be applied by our mind to the representation we make ourselves of that which we judge to be outside it. That we unthinkingly believe that a body on which we concentrate our gaze (a cloud, a table, the needles of a clock) is limited by the lines of its contour, is just fruit of a lightness of our observation, the same that would take us to conclude that the sun is smaller than the palm of the hand. What we call "contour" is no other thing that our discerning capacity applied to distinguishing an object from what is not it. Two volumes that, from our point of sight, are in the same line of vision differ from one another firstly by the characteristic of their color, not by their shape. In fact, at a certain distance, if they were partially superimposed and they both had the same color tonality on their whole surface, we would not accomplish distinguishing them. But every colored surface induces a shape, that of what the color embraces. And to the cerebral memory it results more eloquent the assimilation of that shape with the shapes it already knows; or, said in other words, it is more economical to identify than to discover. Thereof we believe to detect first the shapes than the colors.

The order of priority in the catching of those two factors, first the shape or first the color, is something that no person feels invited to determine, not just because the brevity of the interval in which they succeed each other makes them appear in an almost simultaneous manner, but because in any case the brain integrates them in the whole of a represented image, that of the perceived object. Just the painters, and in particular those that have seen themselves animated by the intention of representing the scenes of the world "such as the eye sees them," have hit upon to the need to take a position in this respect. The impressionists, for example, fascinated by the momentary effect of light, that on incidence over objects produced a myriad of minute shades, executed their works with uncountable touches of the brush over the fabric, to the way of infinitesimal stainlets of color that, in their abundance, would give the sensation of continuity. On their part, artists of the 15^{th} century used to first delineate on the board or on the wall all the significant figures, both those of the characters as well as those of the furniture, the architecture and the landscape, all that after having solved the problem of proportions according to the different planes of perspective, and then indeed dedicate themselves to filling them with color, task that many times they delegated on their young assistants.

To Leonardo, the inexistence of the contours did not obey a technical or stylistic criterion, but a discovery of the manner the spectacle of reality becomes manifest before human eyes. "Do not make the contours of your figures of a color different from that of the field in which they stand out,"[6] prescribes Leonardo to the apprentices of his way of painting. And following, explains: "this is: do not demarcate you figure off its field by means of a prominent stroke." Leonardo considered fallacious the representation of an ordinary object, if to represent it the object was isolated from its surroundings. In fact, his certainty that painting outdid—as art—sculpture came from the capacity of painting to recreate the totality of a scene the eye sees, including in it the variations of color suffered by the surfaces of the bodies, not only with respect of the angle from which light reaches them, but derived from the diverse layers of air interposed between the observer and the contemplated objects, depending on them being more or less distant from his eye. That is to say that, in Leonardo da Vinci's intuition, the world the eyes see was populated of those entities thick within but lacking borders, which in the beginning of the 20^{th} century would be known as open sets.

[6]Codice Urbinas, 46r–v.

Austrian physicist Fritjof Capra is without doubt the scientist that with more lucidity has centered his attention on Leonardo's manuscripts related with topology. In the appendix of one of his more recent books, *The Science of Leonardo*,[7] Capra studies with detail Leonardo's drawings that appear on a big folio of the Codex Atlanticus under the title *De ludo geometrico*[8] *(About the game of geometry)*, and that in the research of the noteworthy artist make part of what more broadly Leonardo denominated "geometry that is demonstrated by movement"[9]. It is about graphic transformations of geometric figures in part rectilinear and in part curved, built with rectangles, triangles and circles, by means of which a figure is transformed into another keeping invariable the area of its surface. They are processes similar to the quadrature of the lunules that centuries back Hippocrates of Chios realized, applied here to an exuberant variation of floral shapes, and taking advantage of resources so ingenious that Capra does not hesitate in qualifying this sort of geometric metamorphoses as "primitive forms of topological transformations"[10], and Veltman, with his formation more oriented towards the graphic computation, qualifies Leonardo for it as "mathematico-morphoses", and his work on surfaces and volumes as "a vision of 2-D and 3-D morphing *avant la lettre*"[11]. This same game of transformations Leonardo practiced in his studies of the human face, realizing variations of the more notorious features, like the diverse classes of nose, with the interest of characterizing facial typology according to the dominant of the mood[12].

It may not be fortuitous that, in his book *The Heritage of Apelles*, Gombrich included, in addition to the already cited article about the "grotesque heads", another essay about Leonardo, this one dedicated to the study of the waters. As has been registered since the most remote antiquity, water is, of the four elements of tradition, the only one that adapts to the shape of the recipient where it lodges. It was to be expected, then, that Leonardo saw in water the perfect means to examine the way in which the shape of a body changes without altering its volume. In incompressible liquids (in general conditions water is so), the preservation of volume entails with it the preservation of mass, so that the phenomenon of continuous transformation of a body in its external appearance, leaving invariant its mass, supposed for the "geometry that is demonstrated by movement", as Leonardo denominated that live science of the changes of nature, a field very rich in exploration. Leonardo saw implicit the continuity of the transformation in the economy of nature while realizing its movements. In folio 85v of the *Codice Arundel* it is read: "Every natural action is carried out by the shortest way."[13] It is all that takes physicist Fritjof Capra to consider this kind of Leonardo's investigations as "primitives of this important field of mathematics" that preceded Poincaré's formalization "by five hundred years".[14]

But not only in his painting was Leonardo consequent with his discovery that, as they are dis-

[7] Doubleday, 2007. References of this book that appear ing the present work have been taken from the text in standard Spanish *La ciencia de Leonardo*, Ed. Anagrama, 2008. Translation is mine.

[8] It is worthwhile to contextualize the sense of "game" as it was employed between the 12^{th} and 16^{th} centuries. While Creator, God was assimilated to the Great Geometer. Thereof Alberti denominated one of his works *Ludi matematici* (1452) and Nicolás de Cusa denominated *De ludo globi* one of his (1463). This position was adopted by artists and philosophers that in their own work did not see themselves as God's rivals but as his emulous. Cf. Veltman, Kim H., *Leonardo da Vinci and Perspective*, 2007, Maastricht McLuhan Institute.

[9] *Geometria che si prova col moto*, Codice Madrid II, folio 107r.

[10] Capra, p. 345.

[11] Veltman, op. cit., Transformational Geometry.

[12] The best reference is the essay *The Grotesque Heads* (1954), by prestigious art historian E. Gombrich, which made part later of his classic book *The Heritage of Apelles* (1976).

[13] "Ogni azione naturale è fatta per la via brevissima".

[14] Capra, op. cit., p. 271.

played in the world, things come devoid of contours.[15] As well as in one of his works, the border zone up to where the delicate veil covering with modesty the head of hair of a woman seems to extend,belongs both to her figure as well as to the remote little path winding between the rocky formations made out from her balcony, or that it may be about an indefinite tenuous zone that does not belong to either of them but to the luminous air of that unreal afternoon, so as well Leonardo abolished the artificial borders that separated the diverse disciplines of investigation. In the conception of those times, that celebrated with joy the rediscovery of the Greek's knowledge, nothing more distant than two contiguous elements, as were air and water. Precisely because their periodic table just consisted of four elements that, arisen since the night of the myth delivered their little splendor in the dawn of science, every natural philosopher knew that merely in the mystery of alchemy could a secret way to pass from one to the other be found. And Leonardo found it. The key was the same secret that life breathes in any of its forms: movement. And in spite of being aware that water and airopposed in a fundamental attribute, since the former is expansible and compressible while, in constant conditions, the latter does not expand nor compress, Leonardo was capable of observing that "the movement of water in the bosom of the water" was similar to that of "air in the bosom of the air".

Facing statements of such amplitude and certainty, even those having shown being more reluctant to grade as scientific Leonardo's investigative work, as is the case of the prestigious mathematician, philosopher and science historian Clifford Truesdell, find themselves compelled to recognize that the tireless and silent artist, that living never got to publish the results of his studies and that received no remuneration whatsoever for doing them, was the founder of the important field known as fluid mechanics.[16] Of course, to Leonardo it turned out to be impossible to make controlled experiments on wind strength. Instead, after scattering colored seeds on a water course, he could observe with much precision the effects produced upon collision with an obstacle in the route, or upon narrowing of the channel banks; and he knew the conclusions he was obtaining kept being valid when applied to the invisible movement of air, for example when it raised its impetus upon being compelled to go through the narrow pass between two mountains. But also the observation of air power led him to make discoveries on water movement, as in his observation that, in the surf of the sea, it is not water that advances. Leonardo writes: "Impetus is much faster than water, but very often runs the wave from where it was created, without water moving from its place, in fashion similar to how happens in May with the waves wind creates over the wheat fields; we see the waves run by the fields without the stems moving from their place."[17]

What had been the finding of *sfumato* in the field of painting, so was the analogy in the field of thought. It is this integrating way of vision, so characteristic of the freedom Leonardo's spirit never renounced, in spite of being in service of one and other and other of the more powerful monarchs, that which allowed the investigator to approach a phenomenon from a plurality of viewpoints. For that

[15]Writes Leonardo: "1, The superficies is a limitation of the body. 2, and the limitation of a body is no part of that body. 3, and the limitation of one body is that which begins another. 4, that which is not part of any body is nothing. Nothing is that which fills no space." British Museum, 131v.

[16]Truesdell, C., Essays in the History in Mechanics, Springer-Verlag, 1968, p. 71. The page here alluded corresponds to the version in standard Spanish, *Ensayos de Historia de la Mecánica*, Ed. Tecnos, Madrid, 1975. In spite of the insurmountable faults he finds with the investigation method followed by Leonardo, Truesdell also recognizes that Leonardo was the first to formulate the principle of communicating vessels, as well as the first to give a statement for the law of free fall. Cf. op. cit., p. 45.

[17]*Codice M*, Institut de France, 87v.

Capra is right in making us see that that not compartmented thought is a real foretaste of what today is known as Complex Thought Theory, promoted by philosopher and sociologist Edgar Morin.Who makes the exercise of going into Leonardo's manuscripts feels entering a surprising–but at the same time very close–universe. On each page will be found some result surprising to the most diverting mind. The closeness is not due so much to the familiarity with the topics–which for their diversity usually exceed the habitual contact today's man might have with them–, but rather to the simple manner, almost innocent, under which Leonardo discovers them at the very moment of treating them. It is that, facing all the treatises that have been published in all times and that teach something about any subject, Leonardo's manuscripts present an essential difference that characterizes them: they have been written in the rough and, in general, have no addressee.

In fact, Leonardo's manuscripts are not redacted to produce an impression in someone but to gather an impression of the mechanism that makes the universe act. Because, daily, as the nomad gatherer previous to our societies, Leonardo's mind went out to face the fruits offered it by the prodigality of this world. The folds on a linen shroud, the shadow of a cloud over the roofs of a church, the length of the forearm, the distribution of the branches on the stem, a mollusk shell at the top of mount Albano, the resistance of a beam, the digestion of food, the stillness of the sun, the membrane of the bat, the neck of the lute, the intrauterine life, the erosion of a barrack tower, the number of stamens of a flower, the relief of a gravel, the shape of the acorn, the sediment of a basin were affairs on which his mind stopped with the same happiness as the dessert dweller when watering through. Each thing was a door that led to the entire orb, not only because each one was associated with the others in the mental map of his personal inquiry but because, to Leonardo, knowing an object did not consist in providing it a sense but in waiting for the object to reveal it to him. Knowledge was not achieved with haste but by inserting oneself in the rhythms of life, because all things of the world came from life. Because of that, he contemplated with the same respect the flight of a butterfly and the womb of a cadaver, and with the same patience wrote down on his notebooks what one and other showed him.

Almost always, a page by Leonardo is an opening of annotations and drawings traced at the live instant of the observation. So that on occasion he writes on the margins, with poor orthography and dubiously, in vertical way, and so that he goes back on his word as he observes better, that he makes amends, cross outs and corrections, so that many of his texts lack the impeccable coherence of the printed treatise after the magnifying glass of revisions. It is not difficult, around one same topic examined by Leonardo in different years, to find discrepancies in his thought and even contradictions in his conception. It was not exactly logical limpidity what concerned Leonardo in the exercise of studying, and little did he care that his notebooks ended seeming a raiment of tatters, so long as under the patches was preserved in pure form the face of truth. In his case, well apply Morin's words: "Employing logic is necessary for intelligibility; surpassing it is necessary for intelligence. Reference to logic is necessary to verification; overcoming logic is necessary to truth."[18] Leonardo's pages are a cross cut to the very act of thinking.

In the thousands of pages of his notebooks almost no registry is found about that molasses of emotions and feelings that is usually denominated "the personal life". And nevertheless, how much life of the author pulsates in each one of them. Observation, even if it is about a moth around a candle, is a finding the contemplator of life thanks since it is the epiphany of his walk, the fashion in which

[18]E. Morin, *La Méthode*, tome 4, Points, p. 207.Translation is mine.

life manifests itself in any whatsoever of its folds. And to that, it just has to be brushed with looking, without imposing on it a meaning, because in pretending to light in artificial form that which in itself is light, its truth is inhibited and retracts, and we will just achieve seeing the spectrum of our own illusion. “What the light of the eye sees is seen by that light,” says Leonardo, “and what the light sees is seen by its pupil.”[19]

The everyday variegation of Leonardo’s notes only on occasion alternated with moments of reflection. It was the diastole of his thought, which found rest sketching the index of treatises he would never publish. And even in those passages appear imbricated the dimensions that interested his chore. If he conceived a treatise on light it would have to contain the thorough studies he had done on the diverse classes of shades—primary, projected, derivative—and the way in which luminous rays propagated by the atmosphere. Moreover, his investigations on optics should appear, since there was explainedthat related to the organ of sight, addressee of light. But then also the science of perspective, that though not ruler of the harmony of the world gives order to vision. And not just linear perspective, but aerial and of color, and that which he called declining, thatdiluted solidity of the bodies made out at great distance. And by account of perspective, astronomy, that is the manner in which man contemplates creation... For that his notes resemble his thought, and one and other resemble that discovery he gave form to in his pictures, and of which only centuries later its topological richness could be seen: that in the world there are no individual things, but that it is inhabited by entities without contours, each one open towards the others. Hence knowing well the nature of any whatsoever of them, being equivalent to knowing the very nature of existing.

[19] Royal Library, Windsor, 19152r.

Illustration1. Codice Trivulziano

Illustration2. La Gioconda, detail

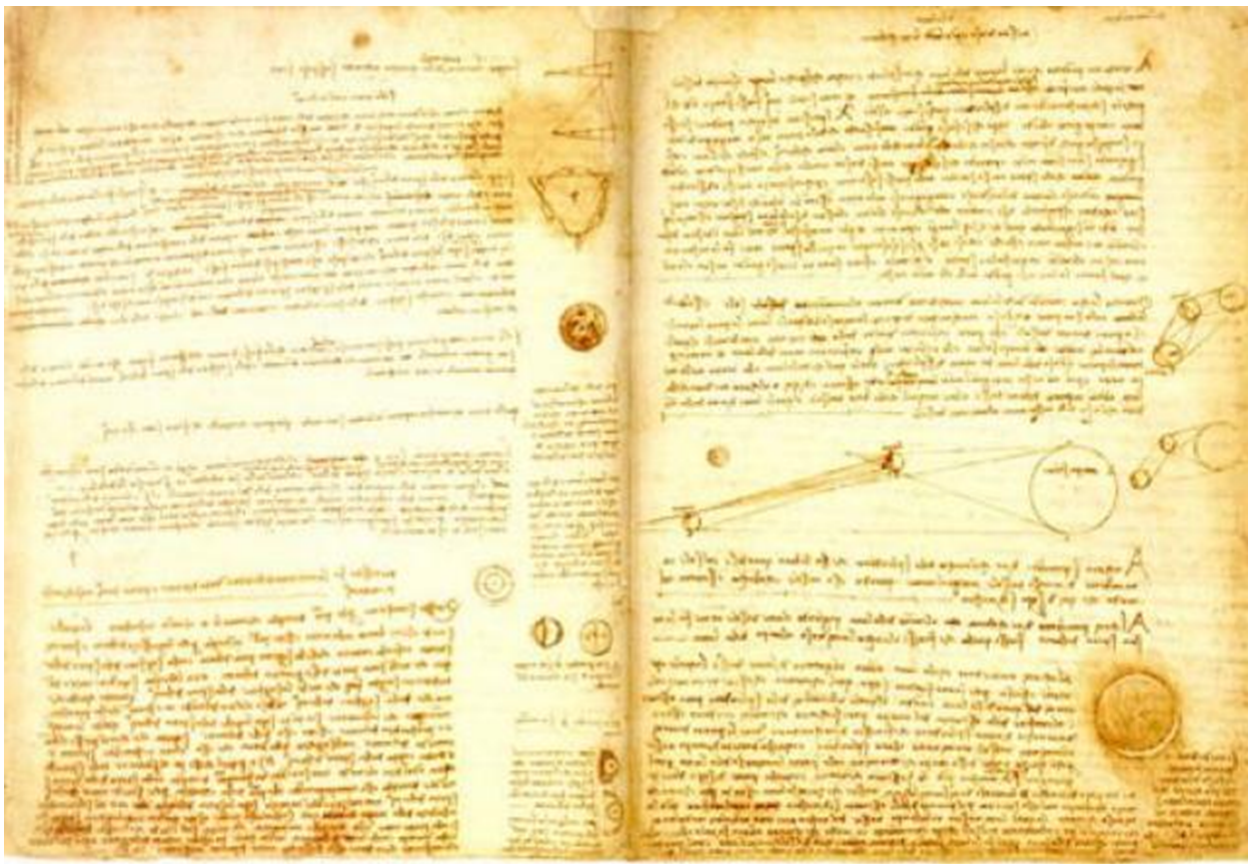

Illustration3. Codice Leicester

Illustration4. Study of water turbulences